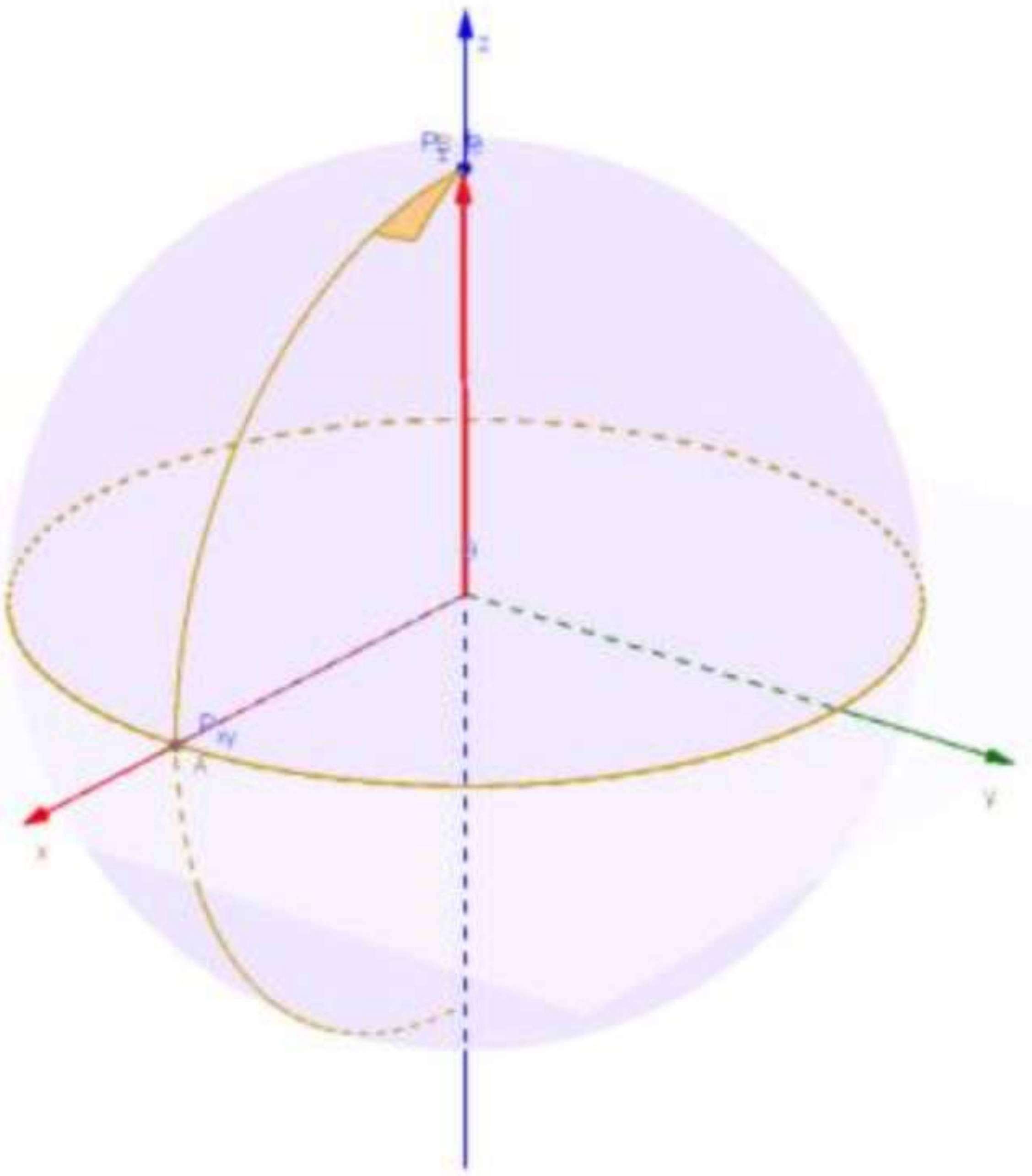
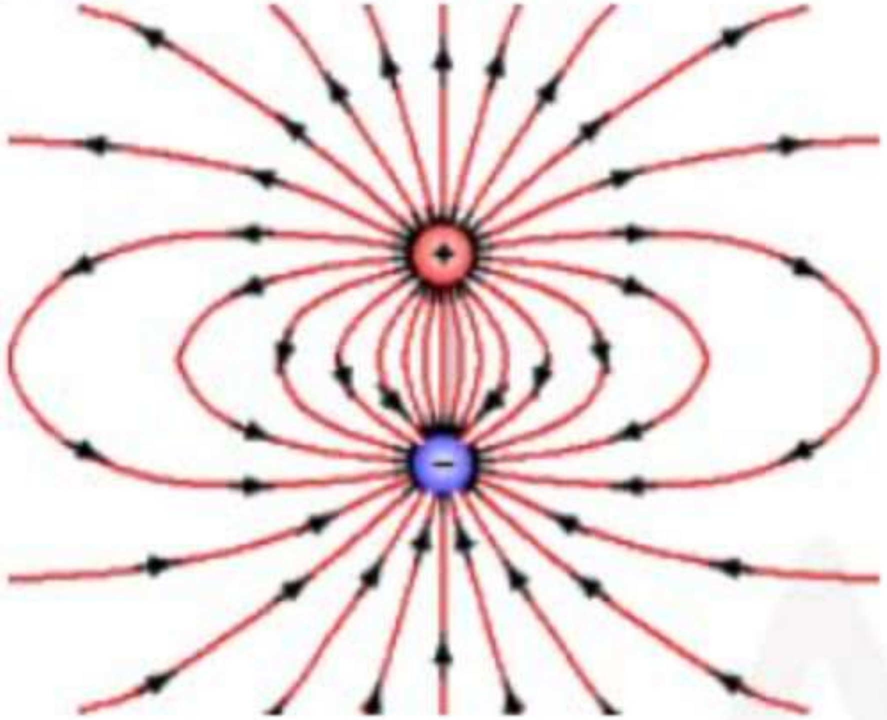
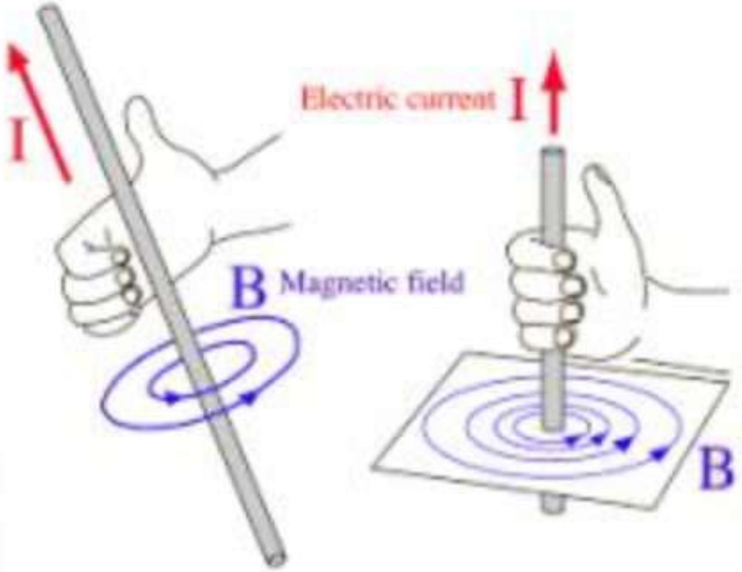
Coordinate systems

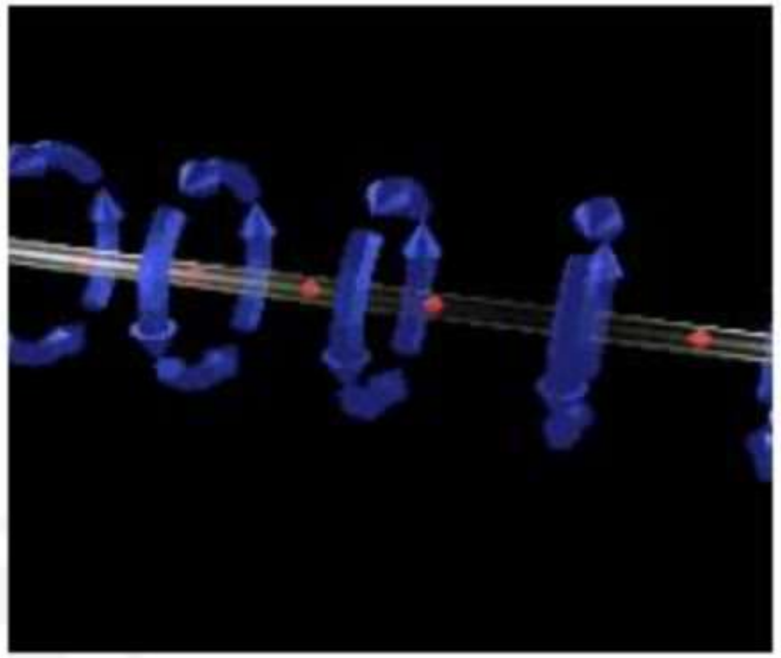
1. **Spherical Coordinate systems**

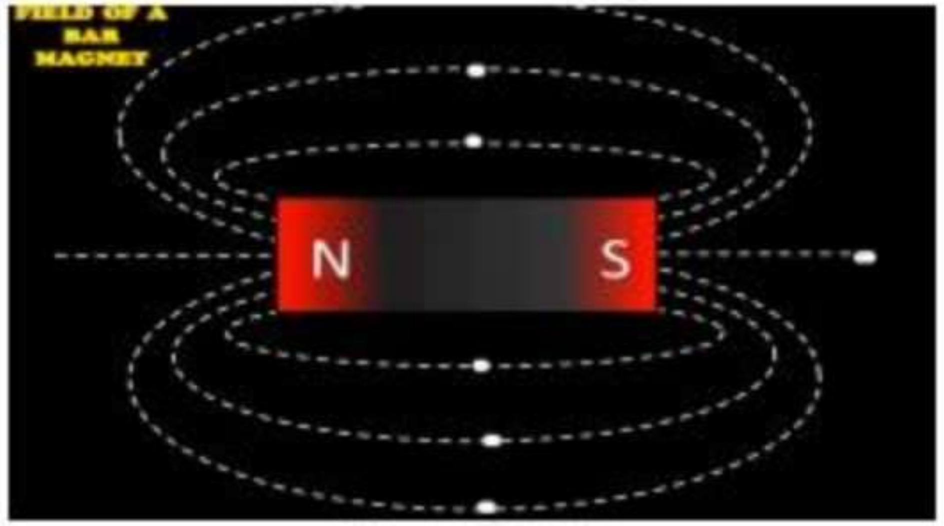


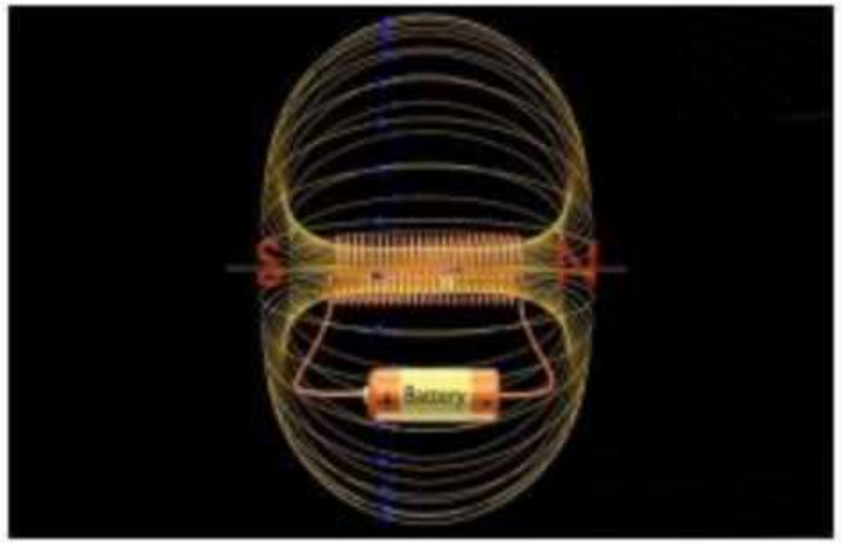
Recap

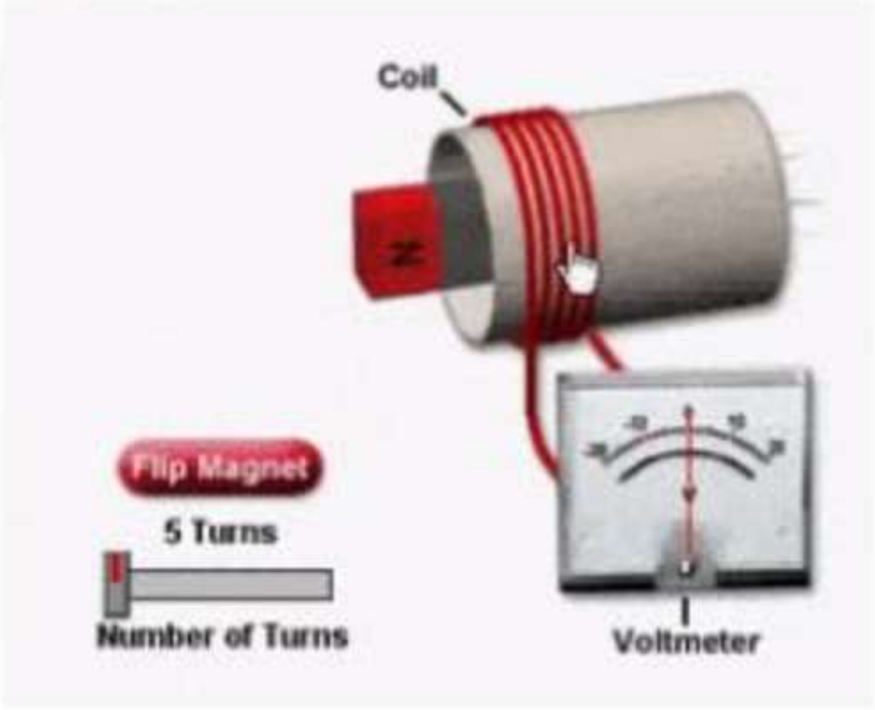



I
Electric current I
B Magnetic field
B

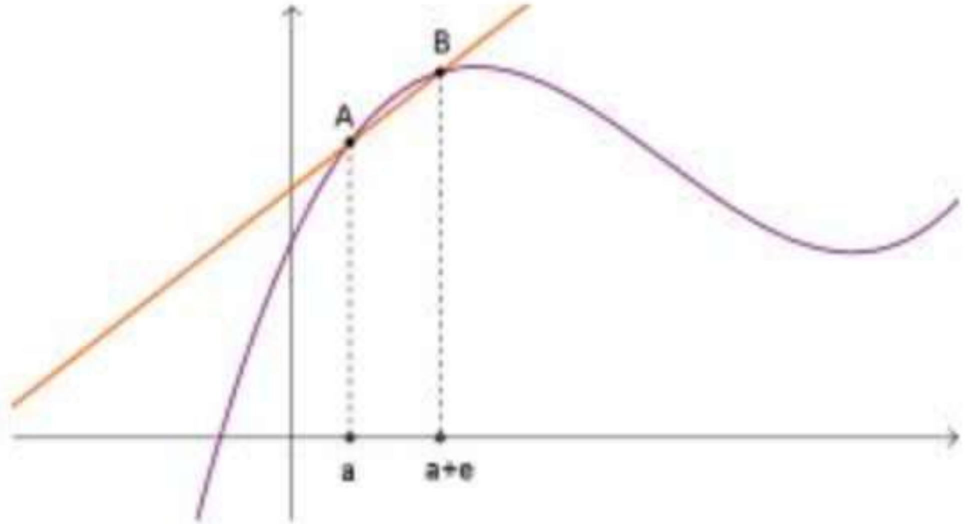



FIELD OF A BAR MAGNET
N
S



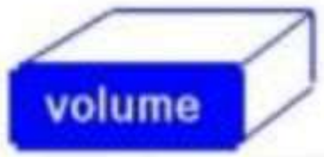

Coil
N
Flip Magnet
5 Turns
Number of Turns
Voltmeter

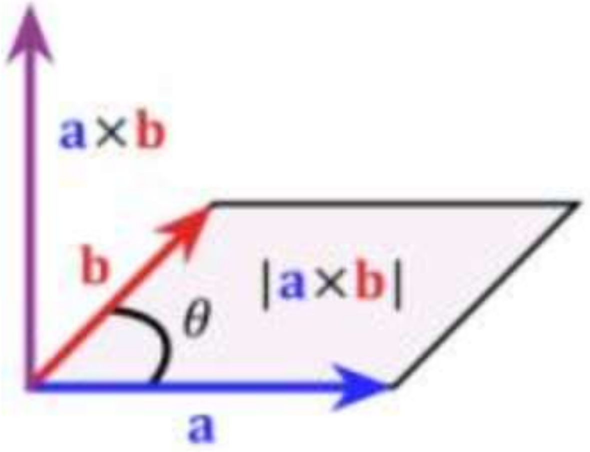
Recap

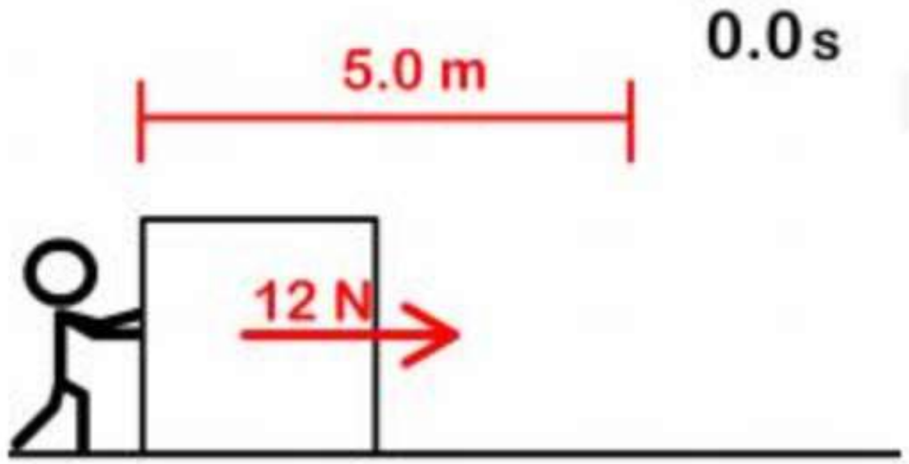


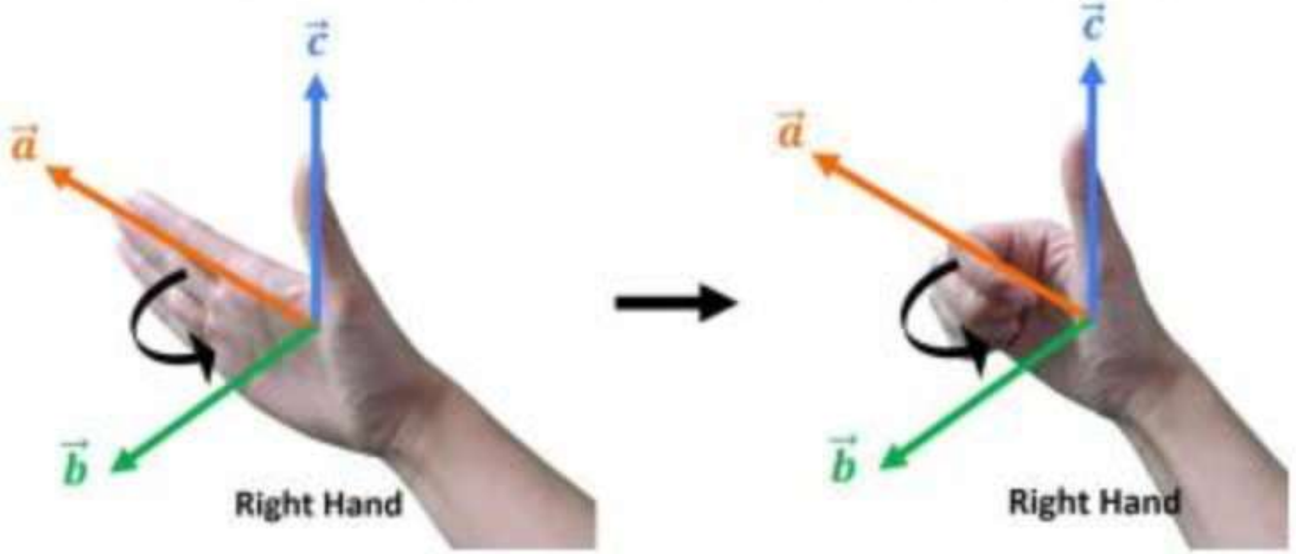
A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.

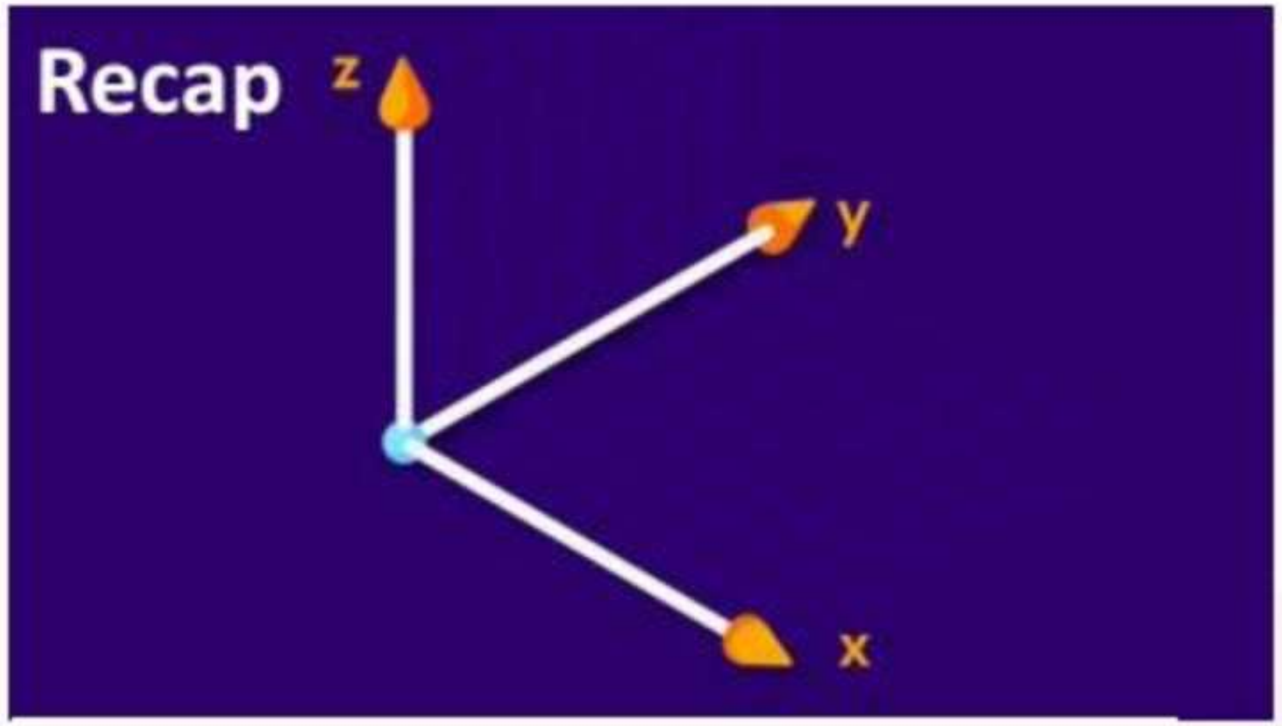
Scalar Quantities	Vector Quantities
length, area, volume	displacement
speed	velocity
mass, density	acceleration
pressure	momentum
temperature	force
energy, entropy	lift, drag, thrust
work, power	weight

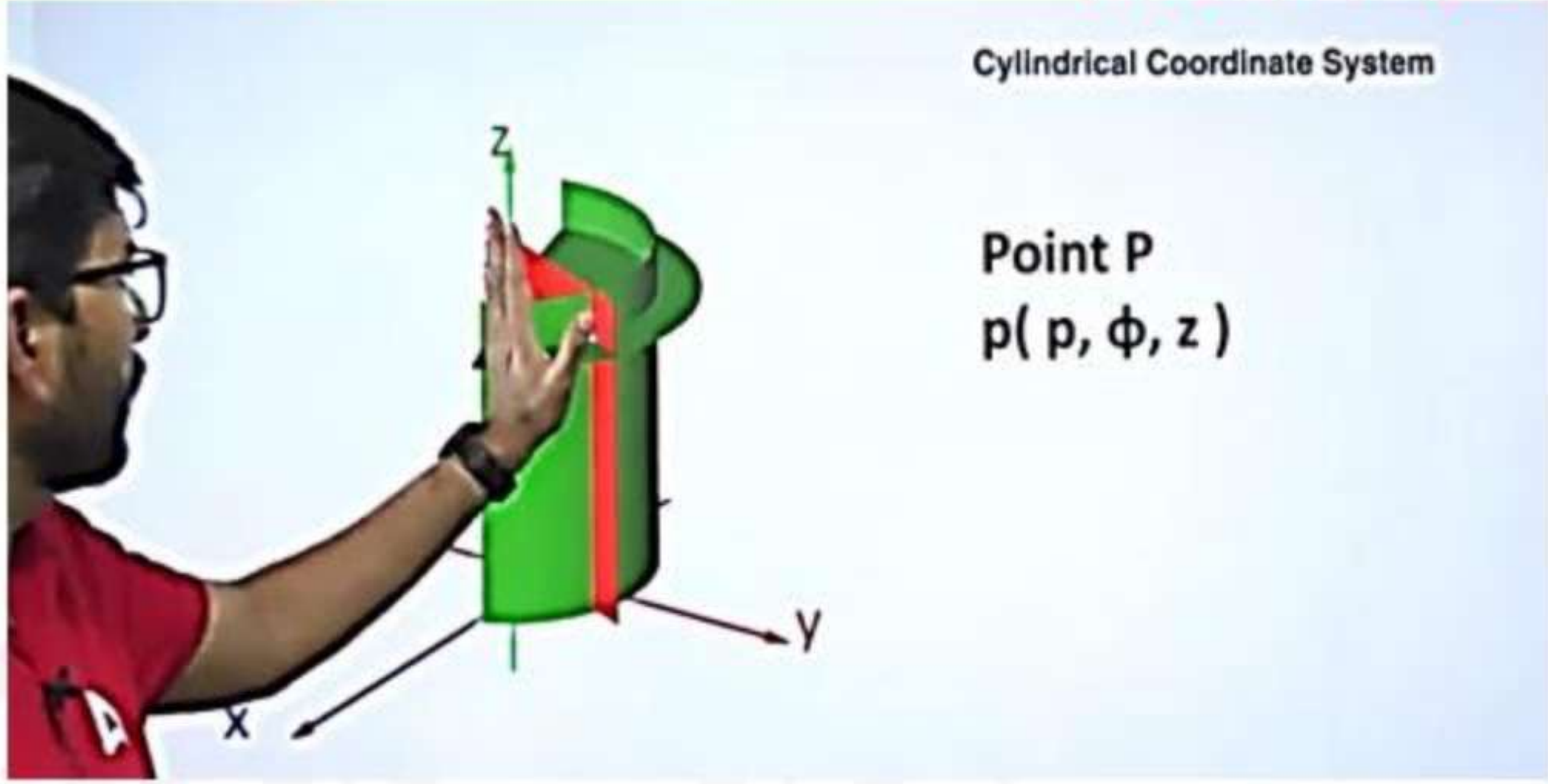


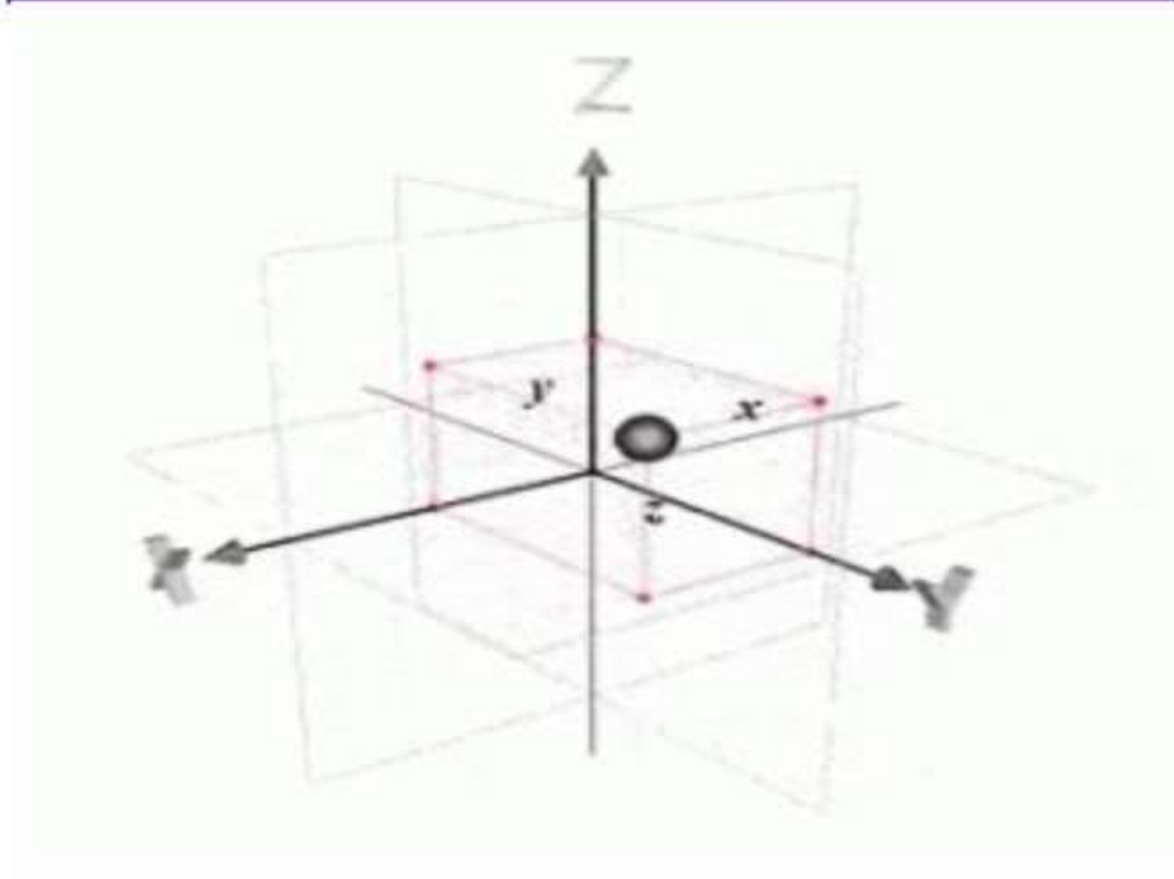


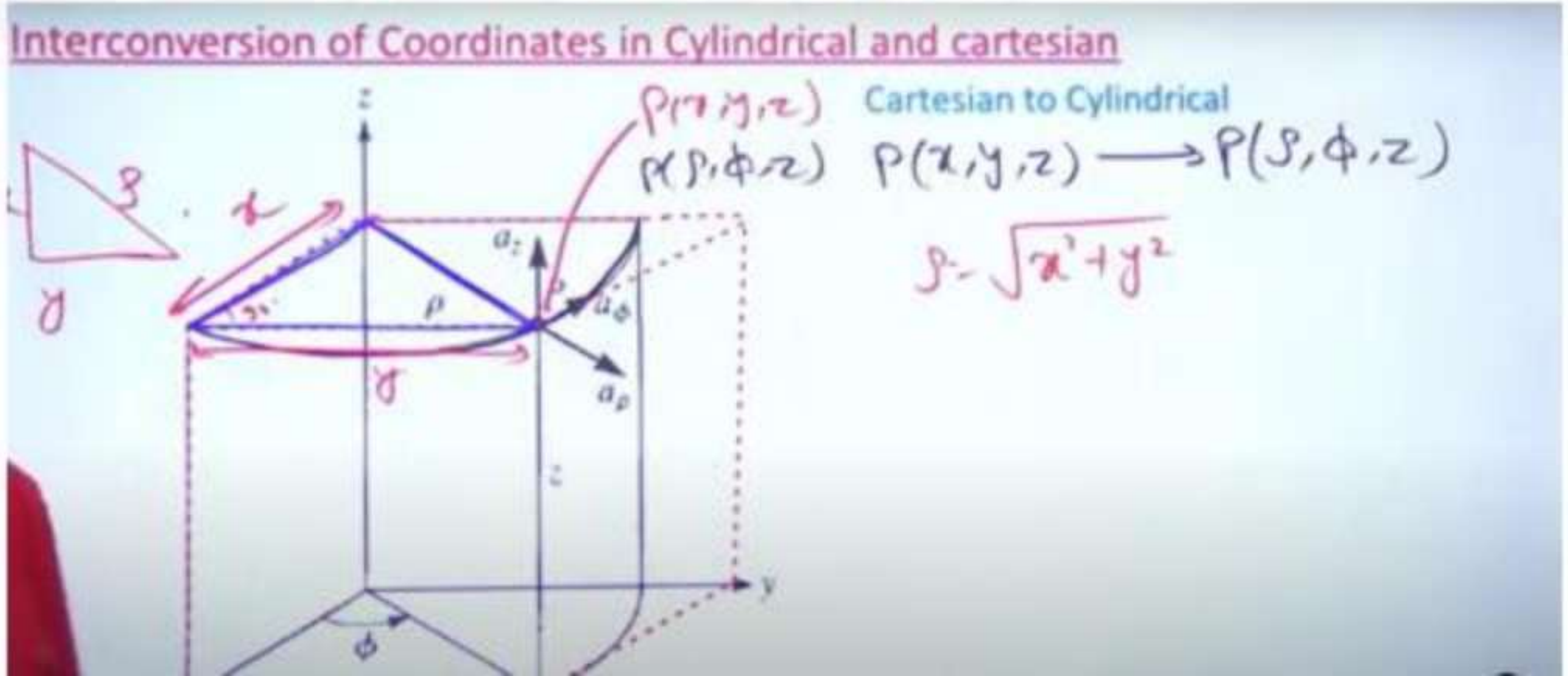





Recap
z
y
x


Cylindrical Coordinate System
Z
Y
X
Point P
p(p, ϕ, z)


Z
y
x
z


Interconversion of Coordinates in Cylindrical and cartesian
Cartesian to Cylindrical
P(x,y,z) ⟶ P(ρ,ϕ,z)
ρ = √(x²+y²)
z
y
ρ
ϕ

Cylindrical Coordinate System

Cylindrical Coordinate System

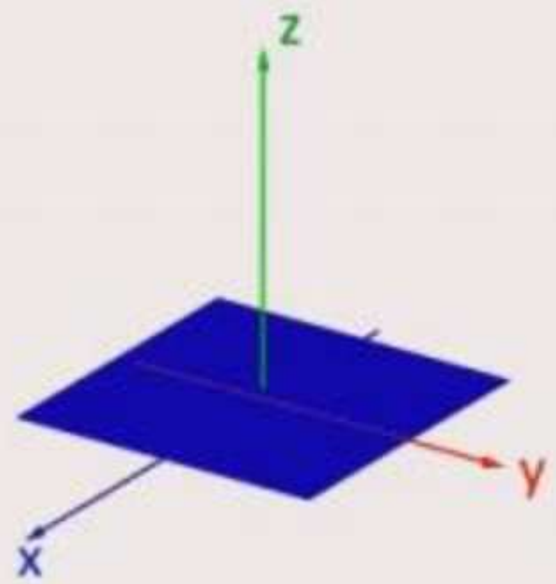
z

y

x

Constant r surface

$R \geq 0^{o}$

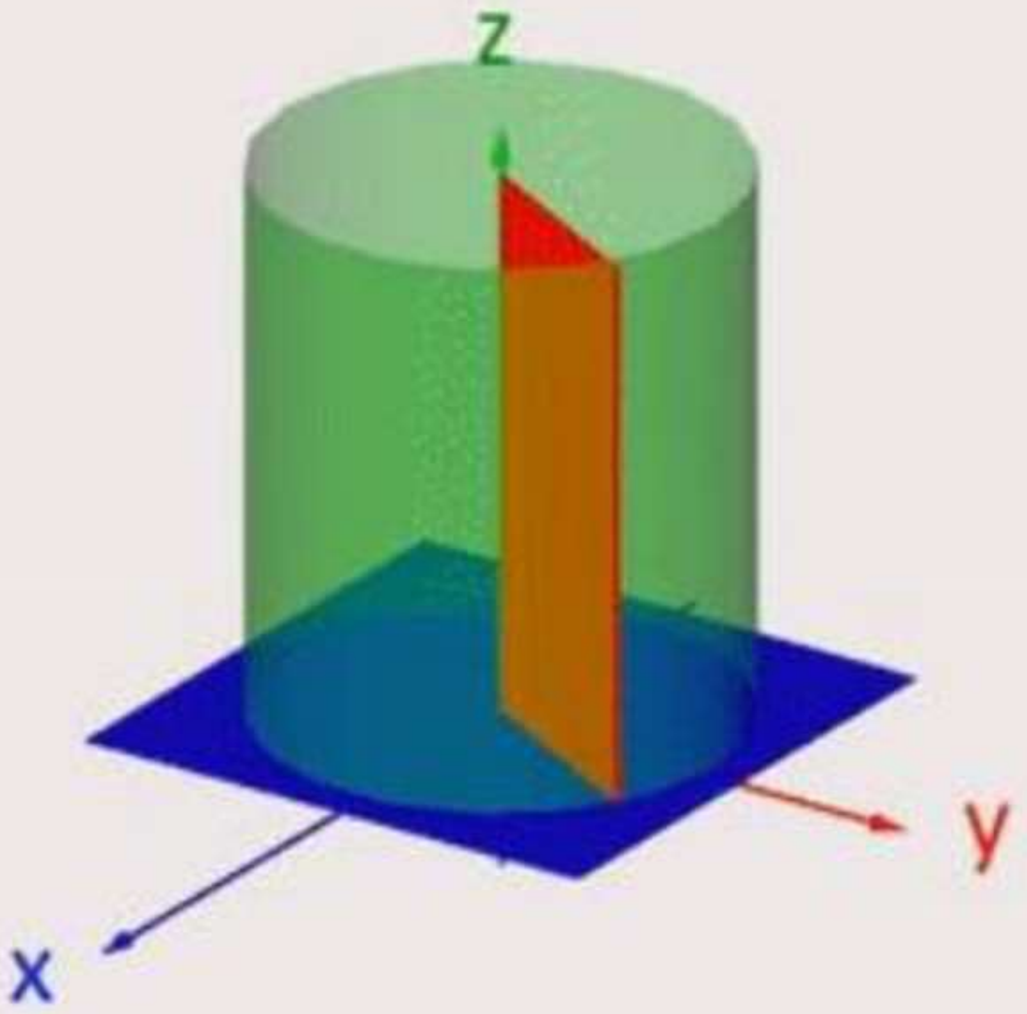


Cylindrical Coordinate System

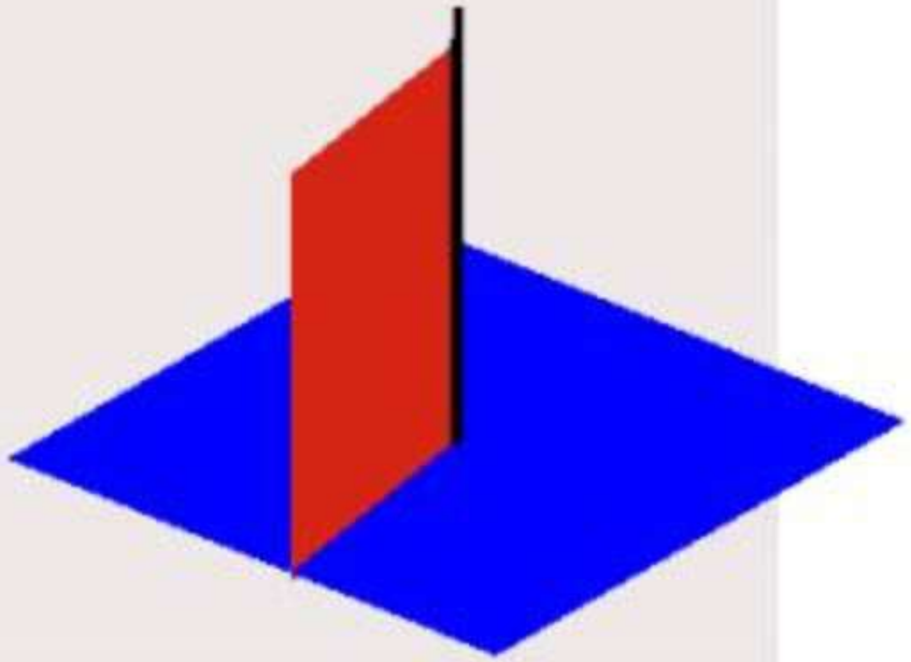
Constant z surface

$-\infty \leq z \leq +\infty$

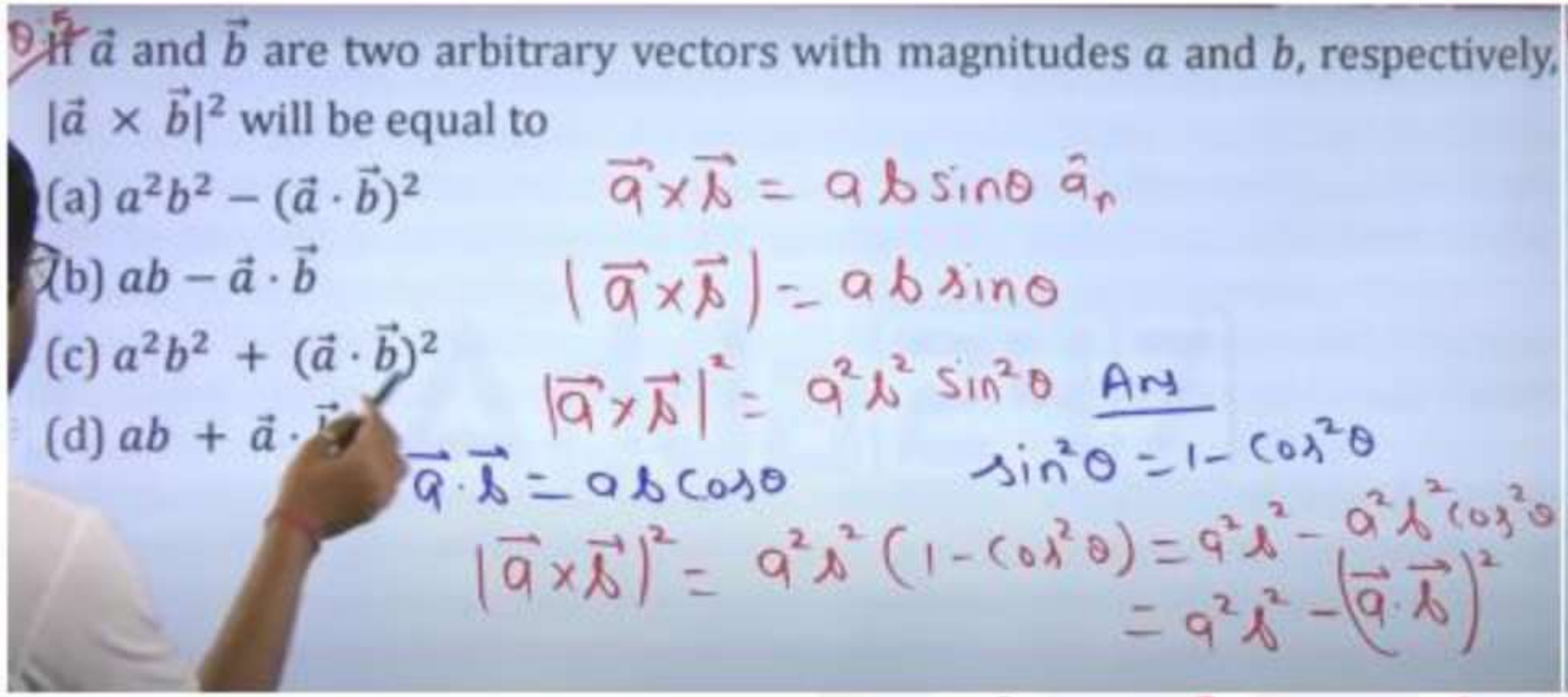
Cylindrical Coordinate System

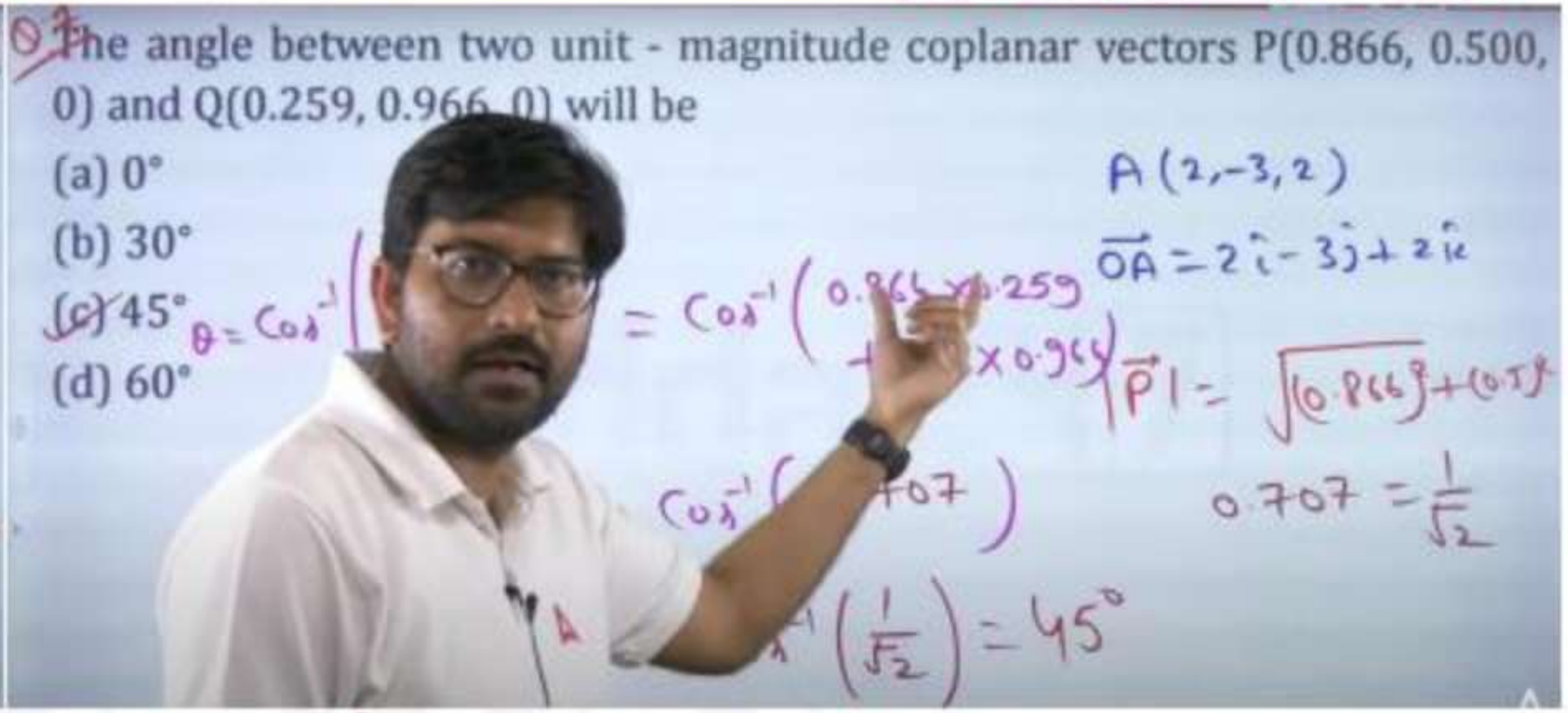


Cylindrical coordinate system planes

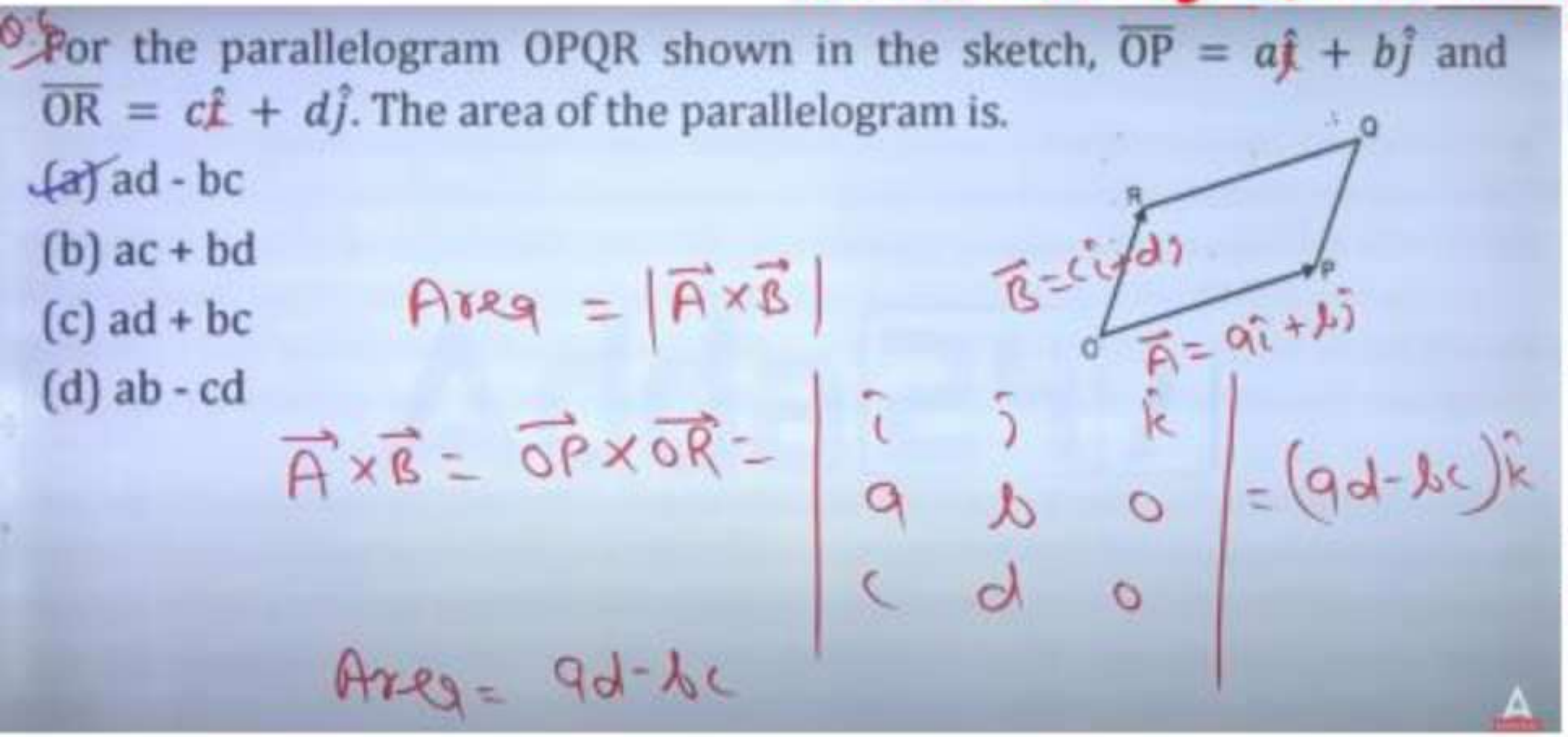


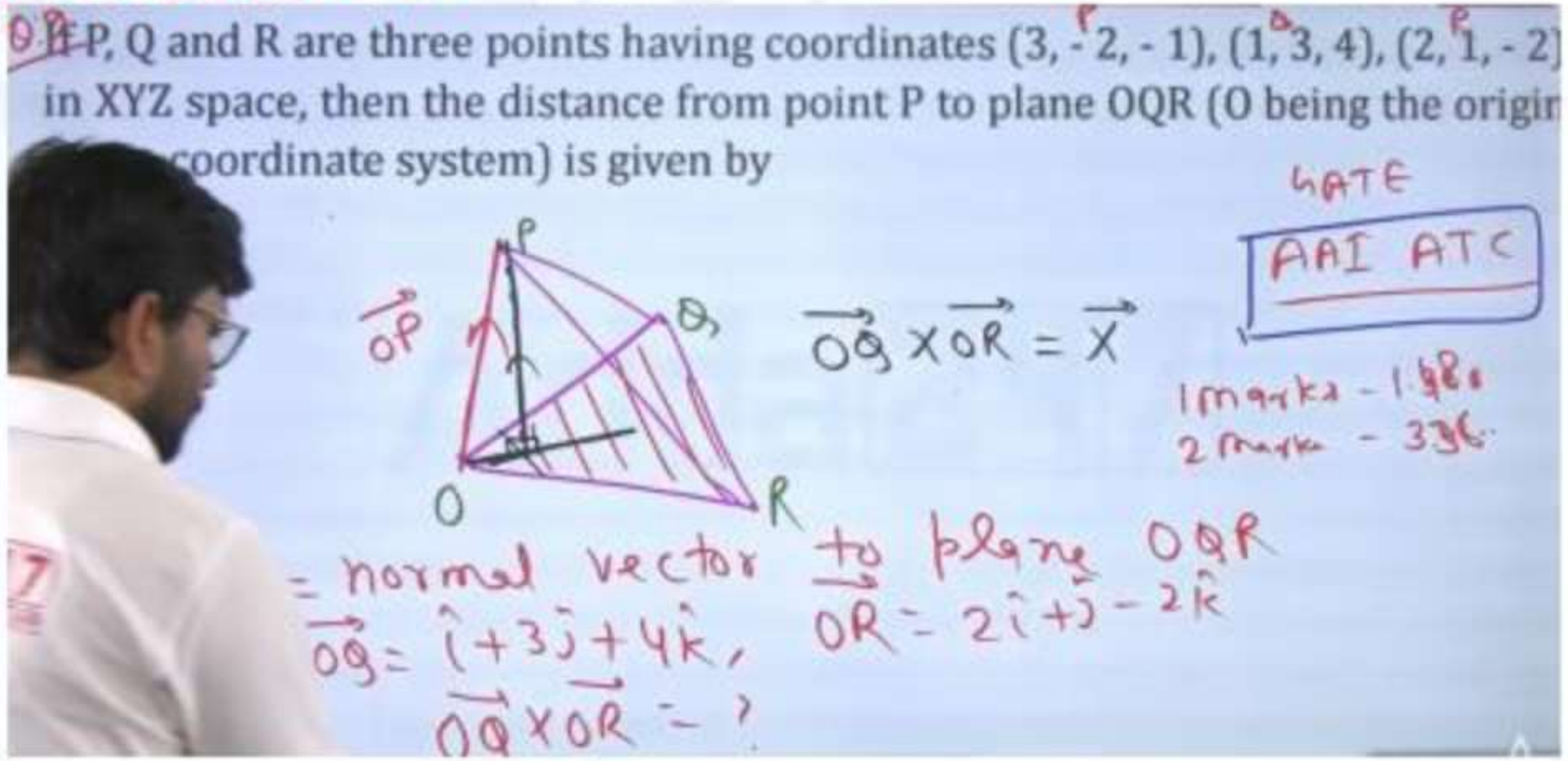
Recap





Number of Questions covered-9





Spherical Coordinate systems

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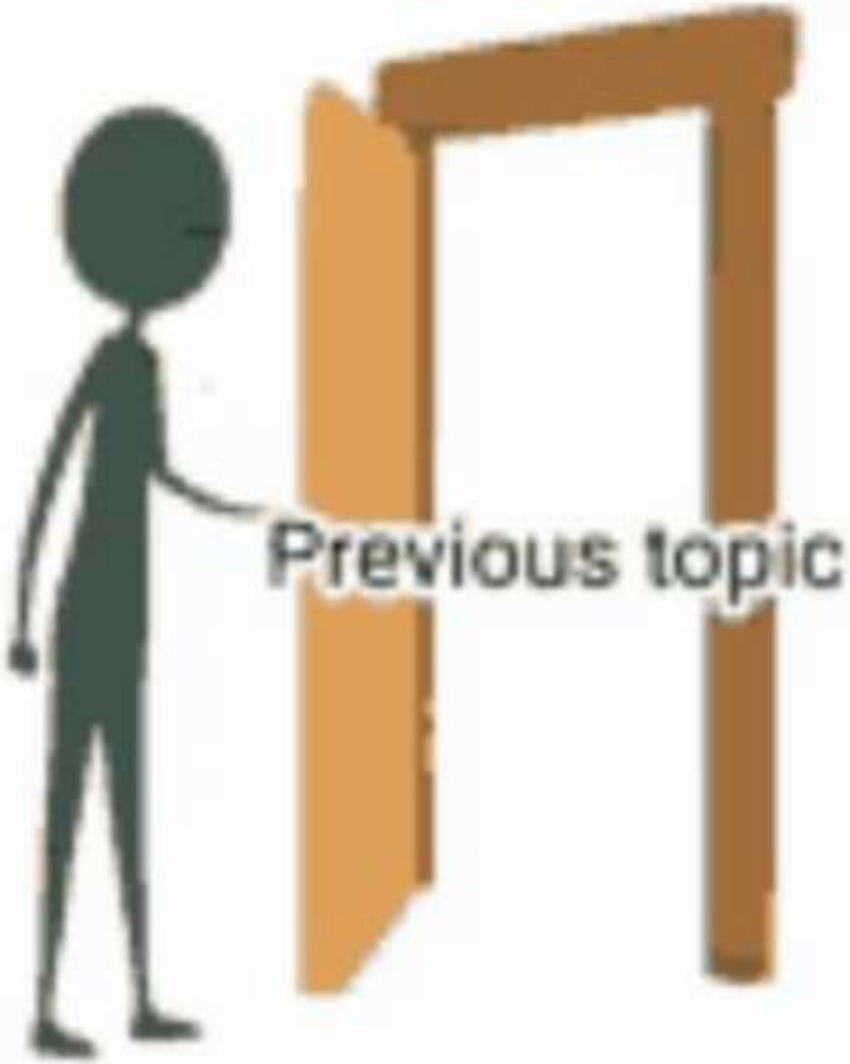
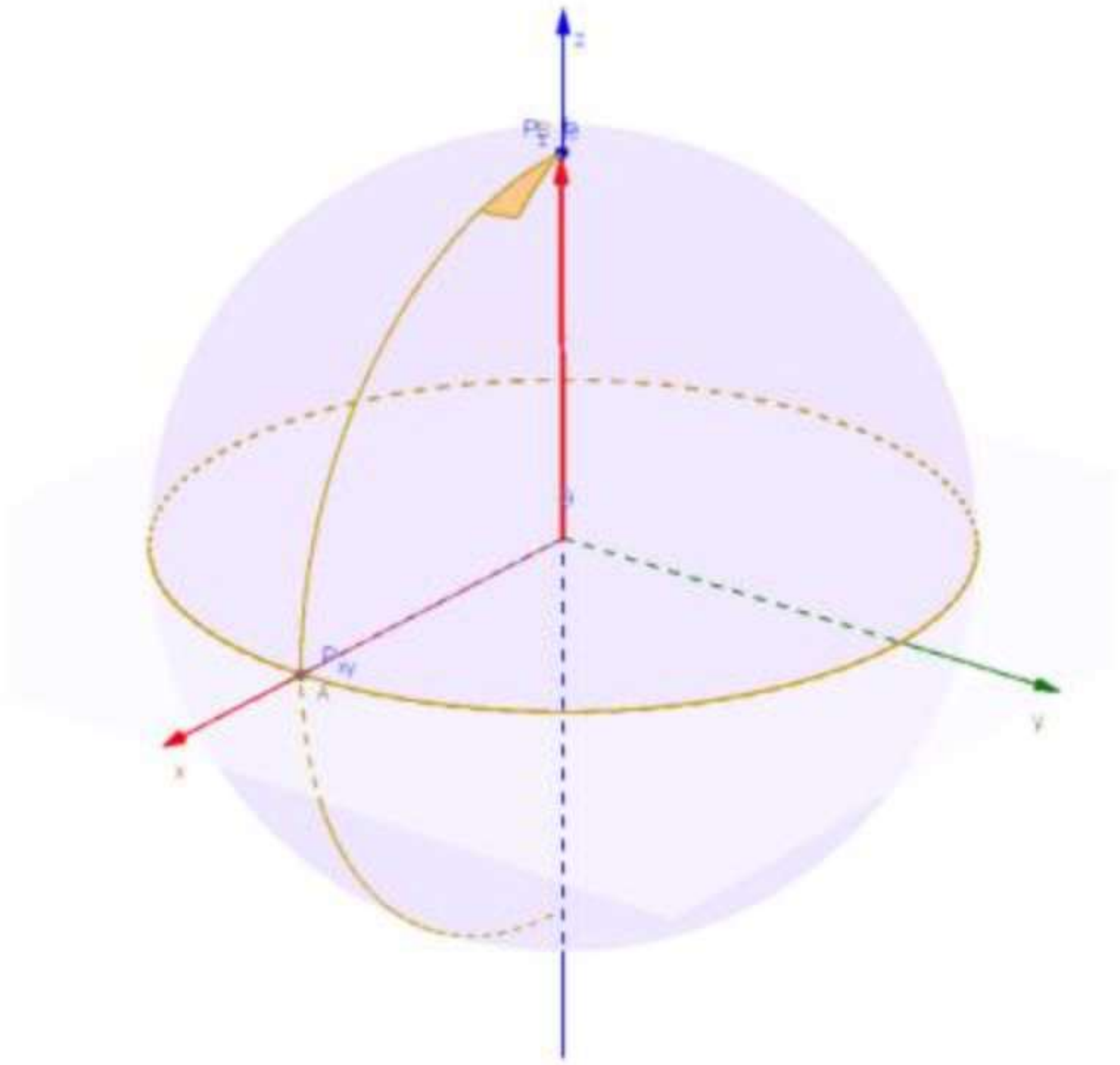
GATE 2024
GATE
प्रचण्ड Batch
Electromagnetic Field Theory
Spherical Coordinate System
LEC-04
EE &ECE



1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical Coordinate systems

Coordinate systems

1. **Spherical Coordinate systems**

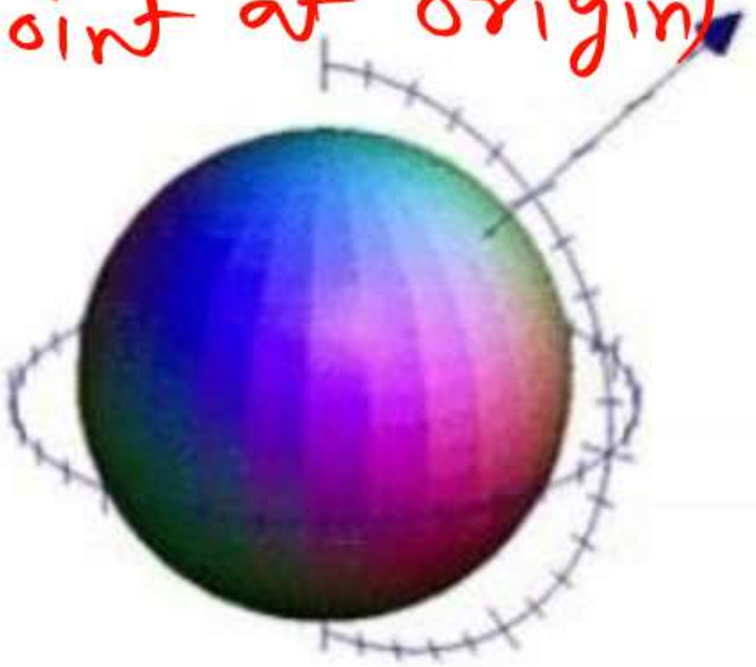


Spherical Coordinates P(r, ϴ, ϕ)

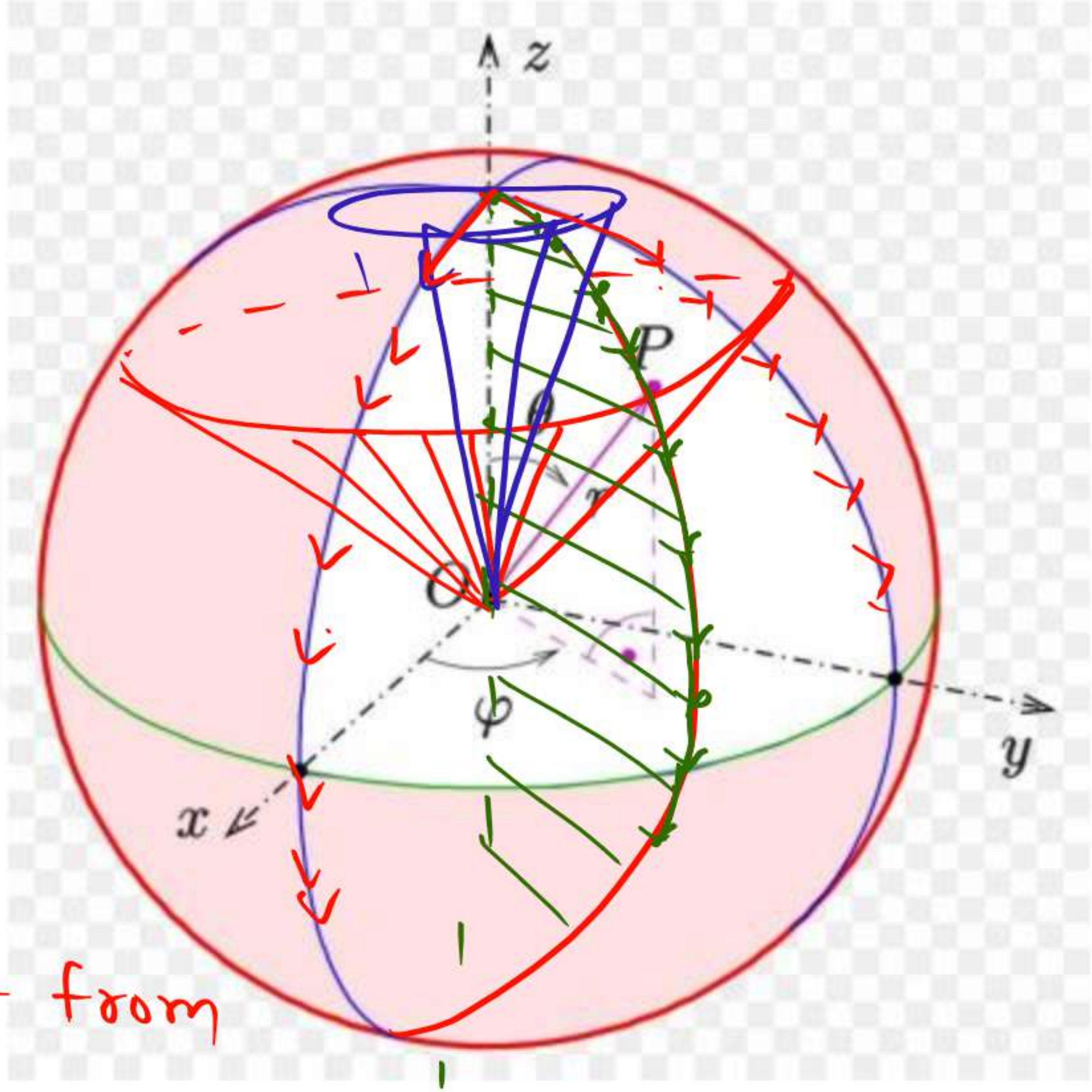
r= Radius of the sphere passing through the Point and centered around the origin

$0 \le r < \infty$

r = 0 [point at origin]

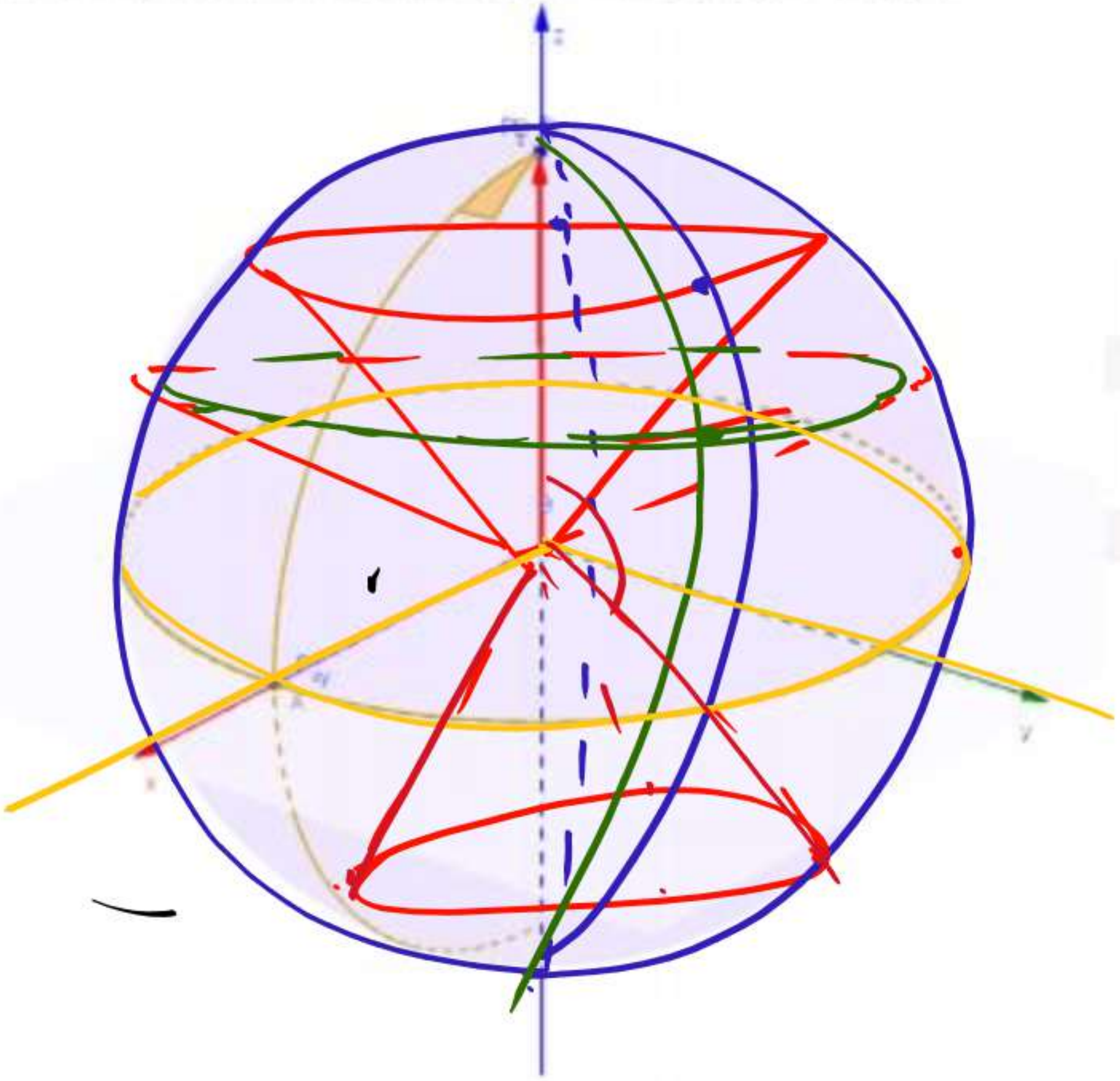


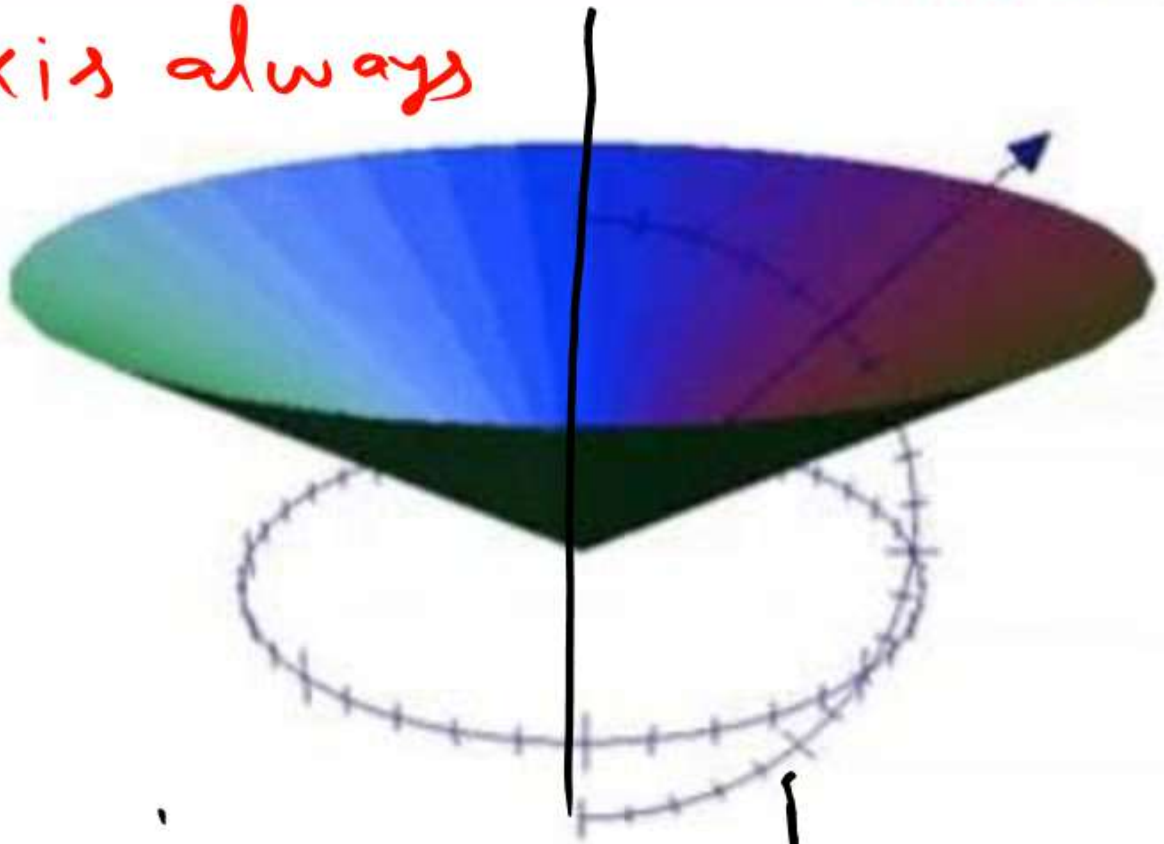
r = radial distance of point from origin.

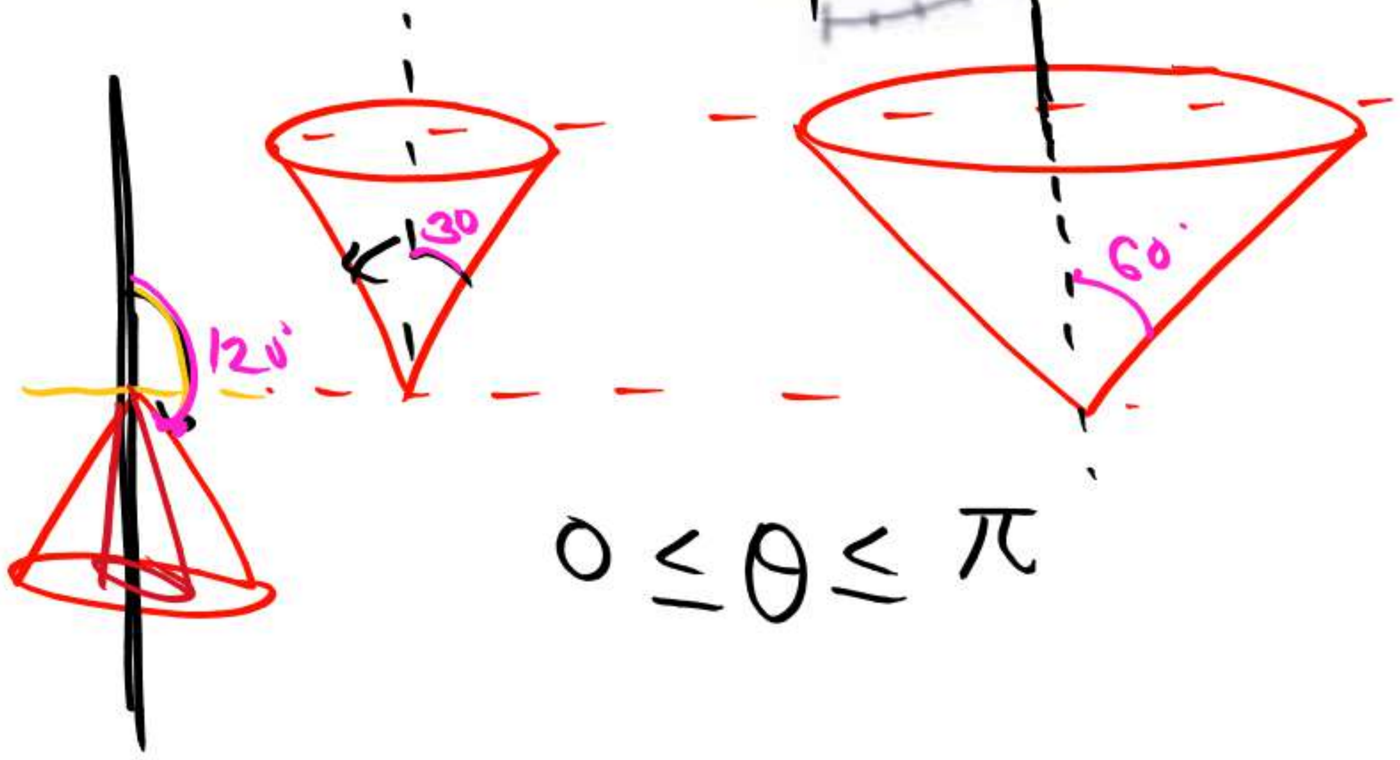


θ = Colatitude angle, from +z axis always

Tilt angle of cone passing through the point and z-axis as central axis

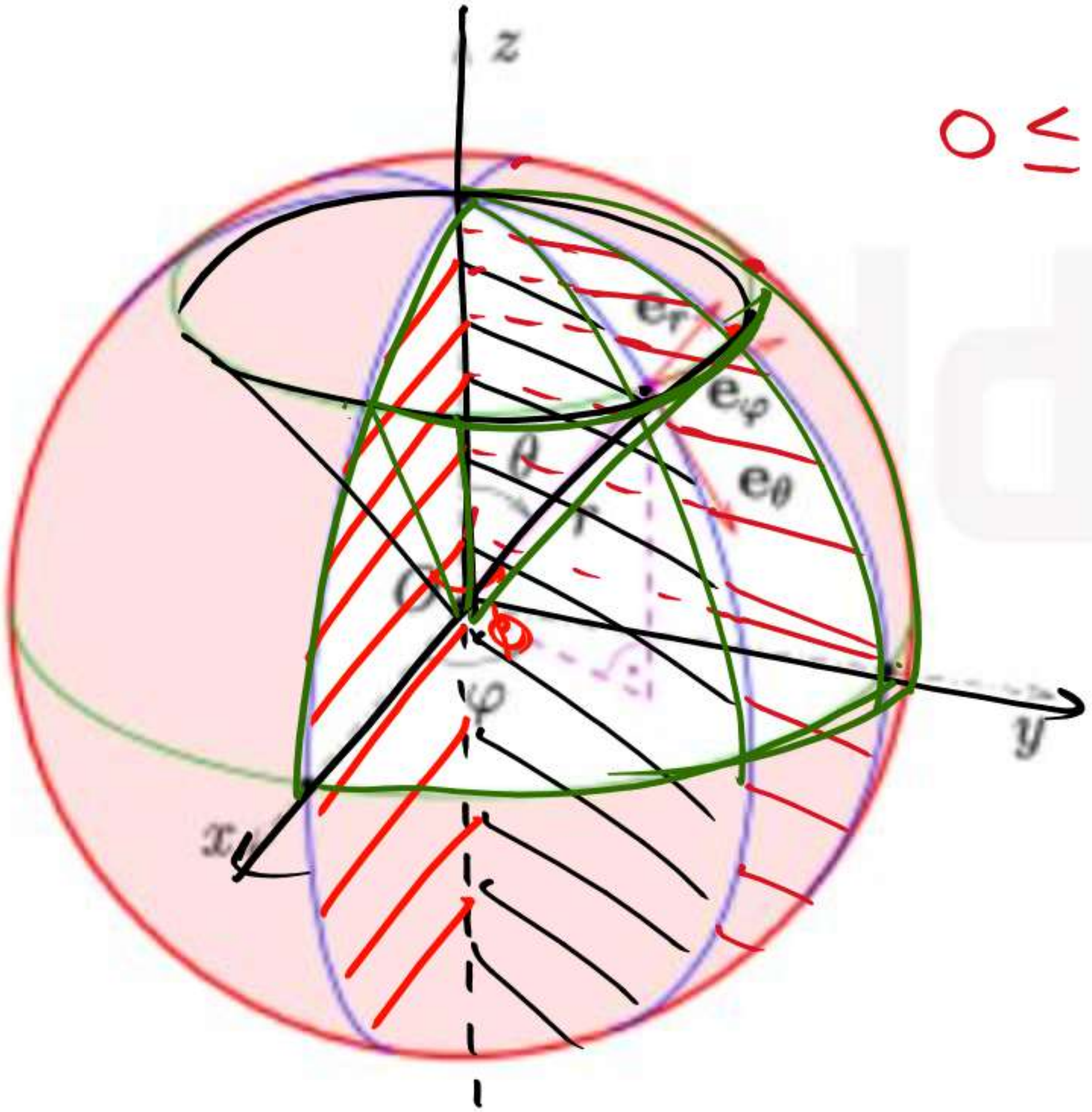




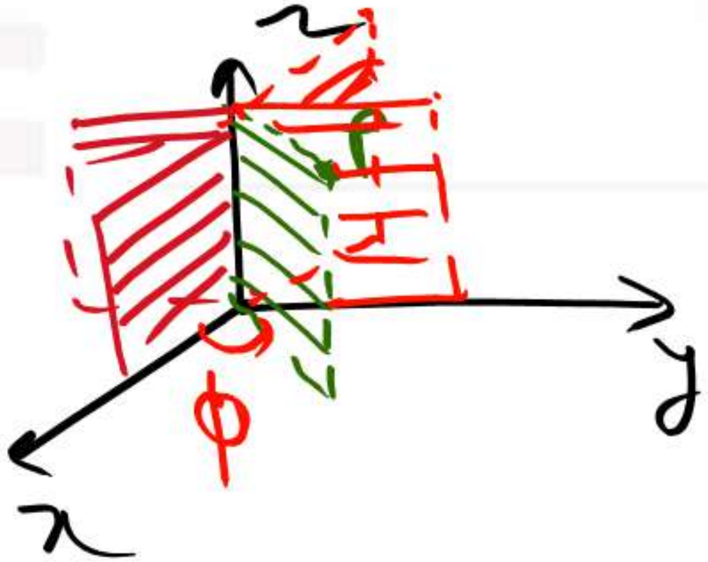


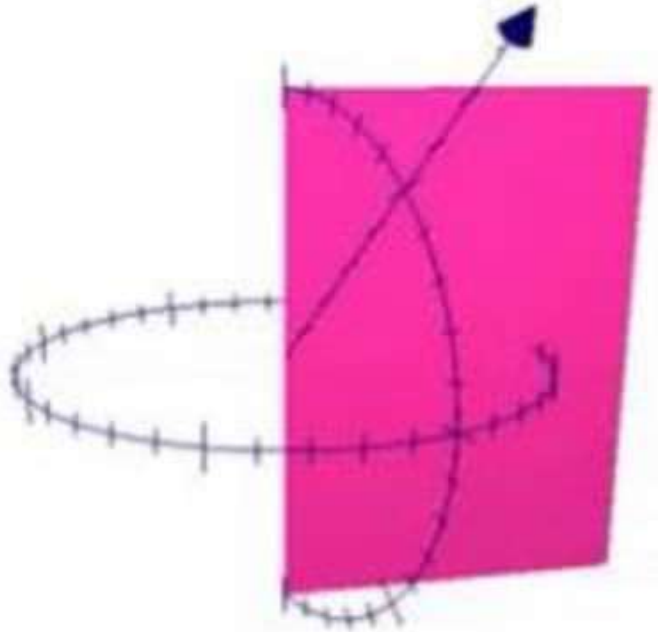
$0 \le \theta \le \pi$

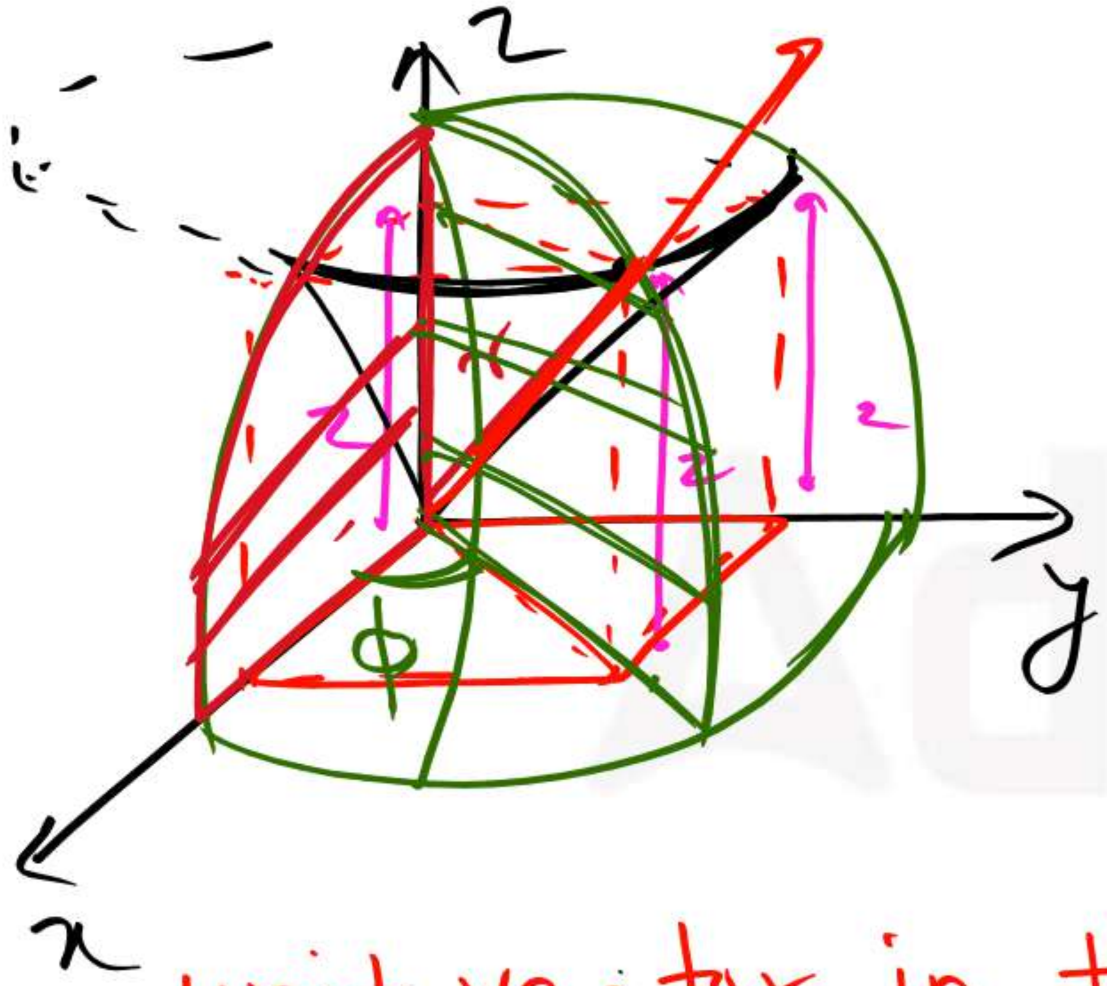
Φ= azimuthal angle, angle of azimuthal plane from XZ-plane

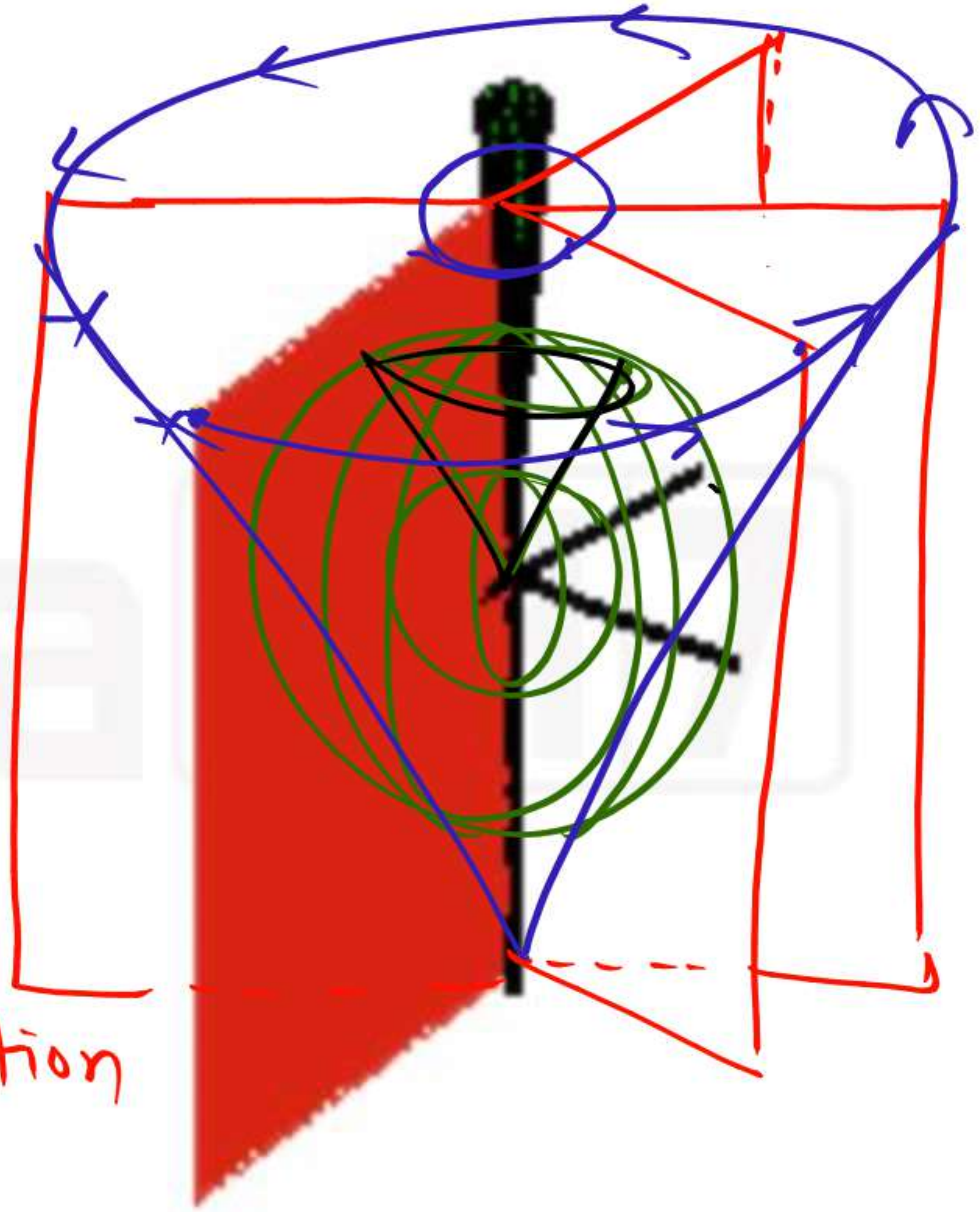


$0 \le \phi \le 2\pi$







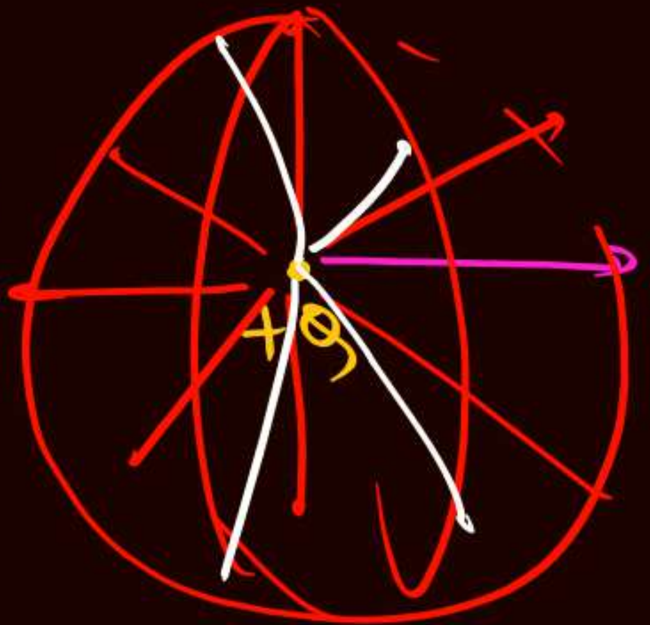


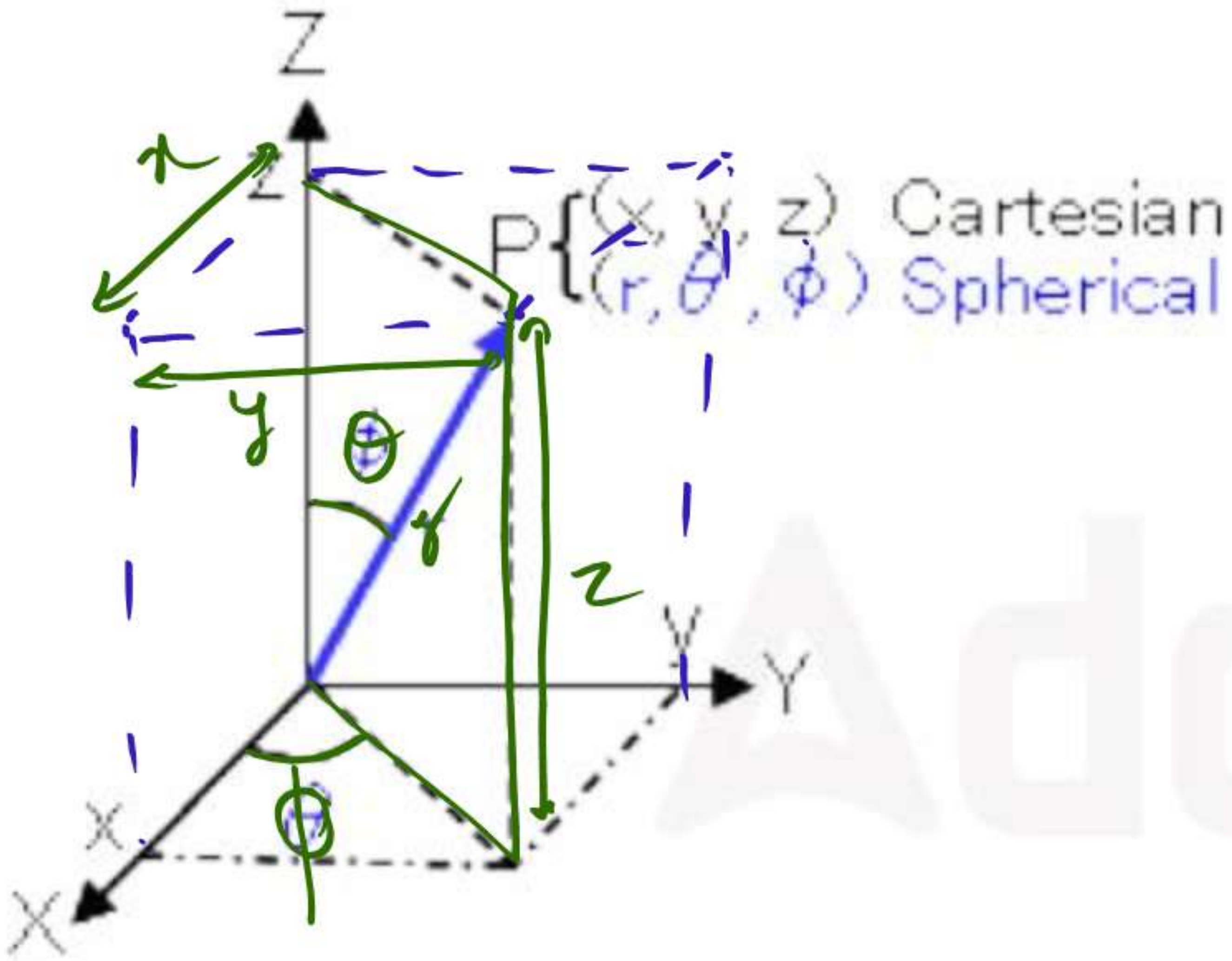
$\hat{a}_r \rightarrow$ unit vector in the radially outward direction from origin.

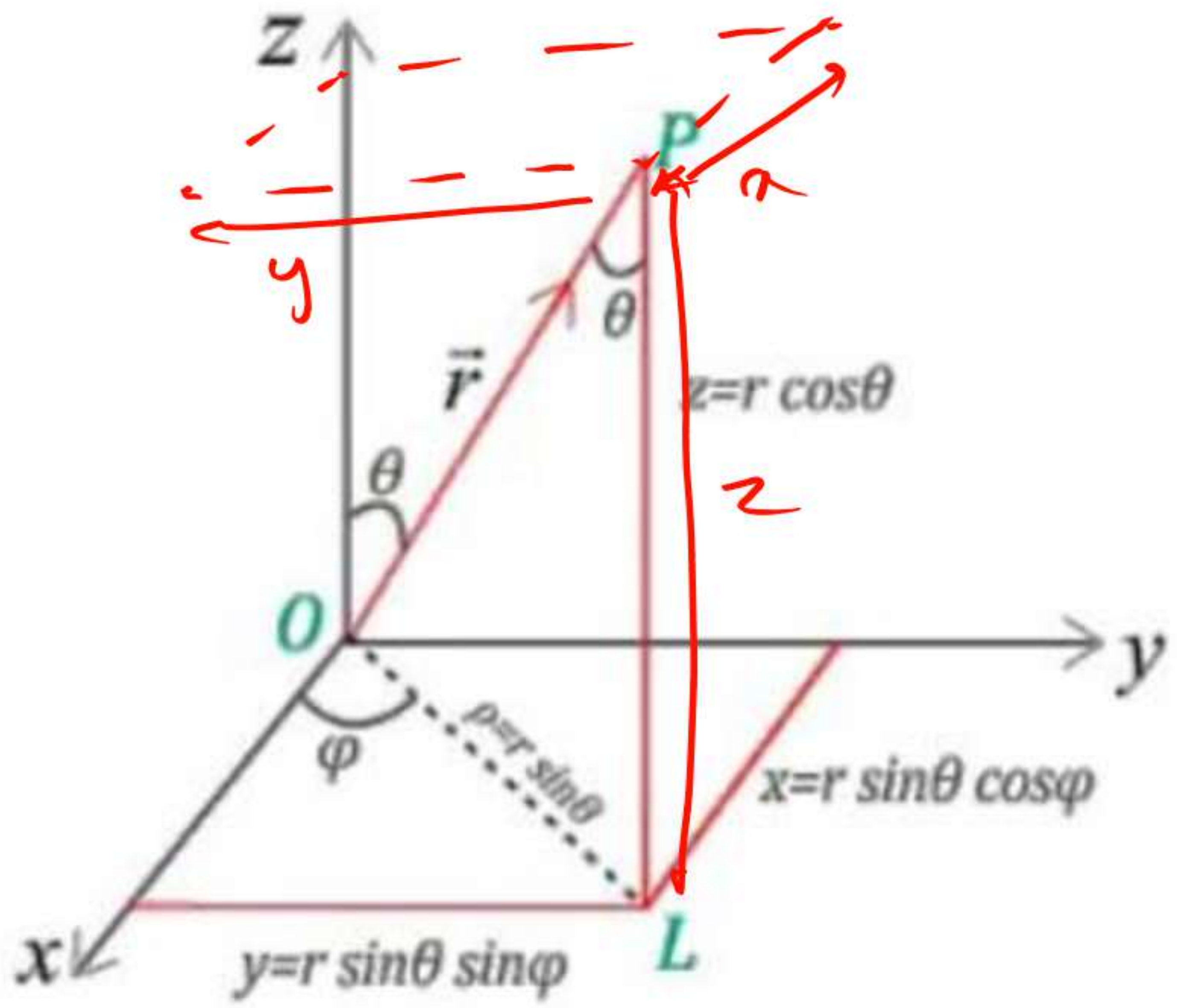
$\hat{a}_\theta \rightarrow$ unit vector in semicircular rotation around origin.

$\hat{a}_\phi \rightarrow$ unit vector in circular rotation around z-axis.

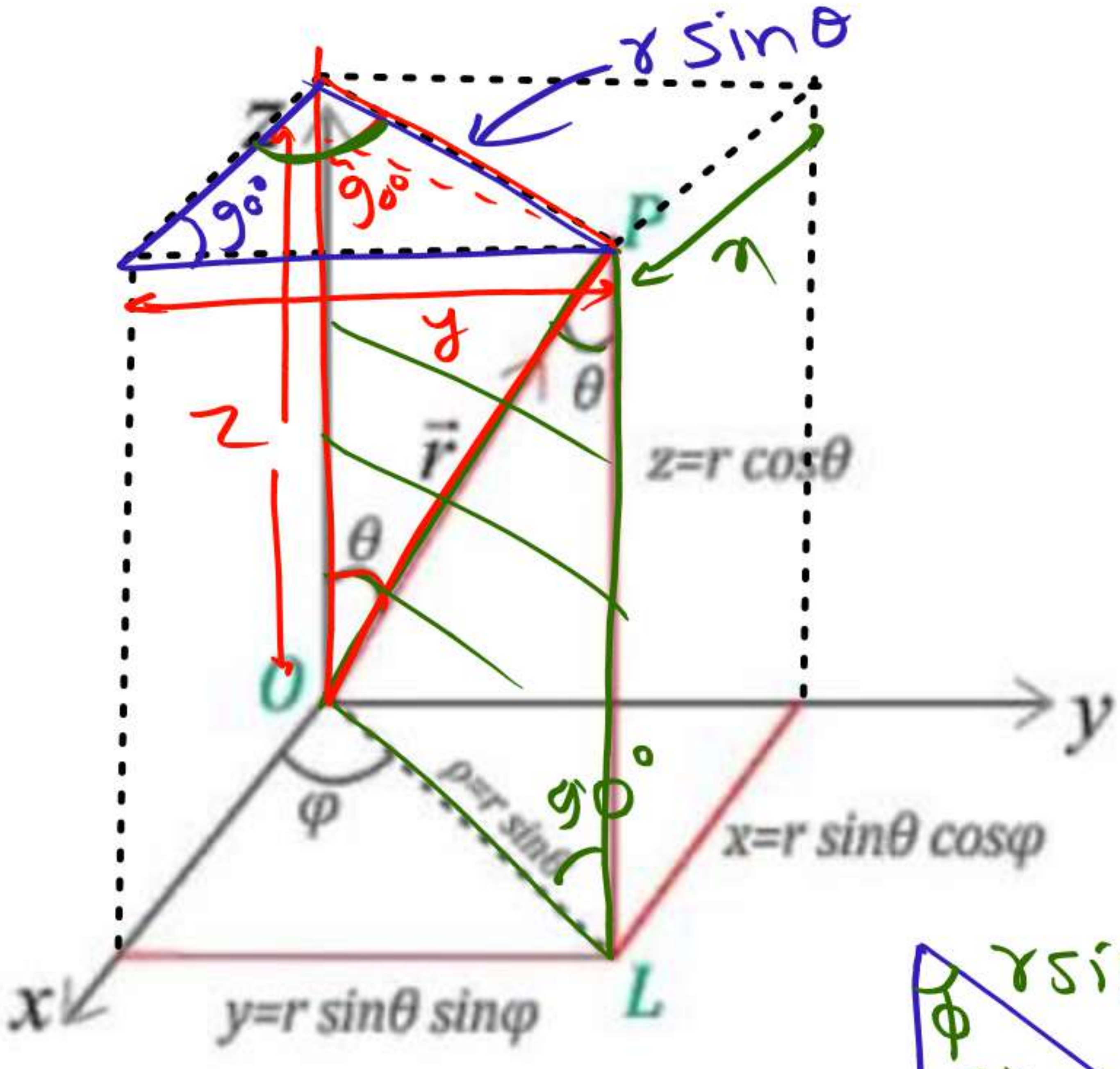
$\vec{H} = $ - ?? ? - - $\hat{a}_\phi$



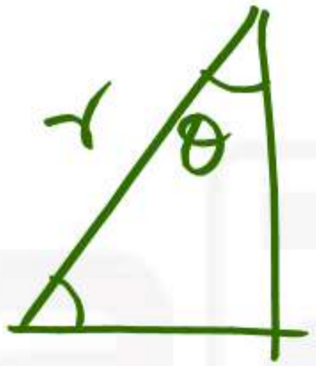

Z
P
(x, y, z) Cartesian
(r, θ, φ) Spherical
Y
X


z
P
θ
r
z=r cosθ
O
y
φ
ρ=r sinθ
x=r sinθ cosφ
x
y=r sinθ sinφ
L

Spherical to Cartesian Conversion



$P(r, \theta, \phi) \longrightarrow P(x, y, z)$

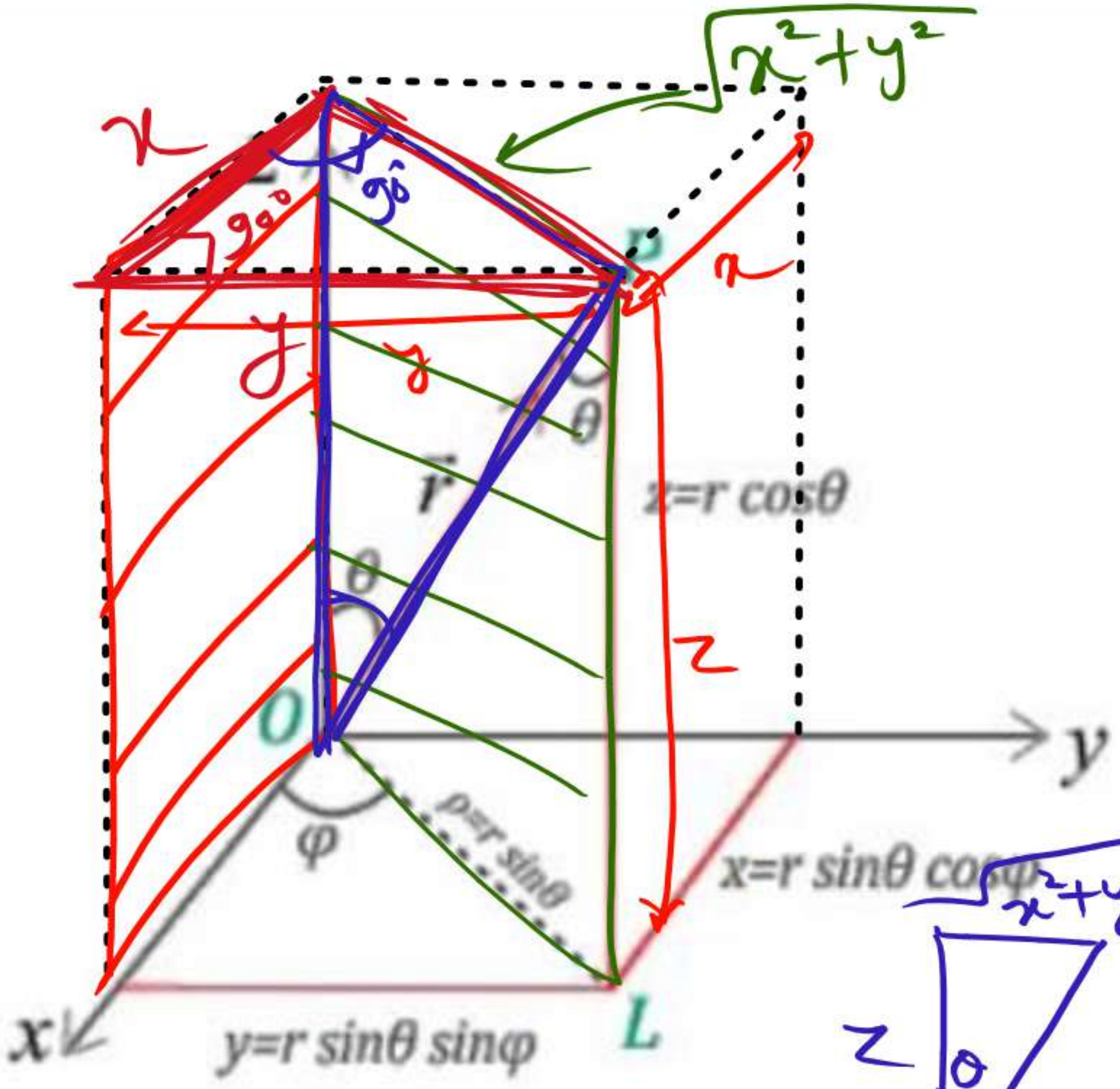


$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

Cartesian to Spherical Conversion

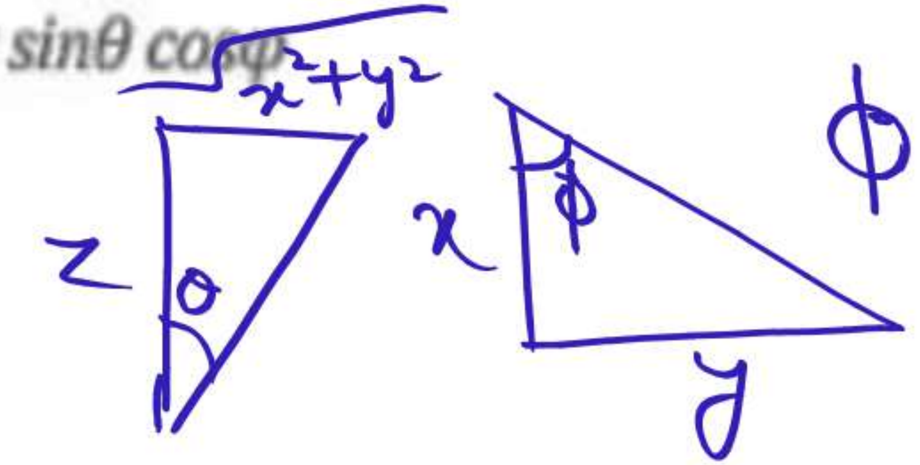


$P(x, y, z) \longrightarrow P(r, \theta, \phi)$

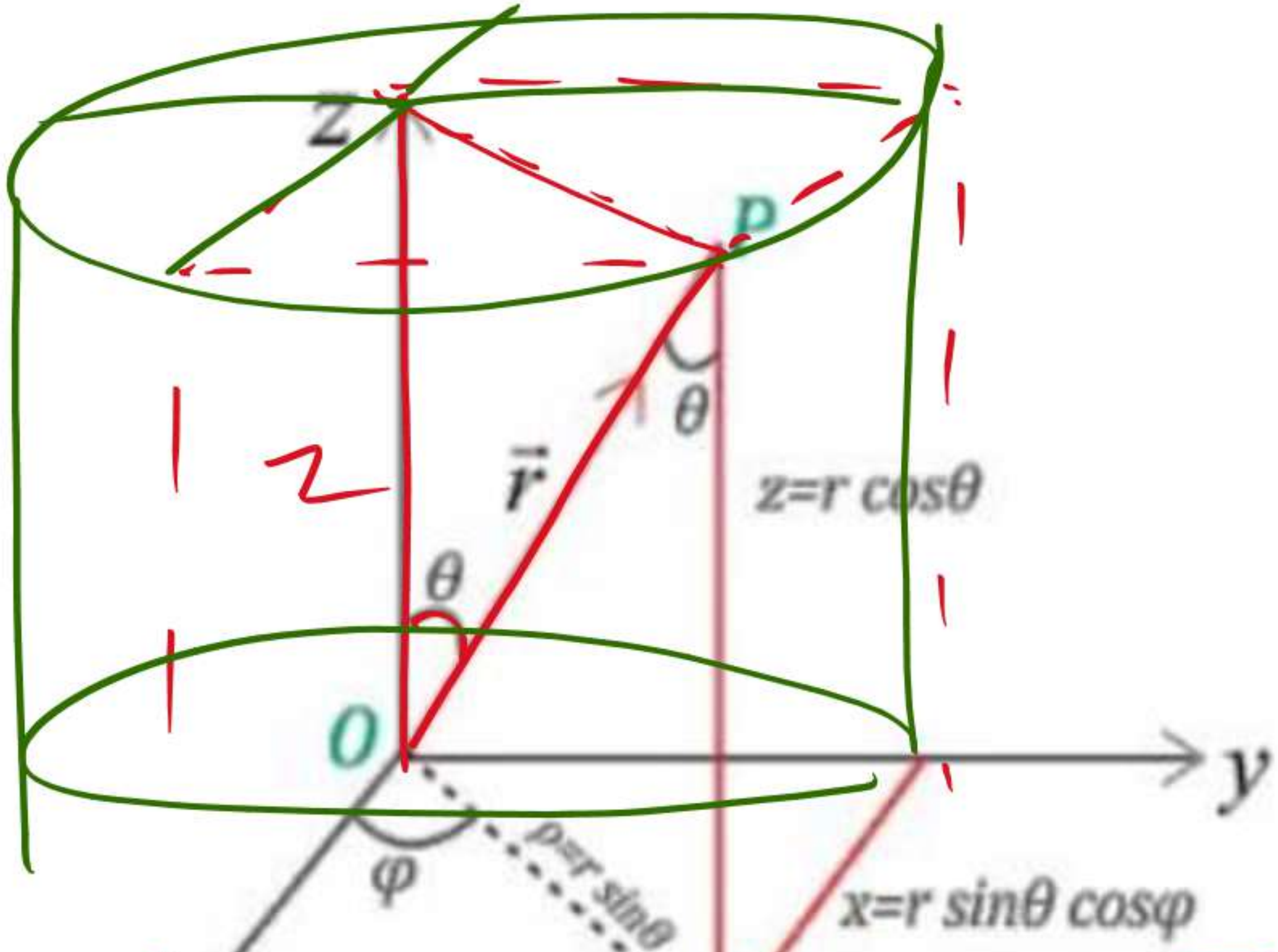
$r = \sqrt{x^2+y^2+z^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$



Spherical to Cylindrical Conversion

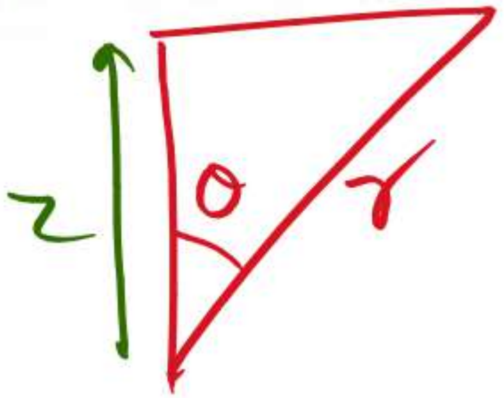


$$P(r, \theta, \phi) \longrightarrow P(\rho, \phi, z)$$

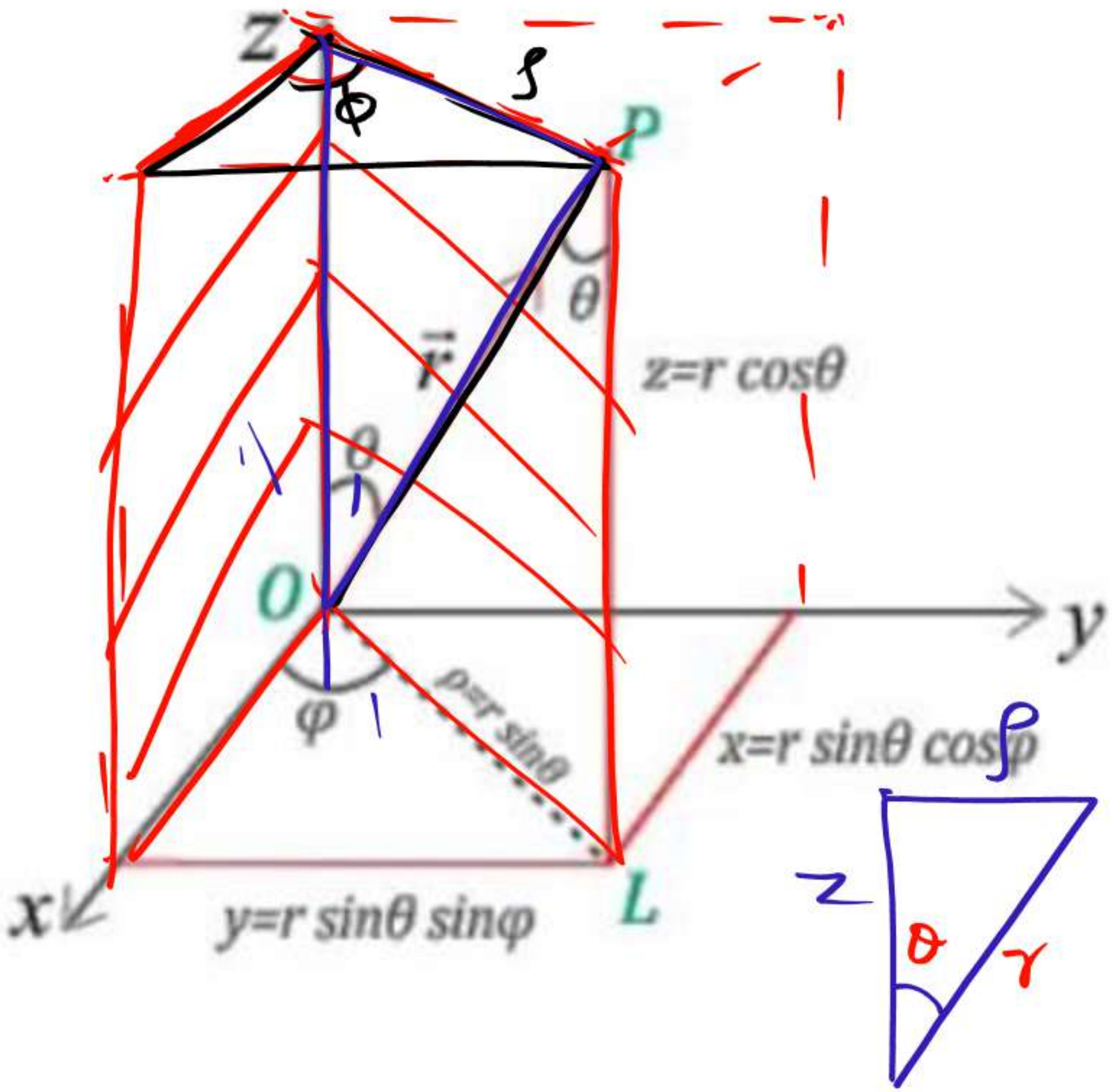
$$\rho = r \sin\theta$$

$$\phi = \phi$$

$$z = r \cos\theta$$



Cylindrical to Spherical Conversion

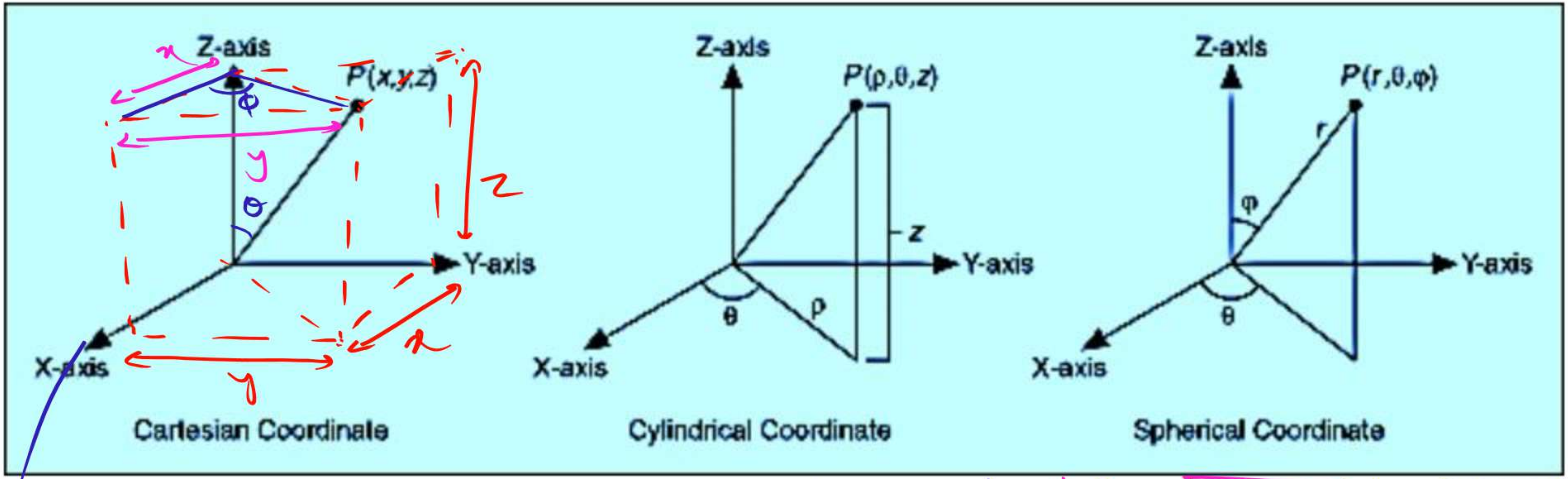


$$P(\rho, \phi, z) \longrightarrow P(r, \theta, \phi)$$

$$r = \sqrt{\rho^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$

$$\phi = \phi$$



$x = \rho \cos\phi$

$y = \rho \sin\phi$

$z = z$

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

$\rho = \sqrt{x^2+y^2}$

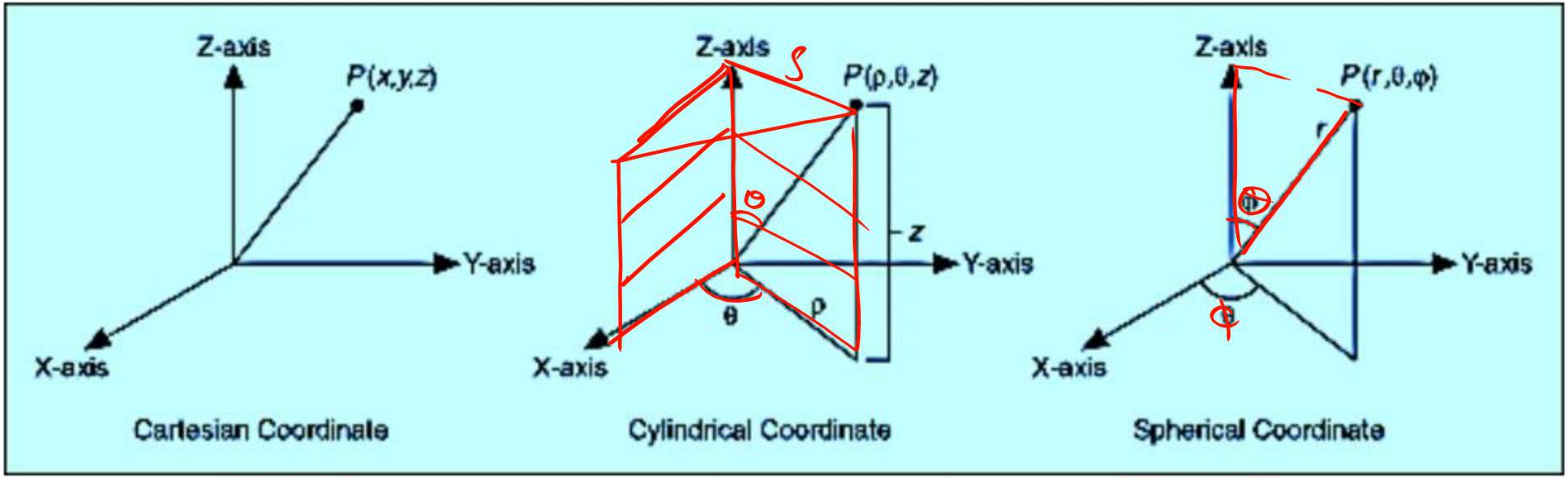
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$



$\rho = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

$\rho = r\sin\theta$

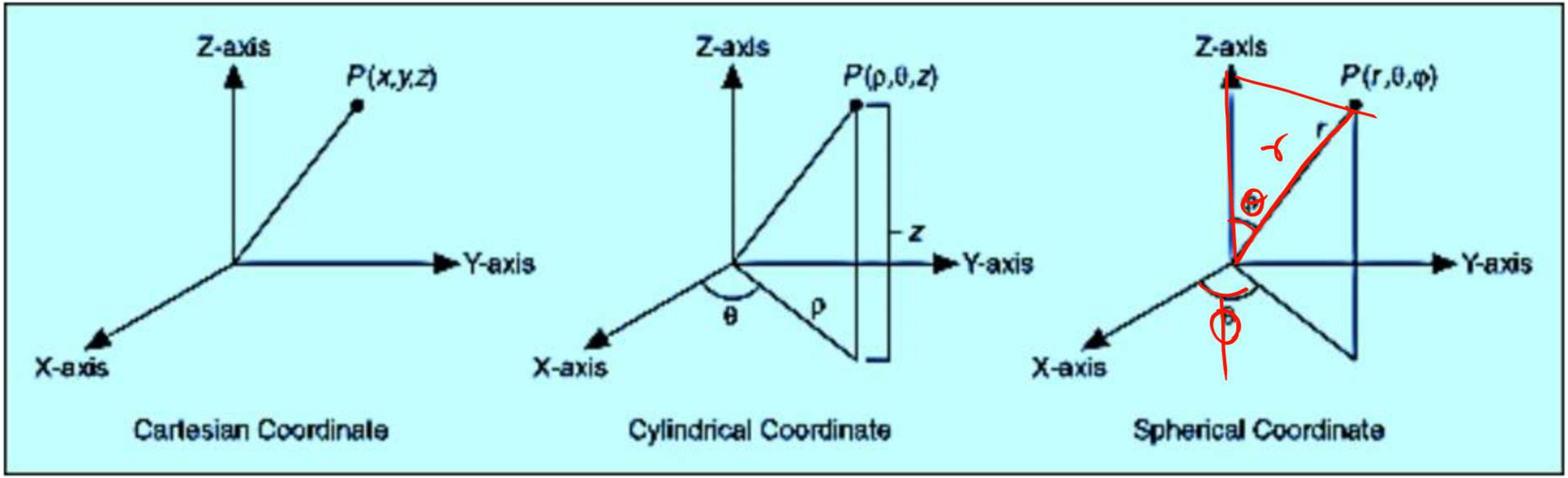
$\phi = \phi$

$z = r\cos\theta$

$r = \sqrt{\rho^2+z^2}$

$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$

$\phi = \phi$



Z-axis
P(x,y,z)
Y-axis
X-axis
Cartesian Coordinate
Z-axis
P(ρ,θ,z)
z
Y-axis
θ
ρ
X-axis
Cylindrical Coordinate
Z-axis
P(r,θ,φ)
r
Y-axis
X-axis
Spherical Coordinate

$(1, 270^\circ, 3)$

Q:10 Find cylindrical coordinates for the point (0,−1,3) written in Cartesian coordinates.

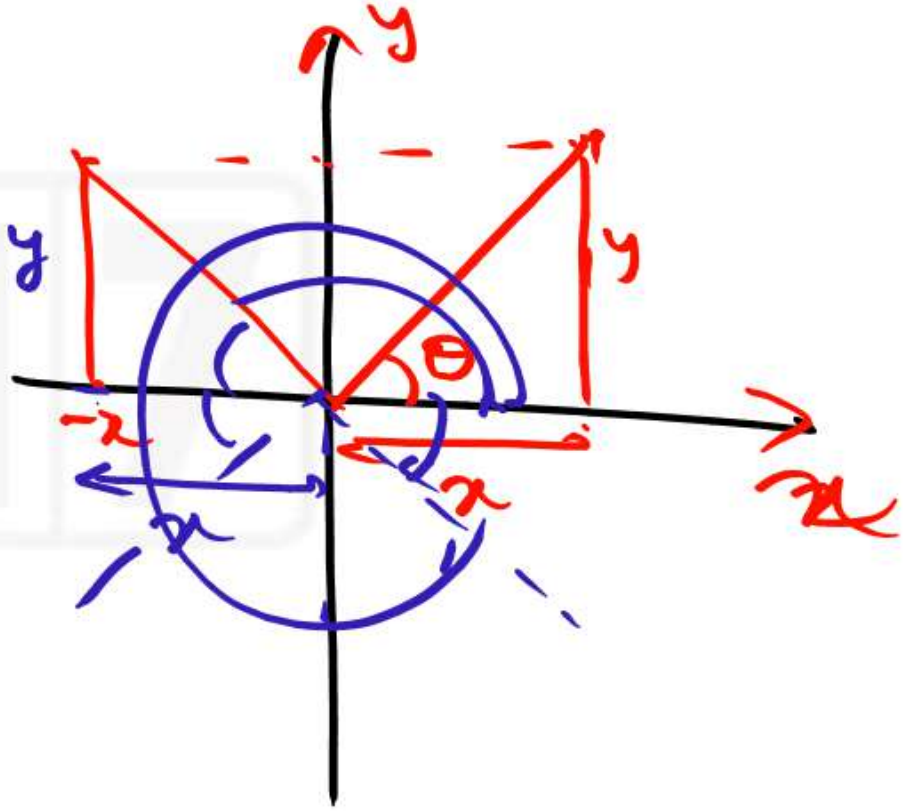
$$P(0,-1,3) \longrightarrow P(\rho_1, \phi_1, z_1)$$

$$\rho = \sqrt{x^2+y^2} = 1$$

$$\phi = \tan^{-1}\left(\frac{|y|}{|x|}\right) = 360^\circ - \tan^{-1}(\infty)$$

$$= 270^\circ \text{ or } -90^\circ$$

$$z = 3$$



Q:11 Consider the surface described in Cartesian coordinates by

$$2z^2 = x^2 + y^2.$$

Describe this surface with an equation in cylindrical coordinates, of the form f(ρ, ϕ, z)=0.

$$2z^2 = x^2 + y^2$$

$$f(\rho, \phi, z) = 0$$

$$x = \rho\cos\theta$$

$$y = \rho\sin\phi$$

$$z = z$$

$$2z^2 = x^2 + y^2$$

$$2z^2 = \rho^2\cos^2\phi + \rho^2\sin^2\phi$$

$$2z^2 - \rho^2 = 0$$

$$f(\rho, \phi, z) = 2z^2 - \rho^2 = 0$$

Problem. 12 Find the Cartesian coordinates of each point, given in cylindrical coordinates.

① $(r, \theta, z) = (1, 1, 1)$

(ρ, ϕ, z) / $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$

$\rho = 1$
$\phi = 1$ radian
$z = 1$

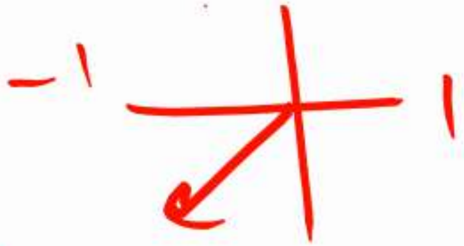
$(x, y, z) =$ [ ]

② $(r, \theta, z) = (\pi, \pi, \pi)$

$x = \pi\cos\pi = -3.14$

$y = \pi\sin\pi = 0$

$z = \pi = 3.14$



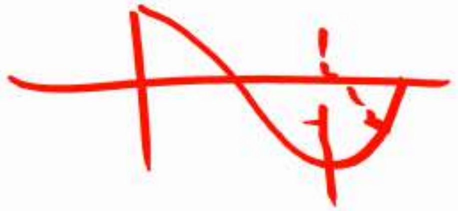
$(x, y, z) =$ $-3.14, 0, 3.14$

③ $(r, \theta, z) = (2, 4\pi/3, -2)$

$x = 2\cos\left(\frac{4\pi}{3}\right) = -\sqrt{3}$

$y = 2\sin\left(\frac{4\pi}{3}\right) = -1$

$z = -2$



$(x, y, z) =$ $-\sqrt{3}, -1, -2$

Problem. 13 Find the cylindrical coordinates of each point, given in Cartesian coordinates.

$(x, y, z) = (1, 1, 1)$

① $\rho = \sqrt{x^2+y^2} = \sqrt{2}$

$\phi = \tan^{-1}(1) = \frac{\pi}{4}$, $z = 1$

$(\rho, \phi, z) =$ $\sqrt{2}, \frac{\pi}{4}, 1$

② $(x, y, z) = (\pi, \pi, \pi)$

$\rho = \sqrt{\pi^2+\pi^2} = \sqrt{2}\,\pi$

$\phi = \frac{\pi}{4}$

$(\rho, \phi, z) =$ $\sqrt{2}\pi, \frac{\pi}{4}, \pi$

③ $(x, y, z) = (2, 2\sqrt{3}, -2)$

$\rho = \sqrt{4+12} = 4$

$\phi = \tan^{-1}(\sqrt{3})$

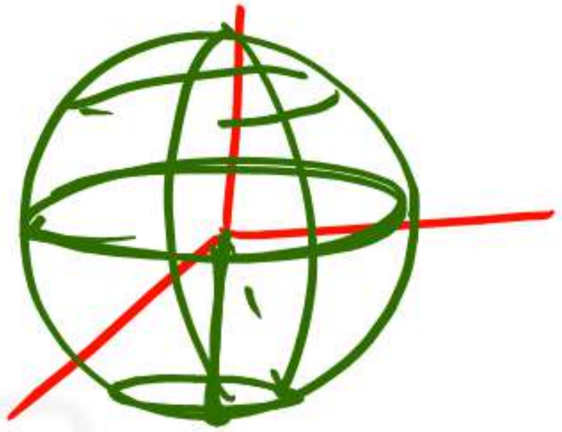
$(\rho, \phi, z) =$ $4, \frac{\pi}{3}, -2$

Q:14 Find the Cartesian coordinates of the point (2,π,π/2) given in spherical coordinates.

$r = 2$

$\theta = \pi$

$\phi = \frac{\pi}{2}$



$x = r\sin\theta\cos\phi = 2\sin\pi\cos\left(\frac{\pi}{2}\right) = 0$

$y = r\sin\theta\sin\phi = 2\sin\pi\sin\left(\frac{\pi}{2}\right) = 0$

$z = r\cos\theta = r\cos(\pi) = -2$

GATE 2024

Sat & Sunday
3 p.m.
All branch

प्रचण्ड Batch

Th, Fr. Saturday
EMFT → 9 pm

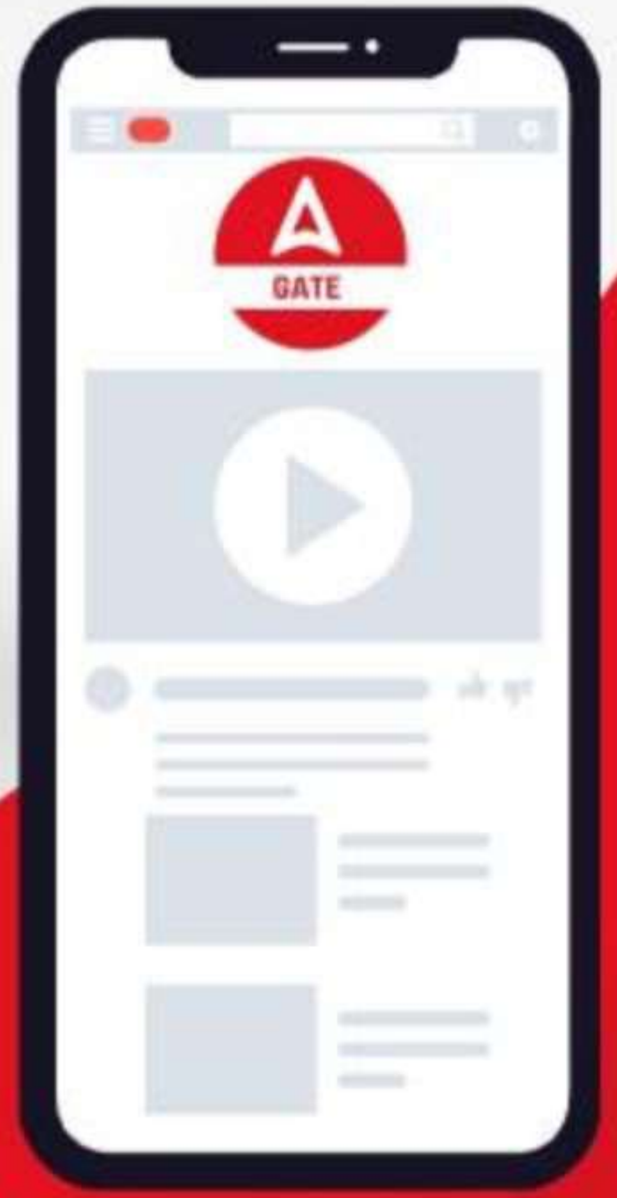
Engineering Mathematics

mon to wed
6 pm → M/W
9 pm → Comm.

LINEAR ALGEBRA

6 p.m.
↑ PSA

OPERATIONS AND CLASSIFICATIONS OF MATRICES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