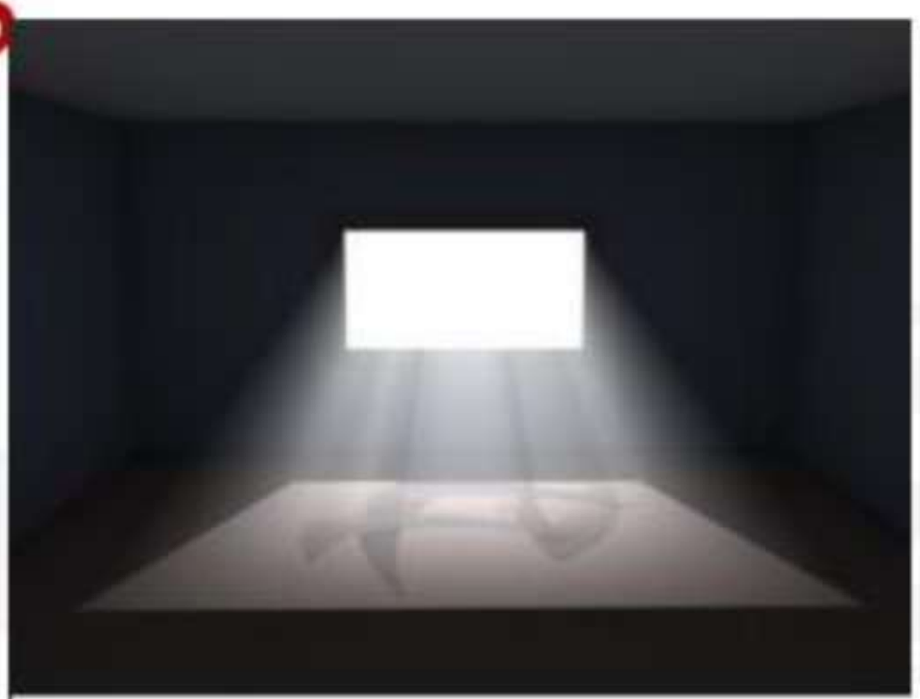
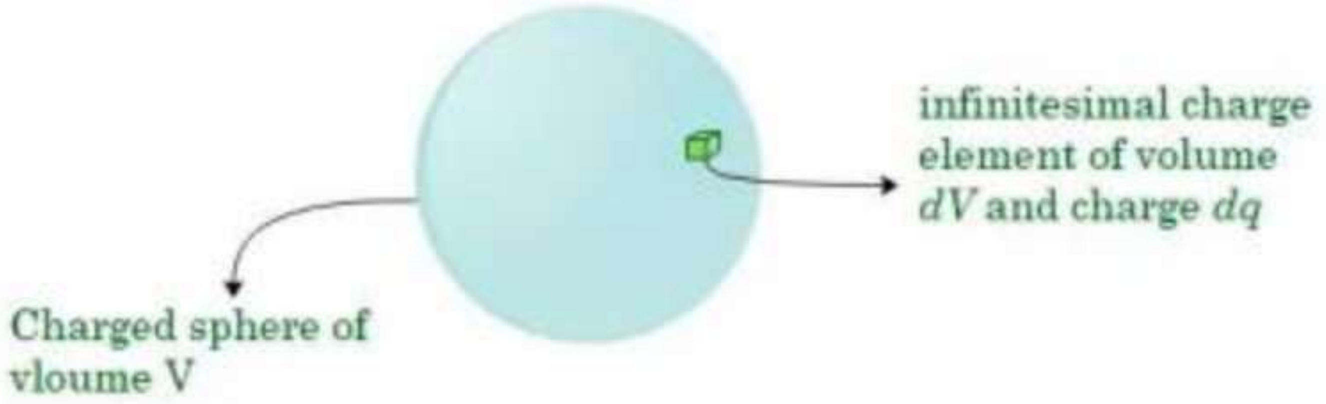
# Coordinate systems

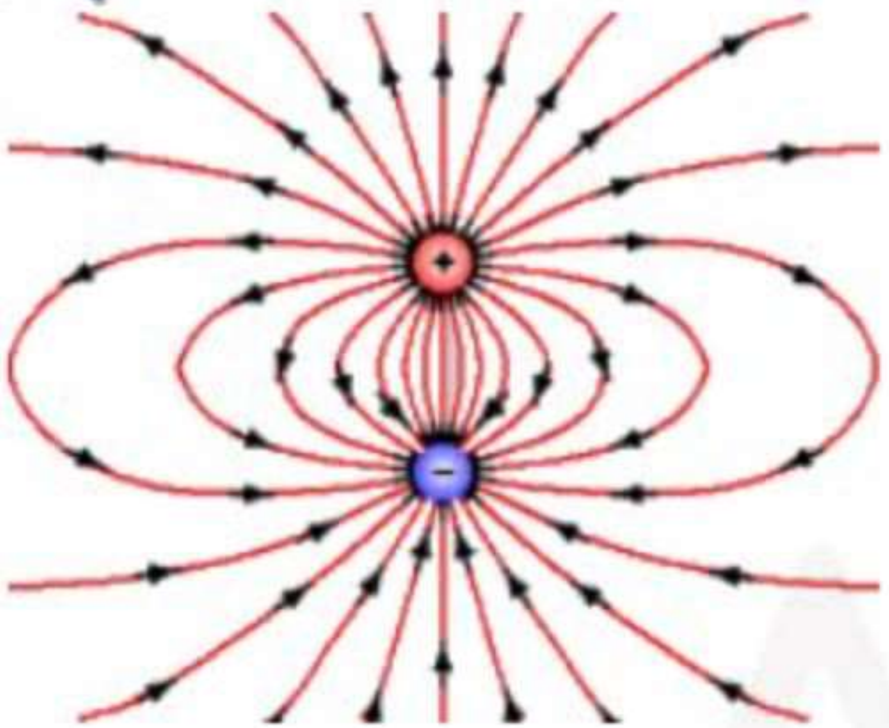

## Vector Integrals (Surface and Volume)

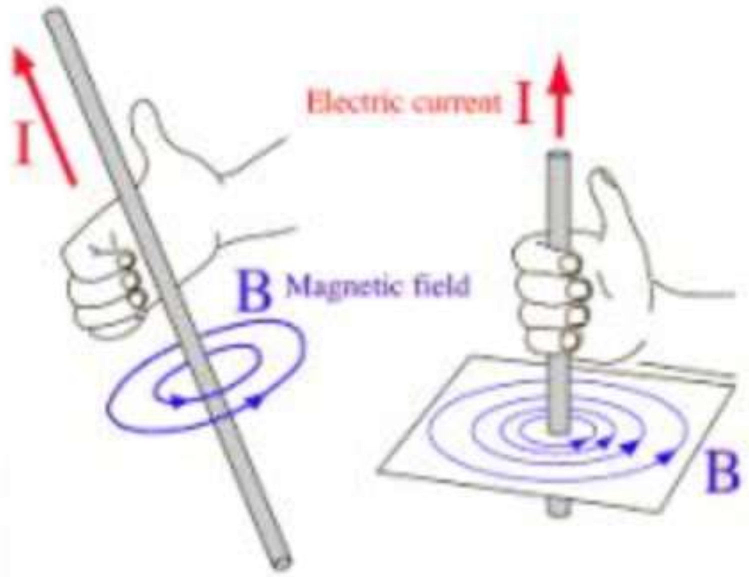


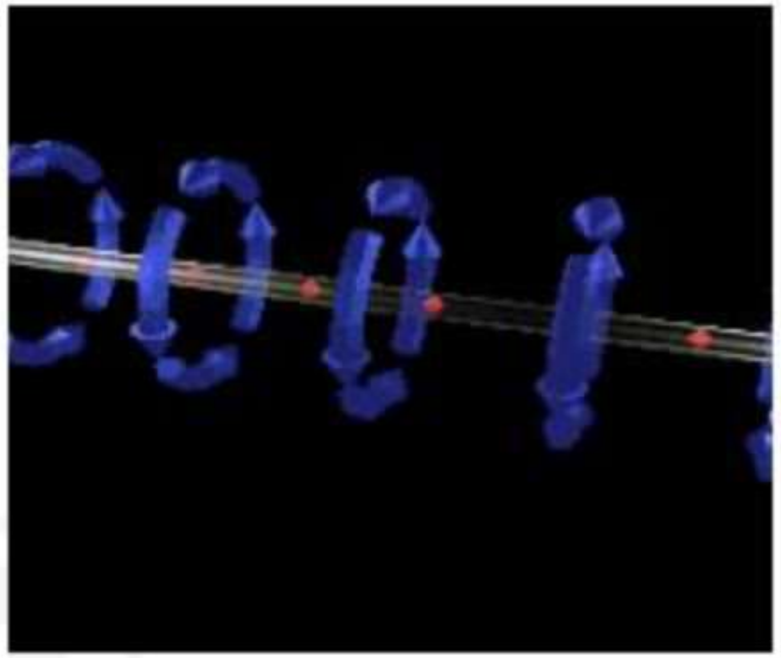


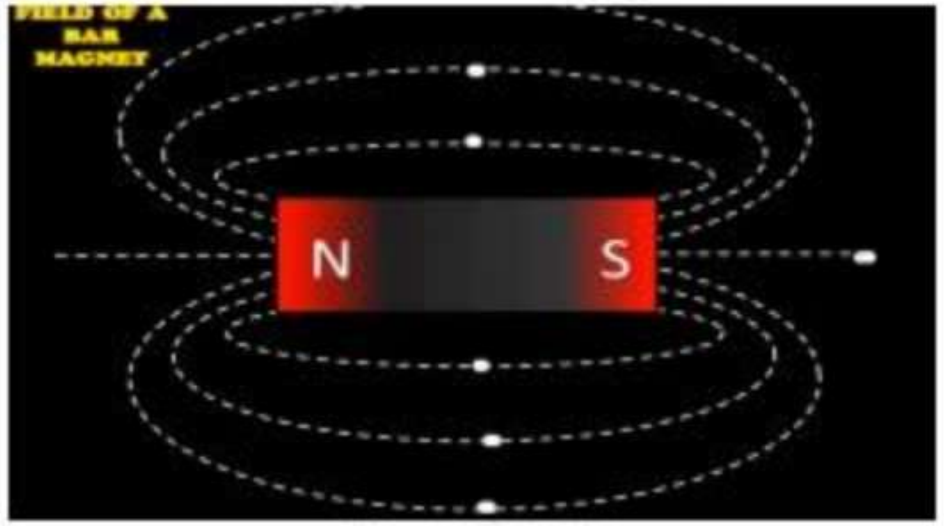

## Question Practice on Vector Integrals

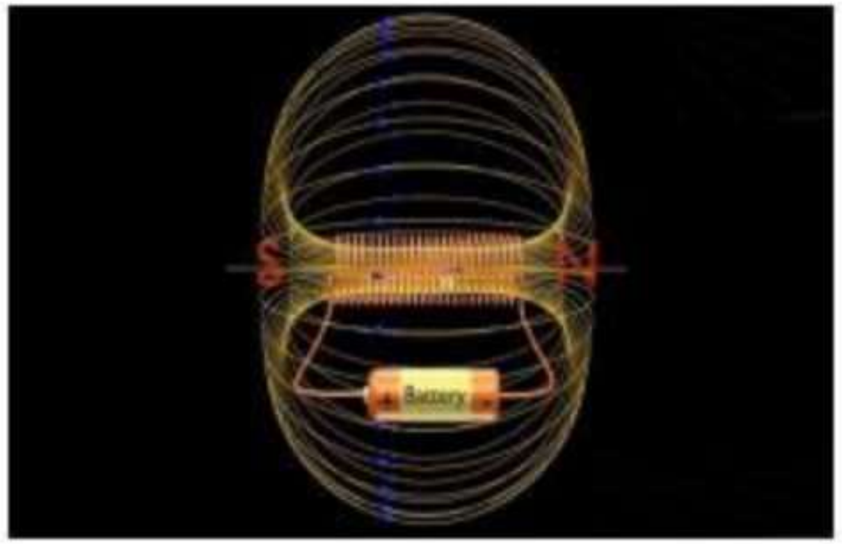
# Recap

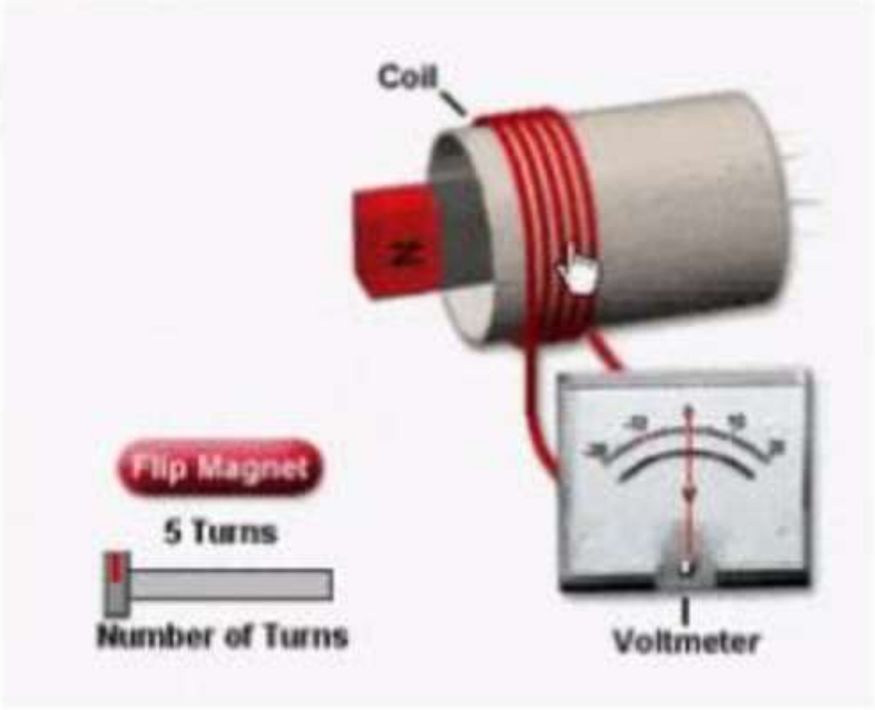



I
Electric current I
B Magnetic field
B

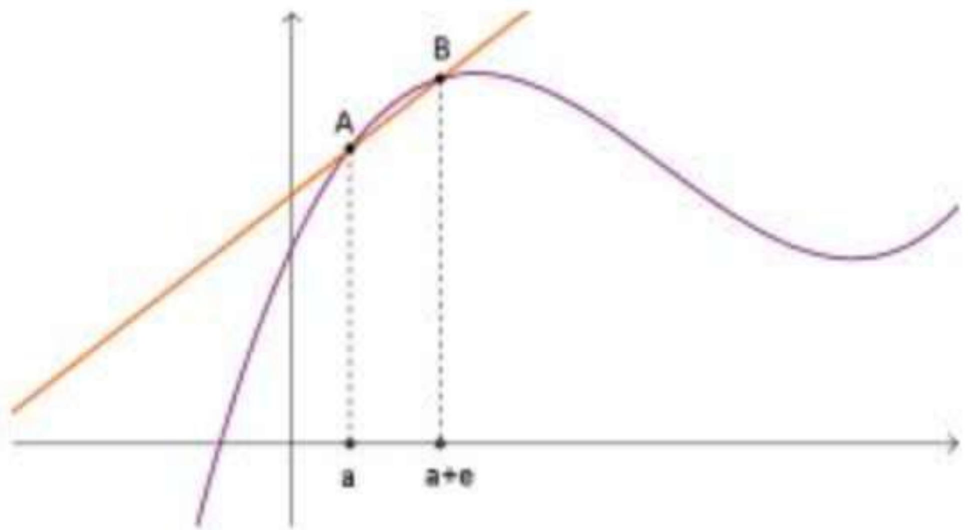



FIELD OF A
BAR
MAGNET
N
S


Battery


Coil
Flip Magnet
5 Turns
Number of Turns
Voltmeter

# Recap



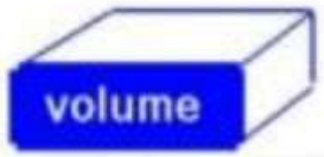

A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.

**Scalar Quantities**

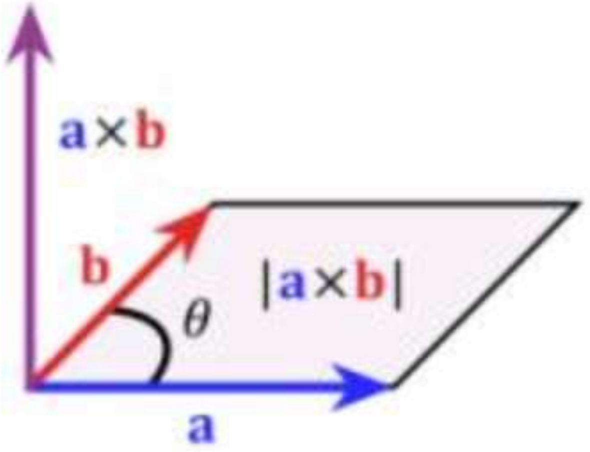
length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power

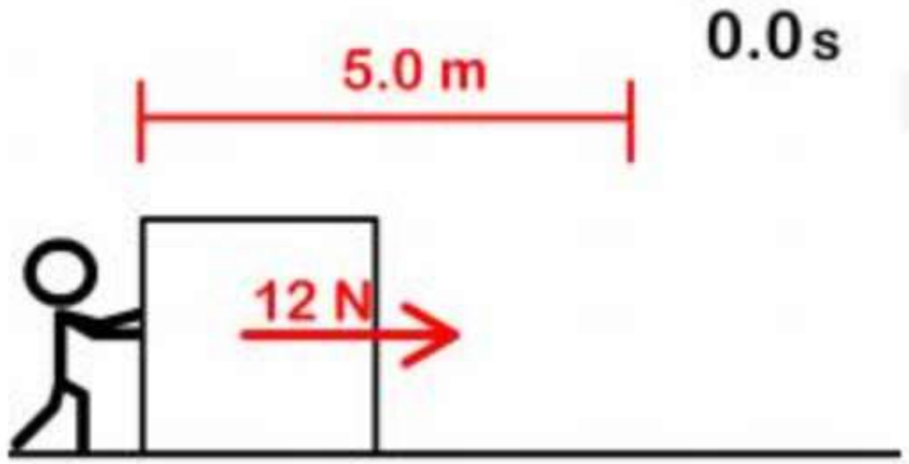
**Vector Quantities**

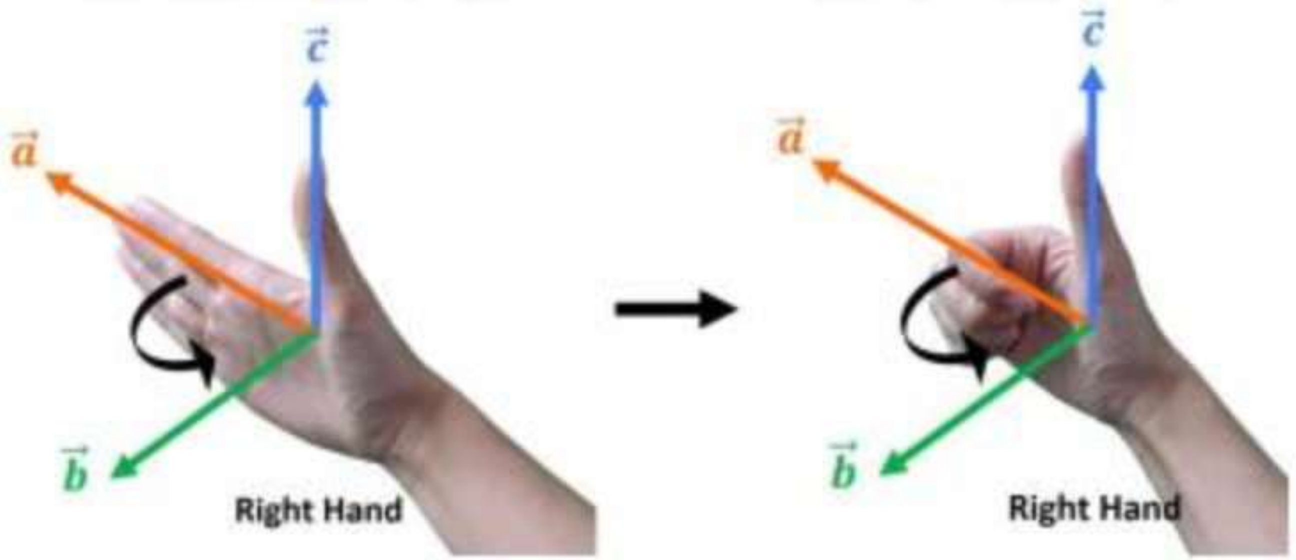
displacement
velocity
acceleration
momentum
force
lift, drag, thrust
weight

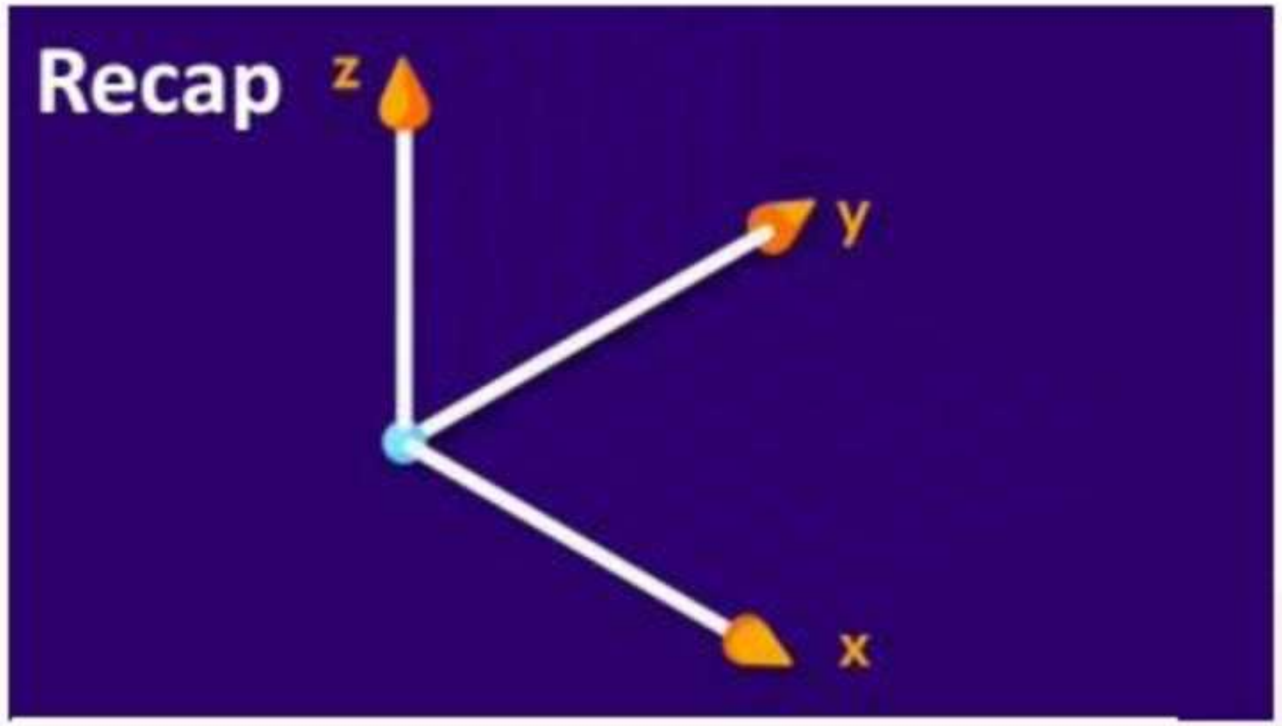


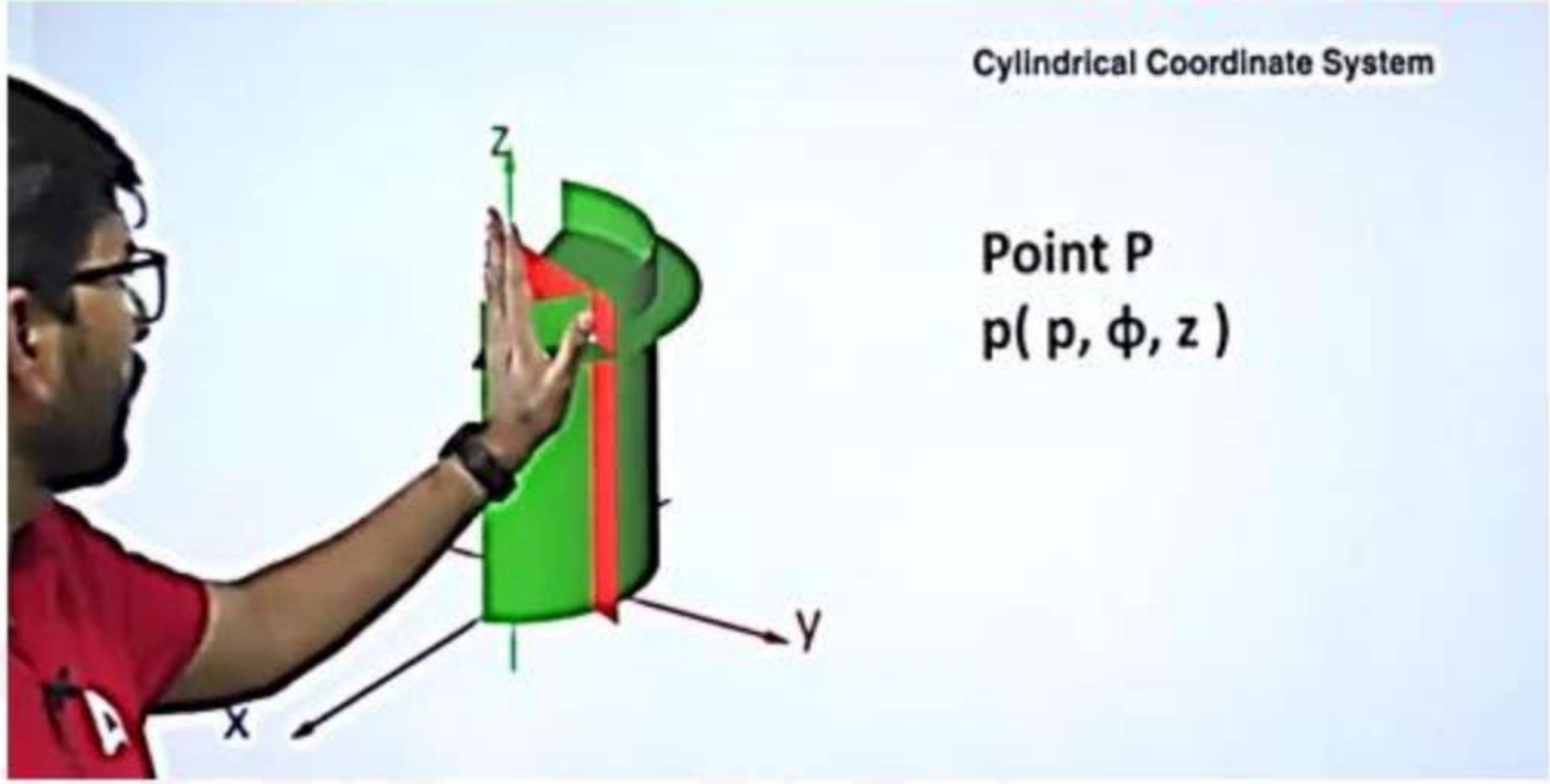


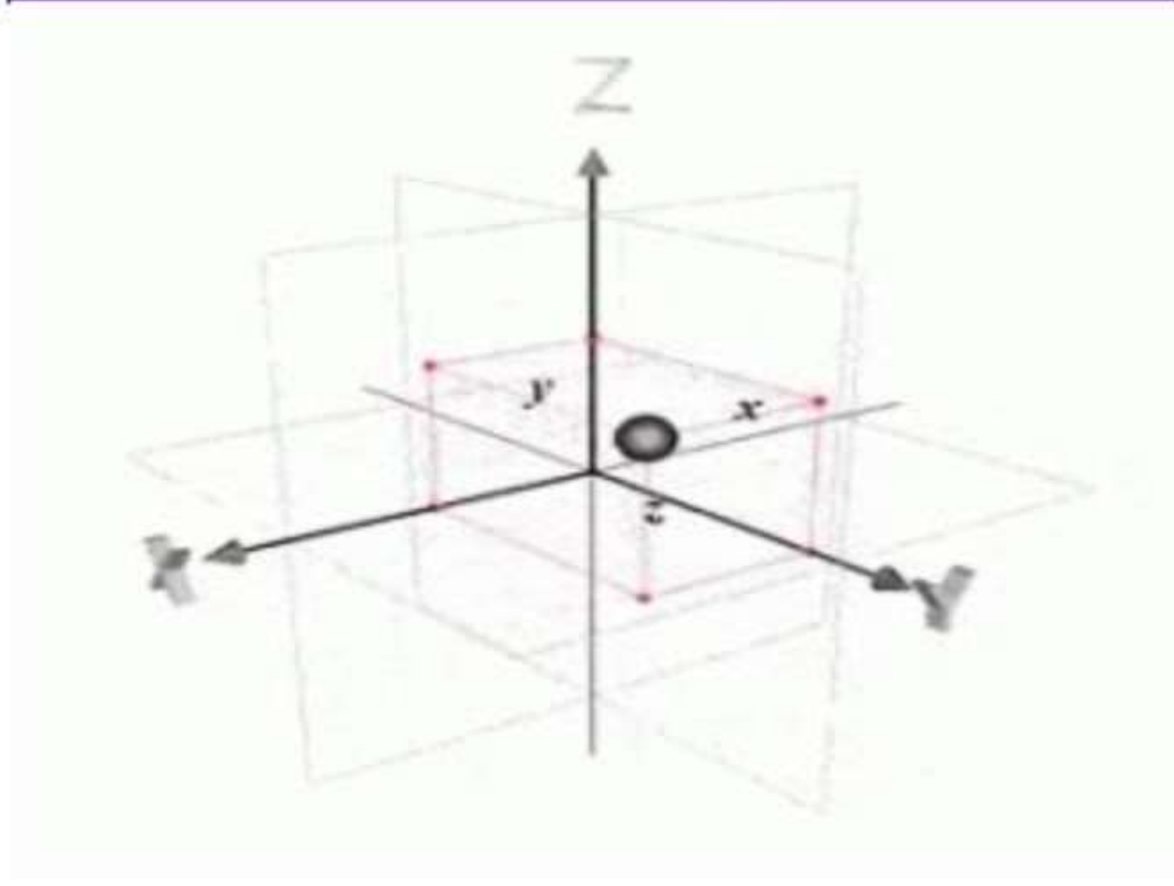


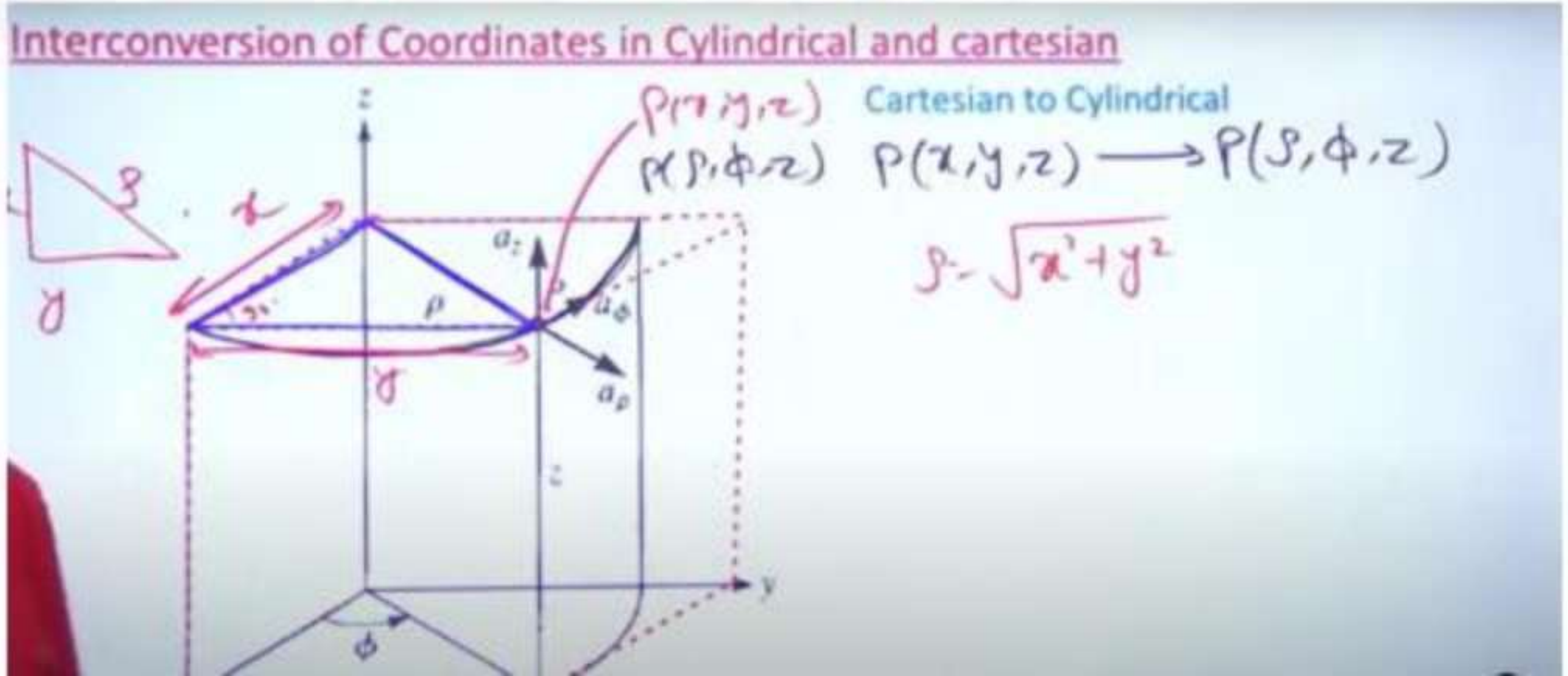




Recap
z
y
x


Cylindrical Coordinate System
Z
Y
X
Point P
p( p, ϕ, z )


Z
y
x
z
X
Y


Interconversion of Coordinates in Cylindrical and cartesian
P(x,y,z)
P(ρ,ϕ,z)
Cartesian to Cylindrical
P(x,y,z) ⟶ P(ρ,ϕ,z)
$\rho = \sqrt{x^2+y^2}$
y
ρ
x
ϕ

Cylindrical Coordinate System

Cylindrical Coordinate System

z

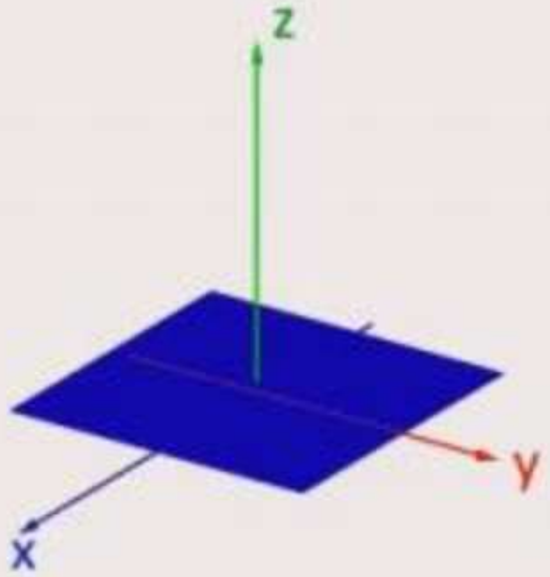
Constant r
surface

$R \geq 0^{o}$

y

x

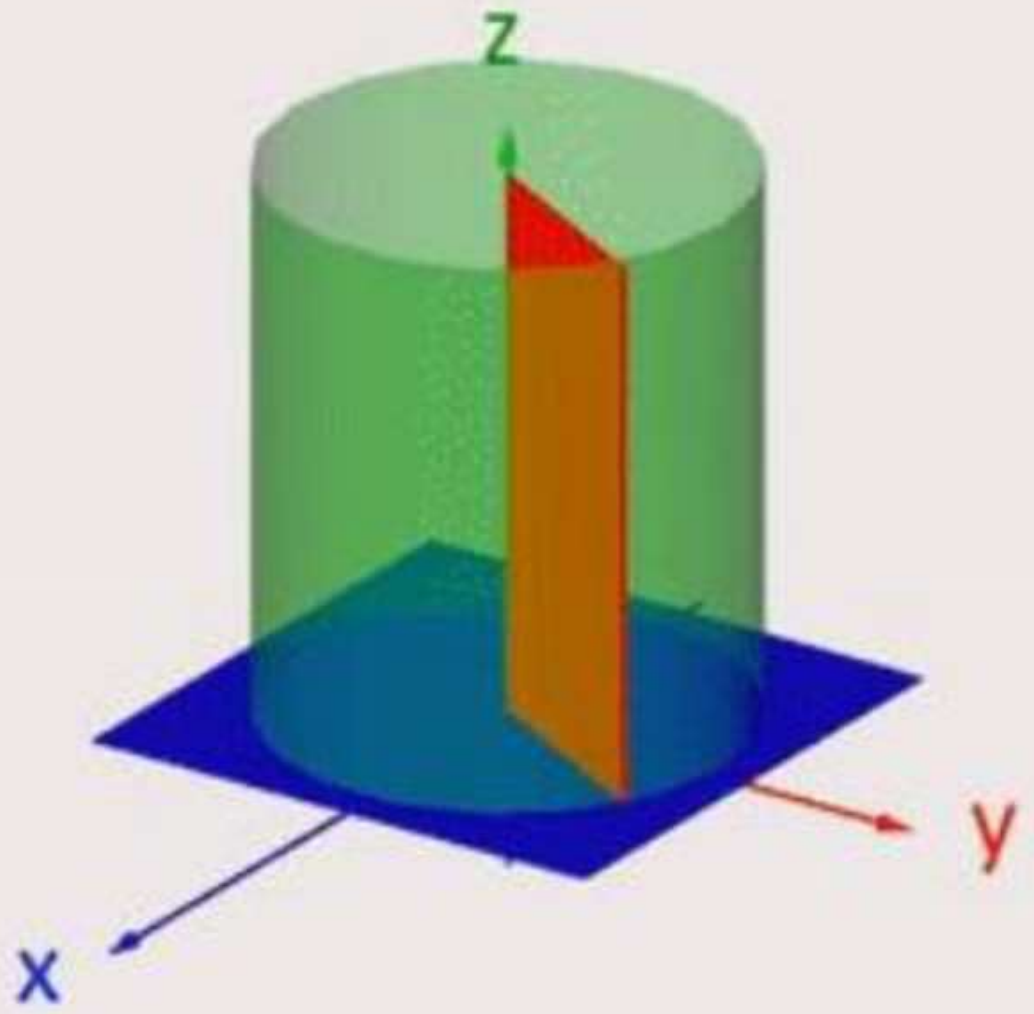
Cylindrical Coordinate System



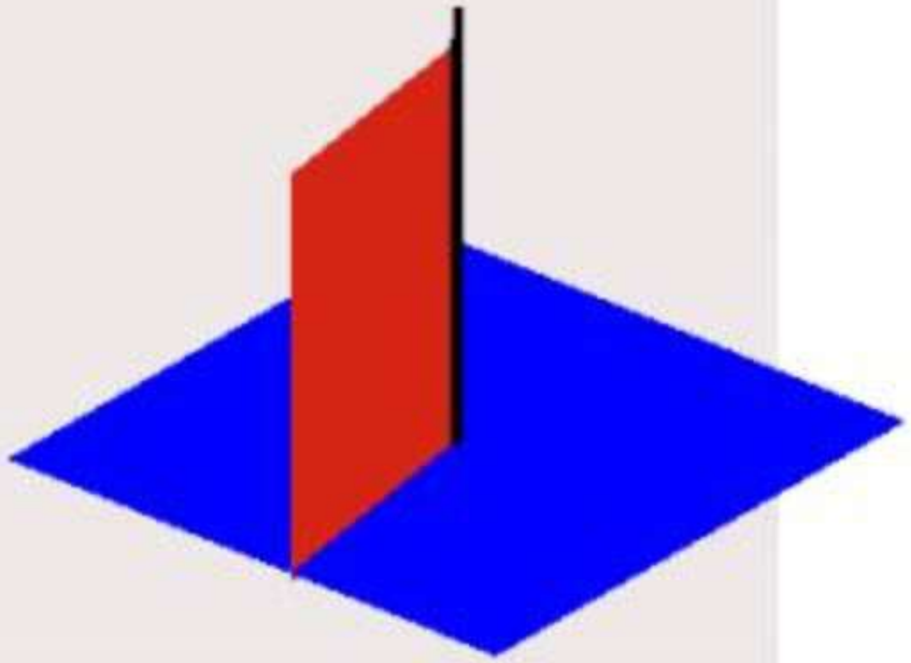

Constant z
surface

$-\infty \leq z \leq +\infty$

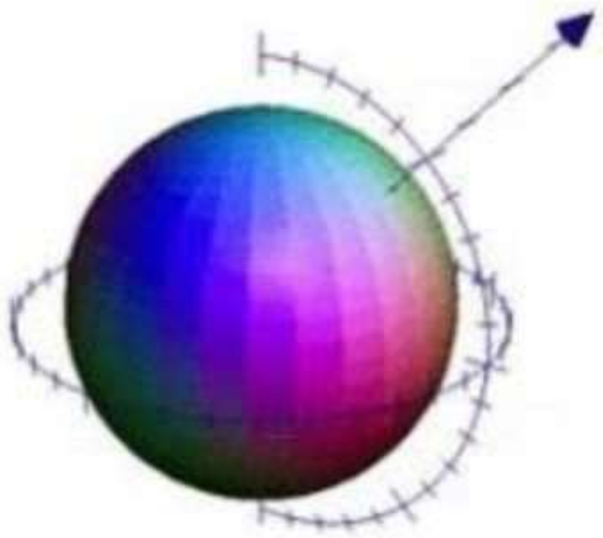
## Cylindrical Coordinate System

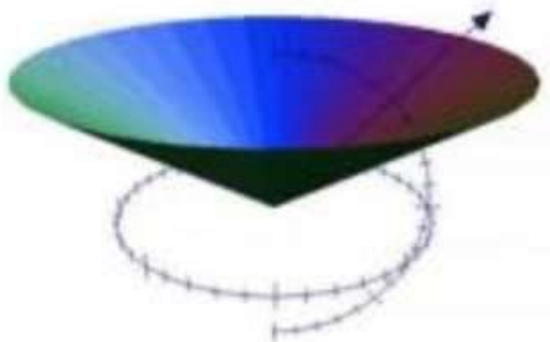


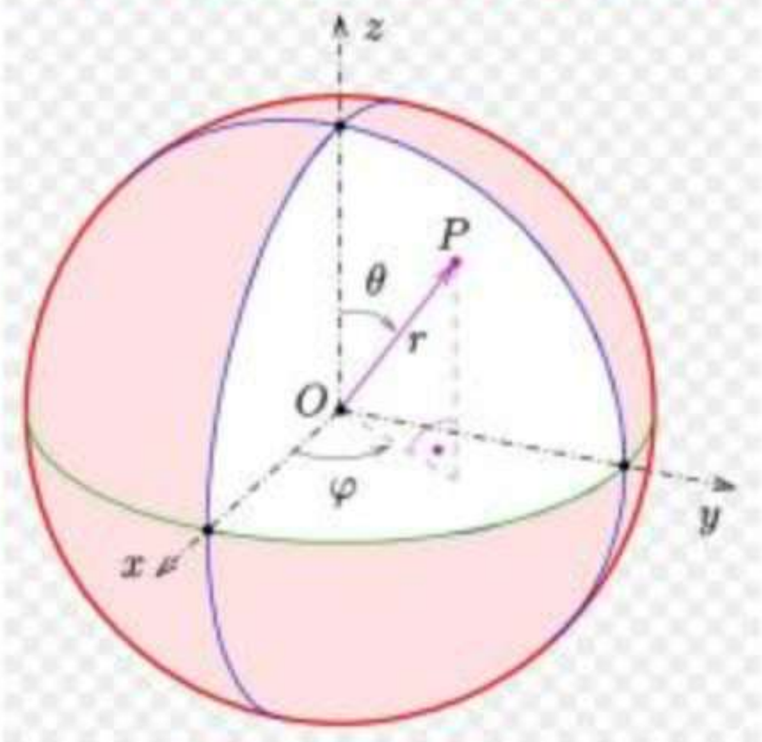

Cylindrical coordinate system planes

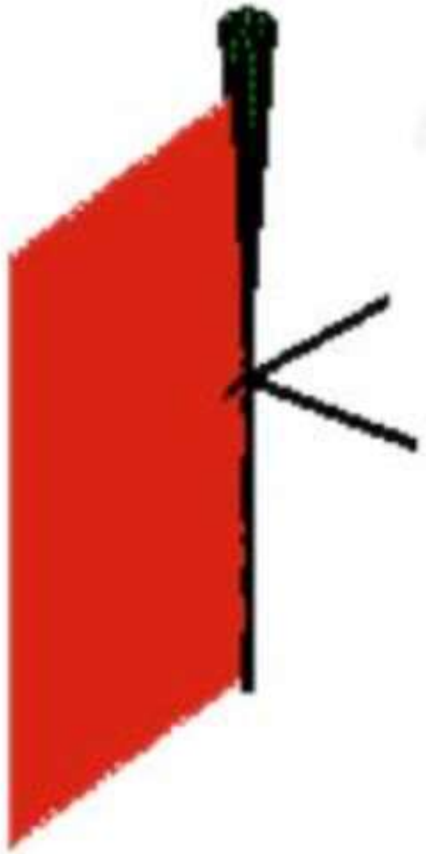


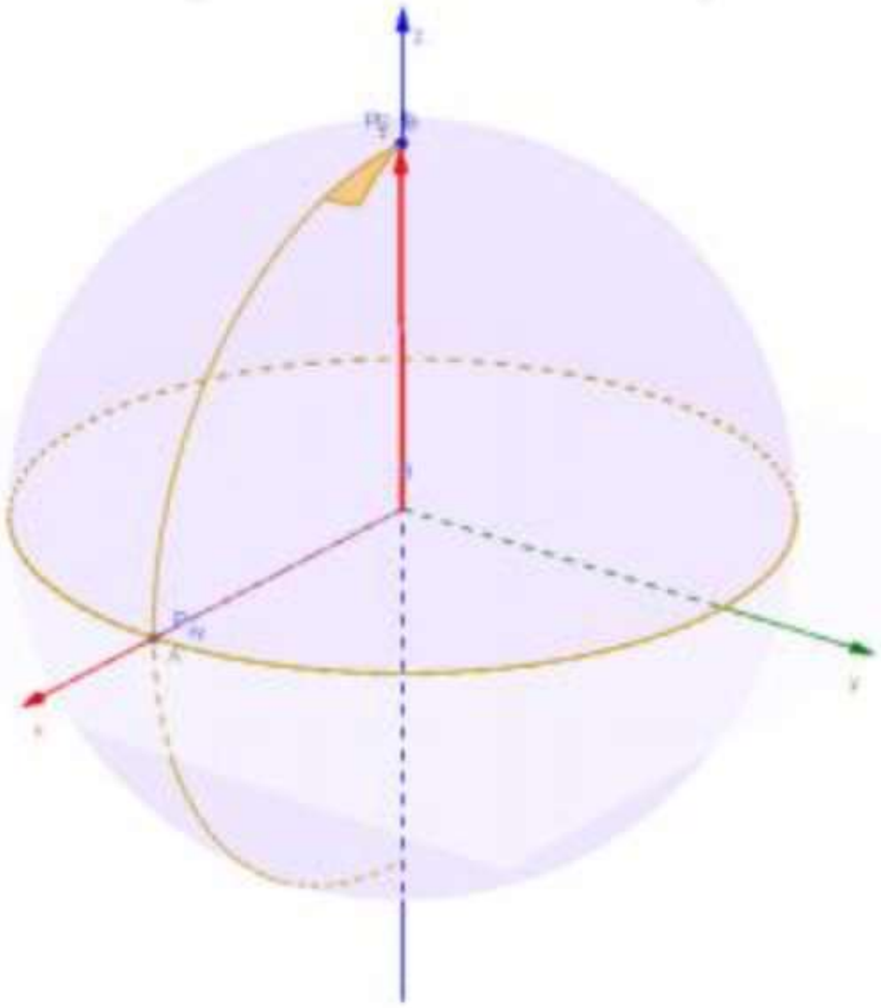
## Spherical Coordinates P(r, θ, ϕ)









# Recap



Q.5 If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes $a$ and $b$, respectively, $|\vec{a} \times \vec{b}|^2$ will be equal to

(a) $a^2b^2 - (\vec{a} \cdot \vec{b})^2$

(b) $ab - \vec{a} \cdot \vec{b}$

(c) $a^2b^2 + (\vec{a} \cdot \vec{b})^2$

(d) $ab + \vec{a} \cdot$

$\vec{a} \times \vec{b} = ab\sin\theta\, \hat{a}_n$

$|\vec{a} \times \vec{b}| = ab\sin\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2\sin^2\theta$ Ans

$\sin^2\theta = 1 - \cos^2\theta$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2(1-\cos^2\theta) = a^2b^2 - a^2b^2\cos^2\theta$

$= a^2b^2 - (\vec{a} \cdot \vec{b})^2$

Q.7 The angle between two unit - magnitude coplanar vectors P(0.866, 0.500, 0) and Q(0.259, 0.966, 0) will be

(a) 0°

(b) 30°

(c) 45°

(d) 60°

$\theta = \cos^{-1}$

$= \cos^{-1}(0.866 \times 0.259 + \ldots \times 0.966)$

A(2, −3, 2)

$\vec{OA} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$|\vec{P}| = \sqrt{(0.866)^2 + (0.5)^2}$

$\cos^{-1}(0.707)$

$0.707 = \frac{1}{\sqrt{2}}$

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

## Number of Questions covered-9

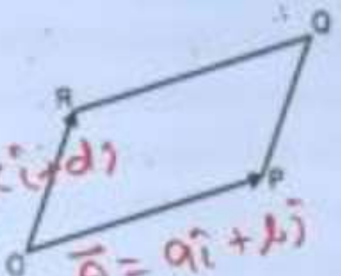
Q.6 For the parallelogram OPQR shown in the sketch, $\overline{OP} = a\hat{i} + b\hat{j}$ and $\overline{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is

(a) ad - bc

(b) ac + bd

(c) ad + bc

(d) ab - cd




$\vec{B} = c\hat{i} + d\hat{j}$

$\vec{A} = a\hat{i} + b\hat{j}$

Area $= |\vec{A} \times \vec{B}|$

$$\vec{A} \times \vec{B} = \vec{OP} \times \vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = (ad - bc)\hat{k}$$

Area = ad − bc

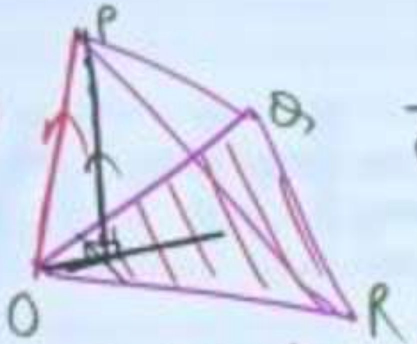
Q.8 If P, Q and R are three points having coordinates (3, - 2, - 1), (1, 3, 4), (2, 1, - 2) in XYZ space, then the distance from point P to plane OQR (O being the origin ... coordinate system) is given by

GATE

AAI ATC

1 marks - 1.98

2 marks - 3.96




$\vec{OP}$

$\vec{OQ} \times \vec{OR} = \vec{X}$

= normal vector to plane OQR

$\vec{OQ} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{OR} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{OQ} \times \vec{OR} = ?$

# Spherical Coordinate systems

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# GATE 2024

GATE

प्रचण्ड Batch

## Electromagnetic Field Theory

## QUESTION PRACTICE

# VECTOR INTEGRALS

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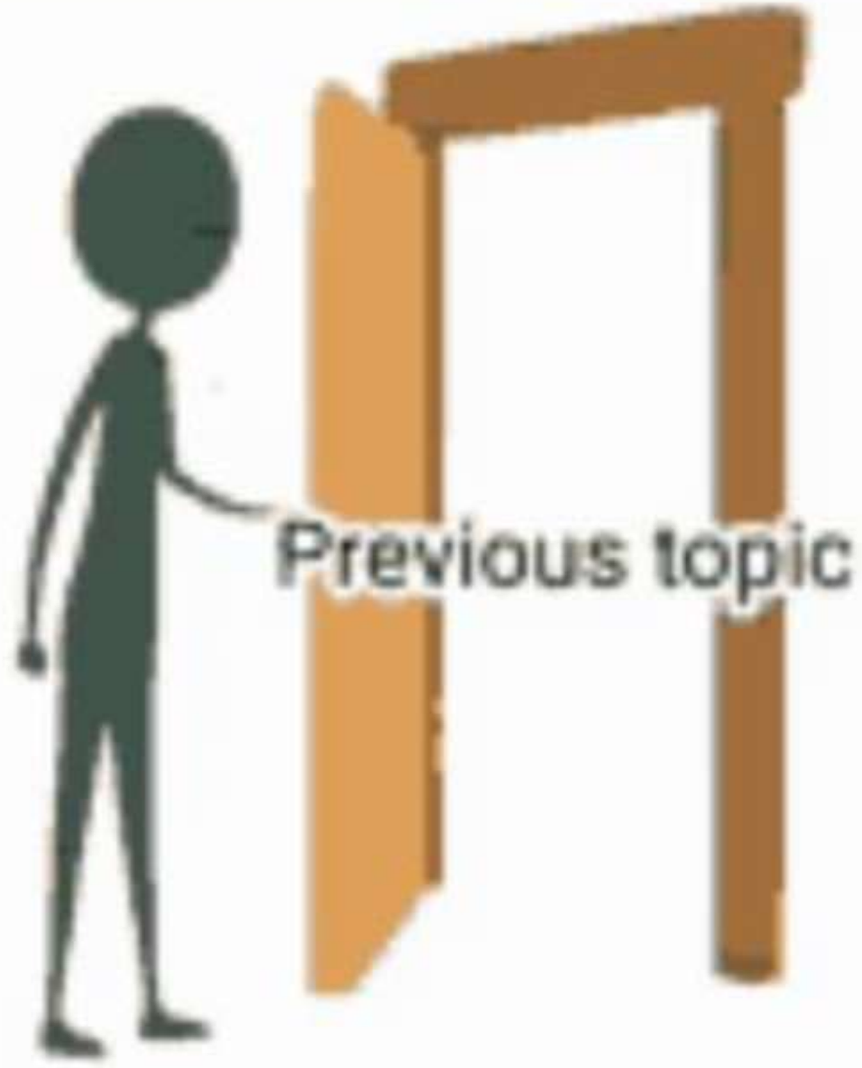


1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical and Spherical Coordinate systems
7. Vector integrals( Line and closed line)

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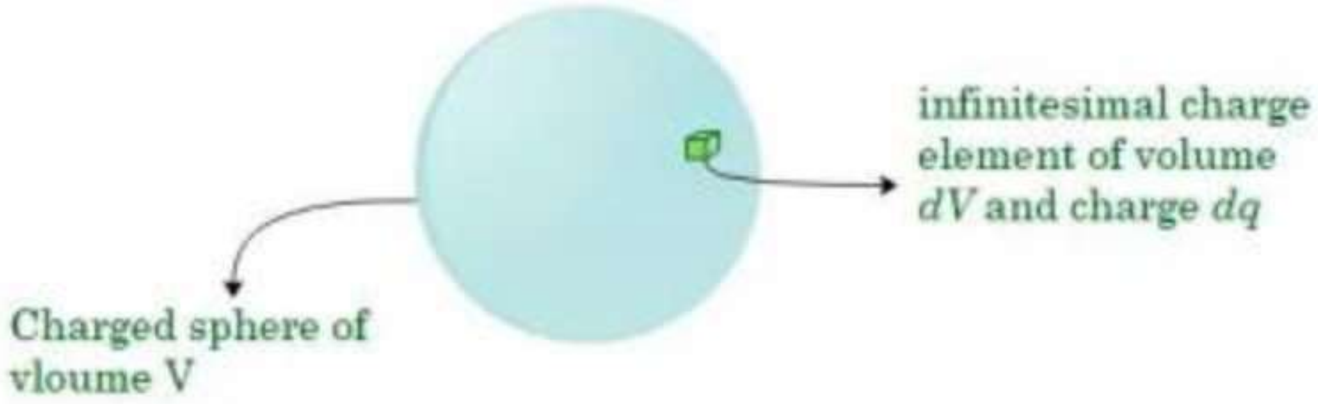
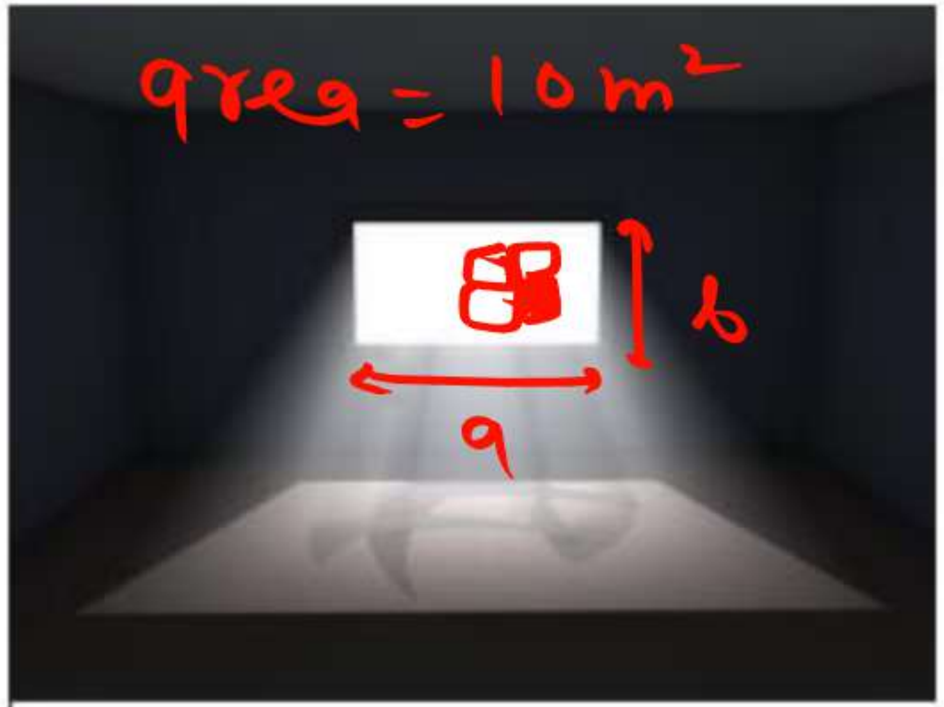
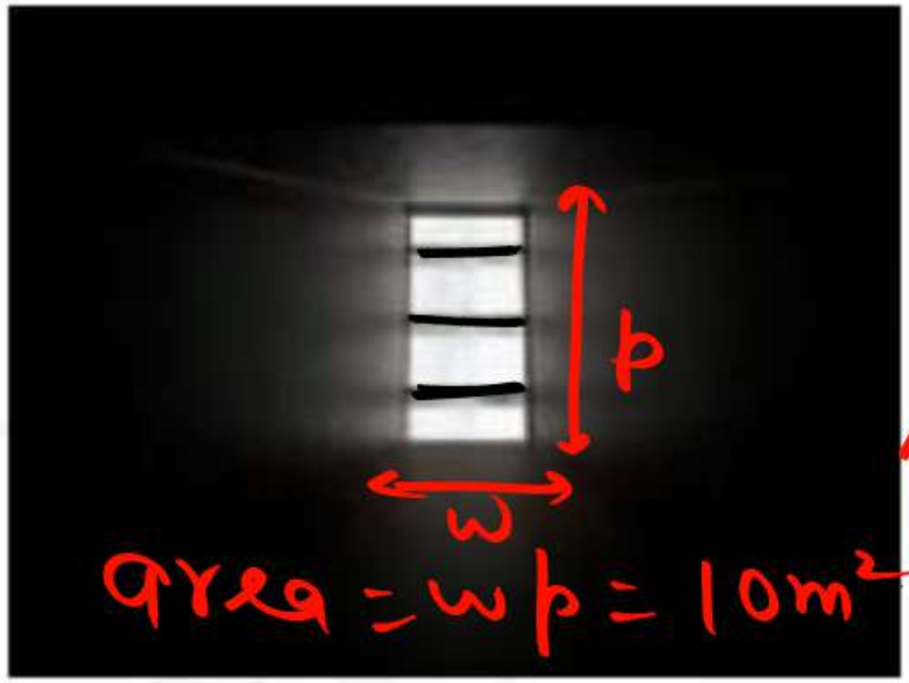
## Vector Integrals (Surface and Volume)

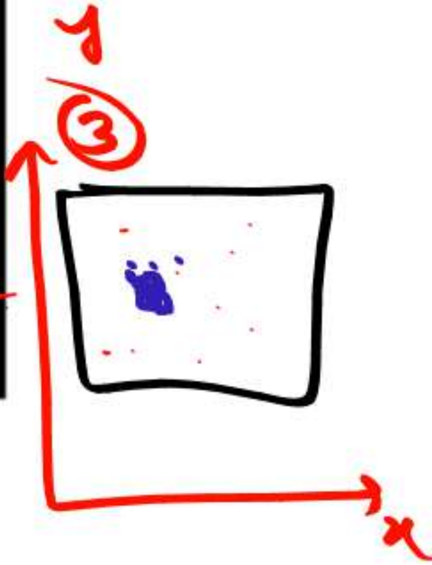



## Question Practice on Vector Integrals

## Surface Integrals




light intensity = 5 cd/m²

total light from window = 50 cd

total light = ??

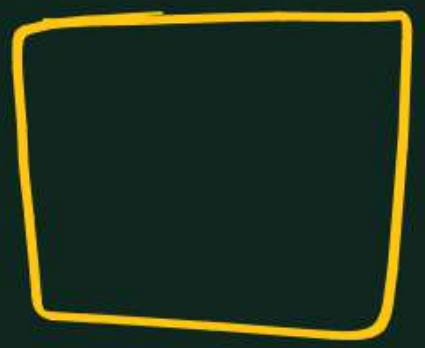
light intensity = $5x^2y$ cd/m²

total light = $50x^2y$ cd

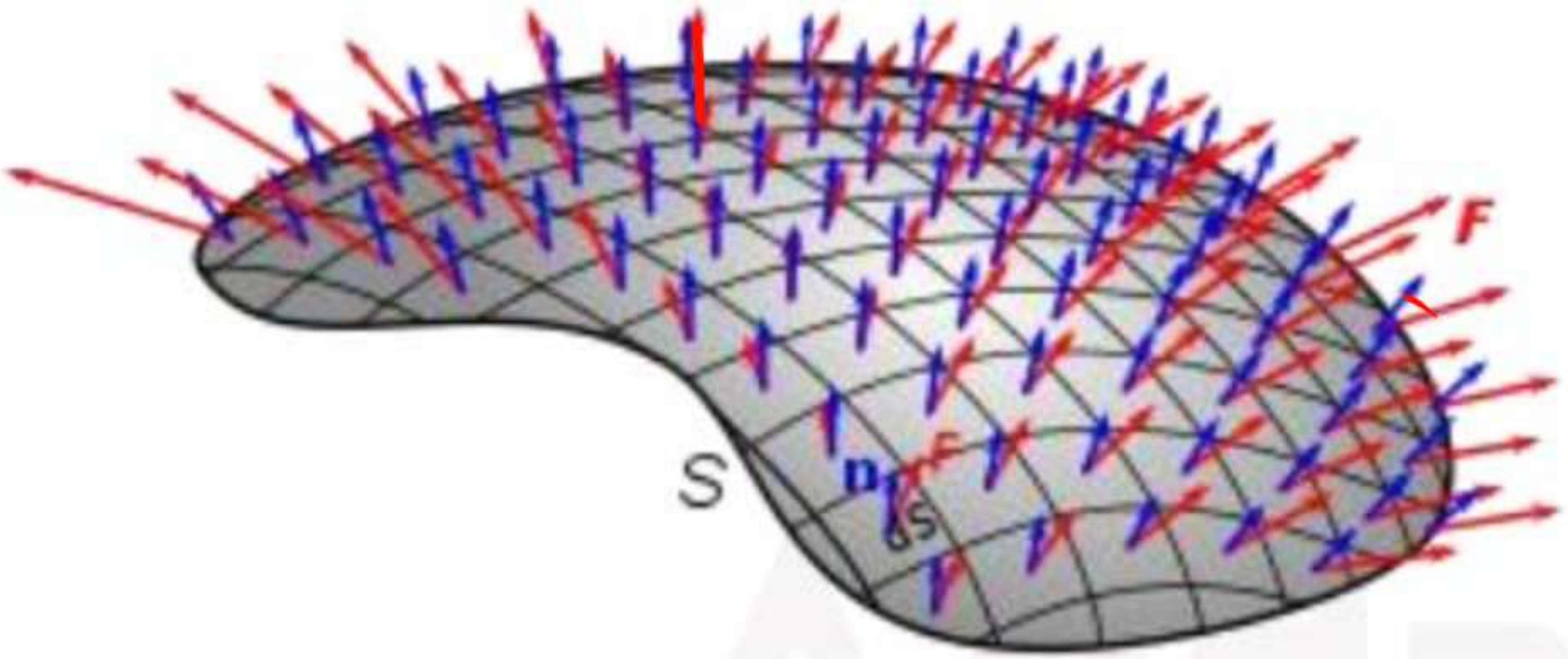
$\int \vec{A} \cdot \vec{ds}$ = total light

$$\int \vec{A} \cdot \vec{ds} = \text{flux}$$

flux density (per unit area)

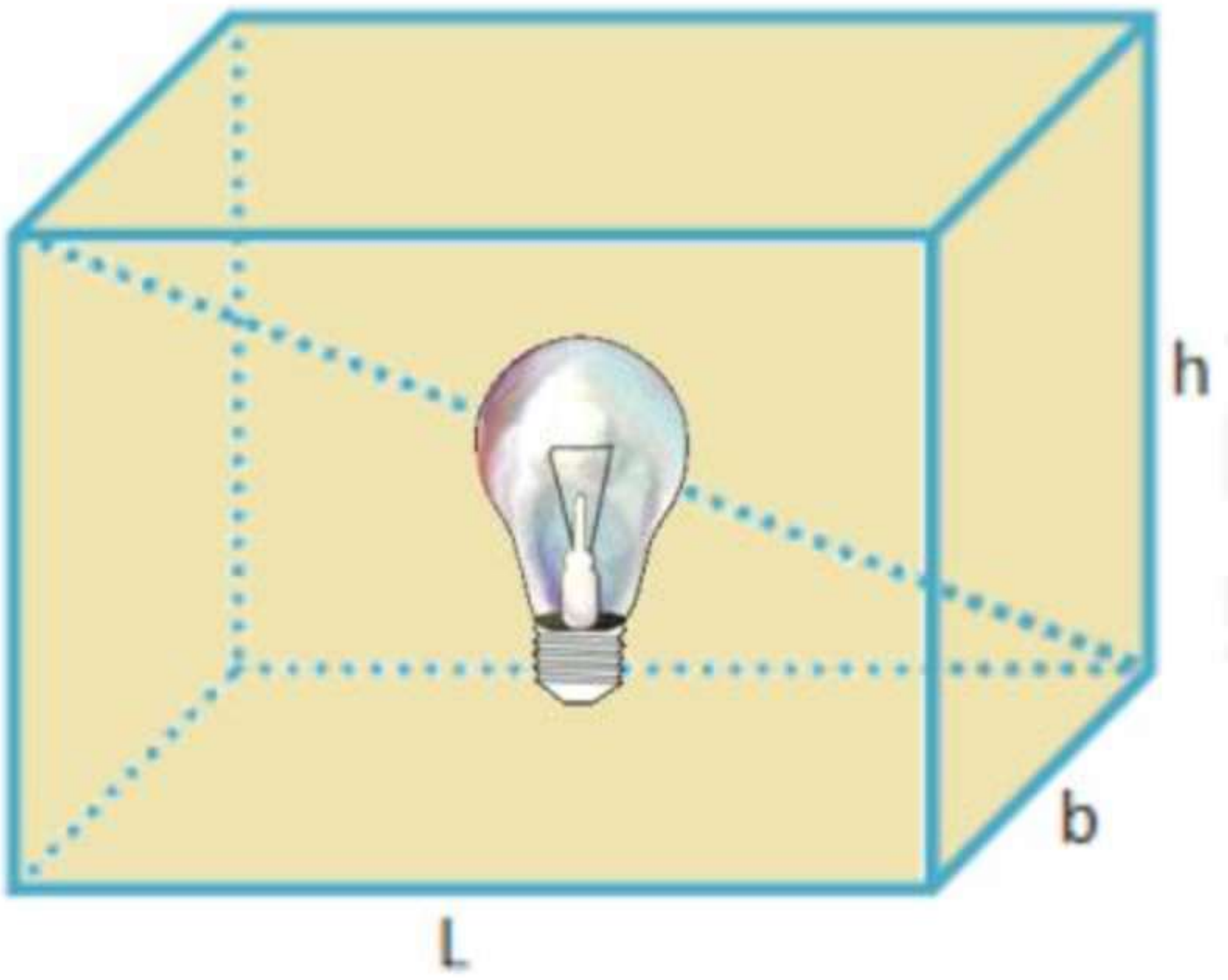


$\vec{A} \cdot \vec{ds} \rightarrow$ flux from small surface area.



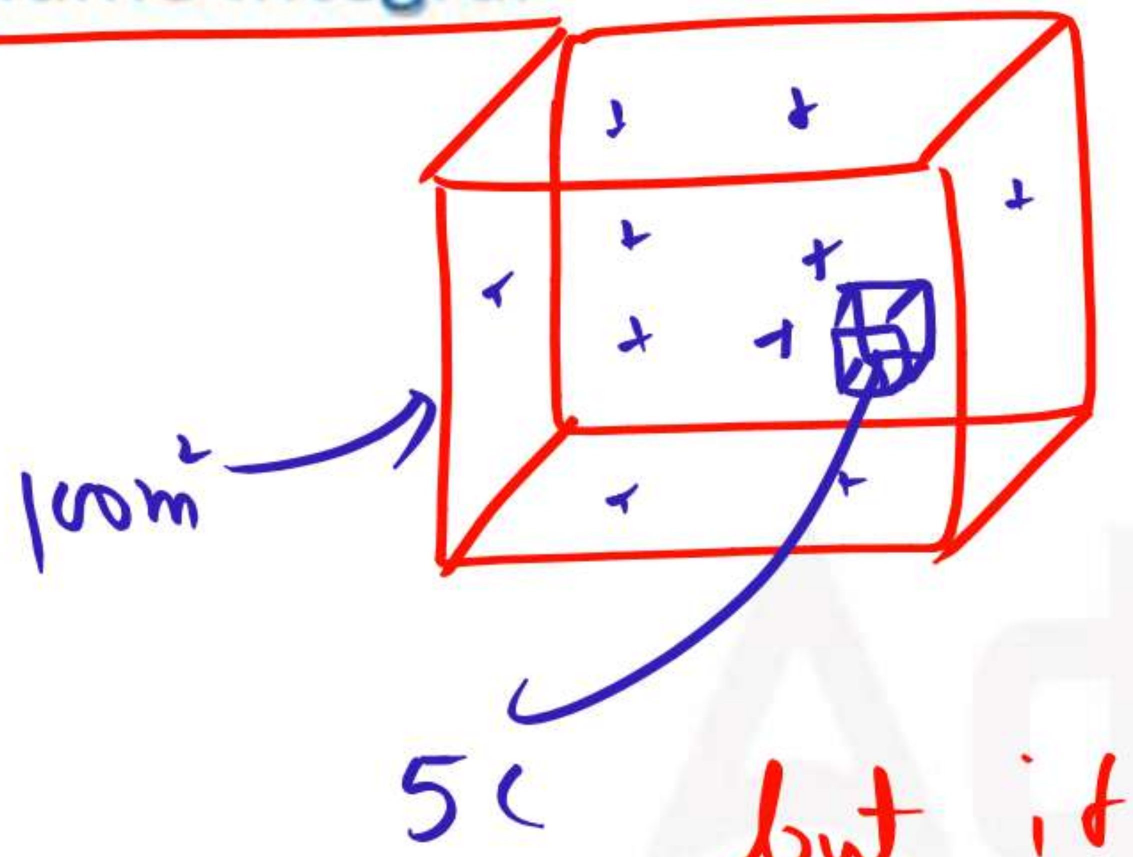

$$\int \vec{F} \cdot \vec{ds}$$

Closed surface integral




$\oint \vec{A} \cdot \vec{ds} \rightarrow$ net outward flux

## Volume Integral



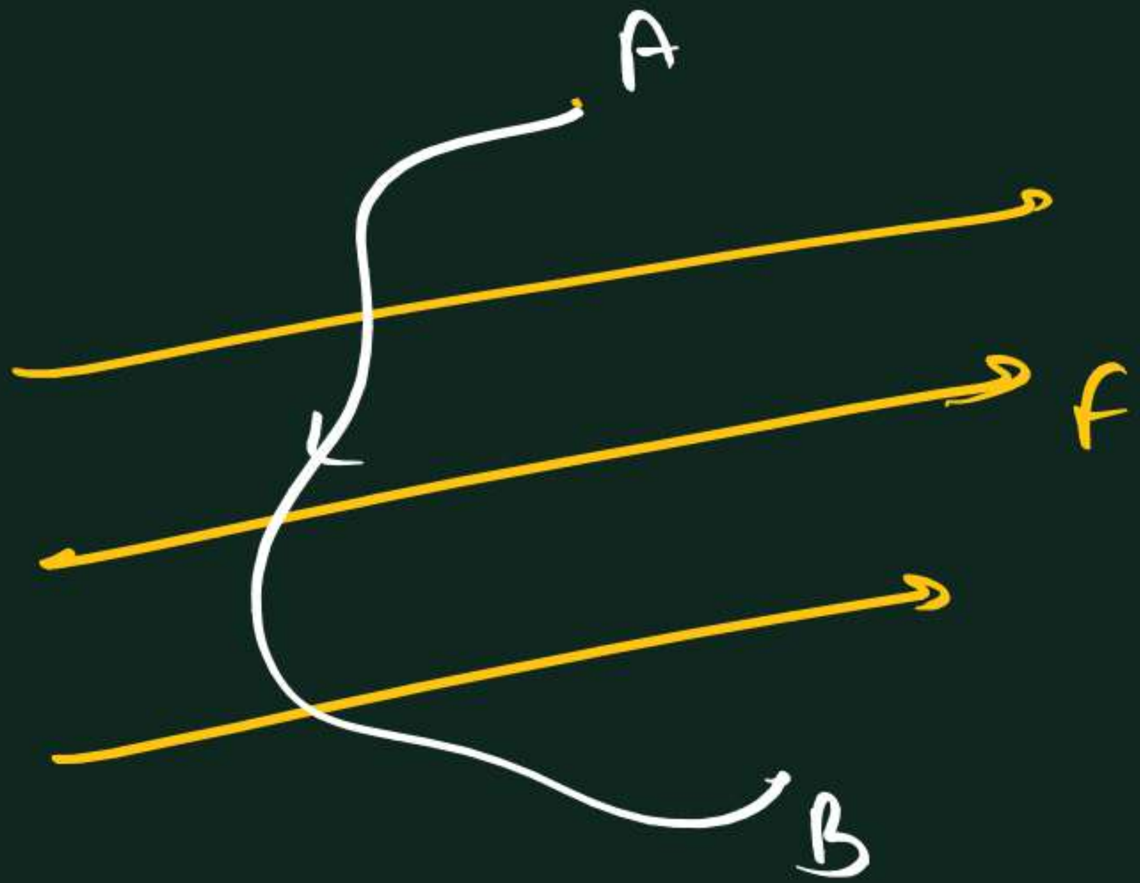

$\rho_v \rightarrow 5\,C/m^3$ charge density.

total charge = 500 Coulomb

but if $\rho_v = 5xy^2 - 2xz \; C/m^3$

amount of charge in small volume = dv = $\rho_v dv$

total enclosed charge = $\int \rho_v dv$

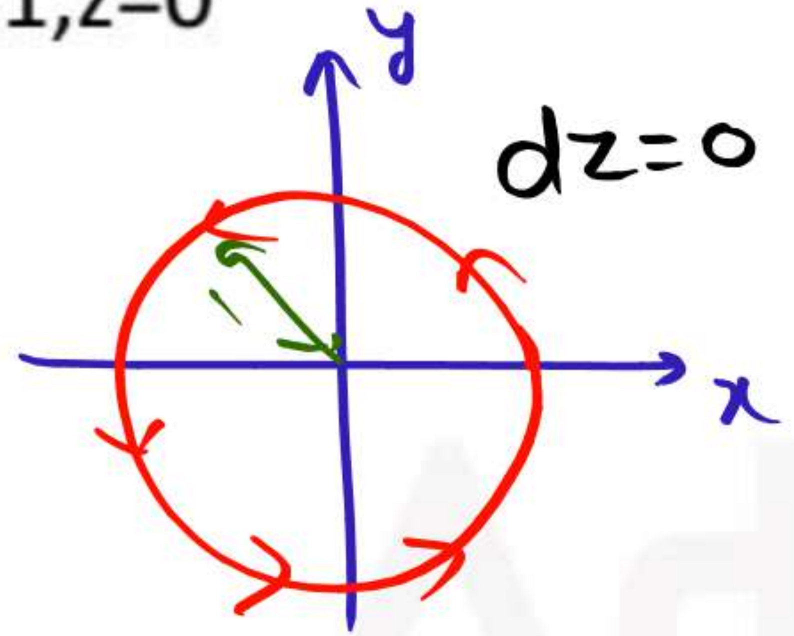



$$-\vec{F}\cdot\vec{dl} = dw$$

$$-\int_A^B \vec{F}\cdot\vec{dl}$$

Q:15 Find the circulation of $\vec{F}$ around the curve c where $\vec{F}$= yi+ zj+ xk and c is the circle x^2 + y^2 =1,z=0

Sol!




$dz=0$

$$\text{Circulation} = \oint_c \vec{F}\cdot\vec{dl}$$

$$= \oint_c (y\hat{i}+z\hat{j}+x\hat{k})\cdot(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k})$$

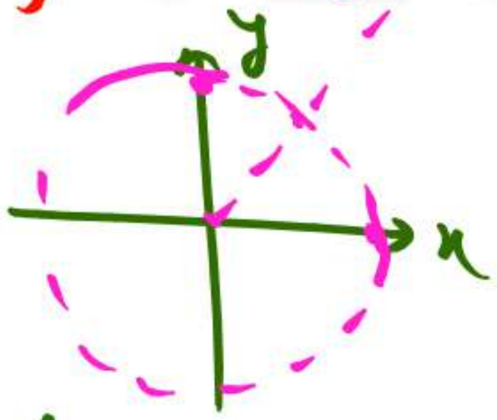
$$\oint_c y\,dx + z\,\cancel{dy} + x\,\cancel{dz}$$

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$x = r\cos\theta$

$y = r\sin\theta$

$dx = -r\sin\theta\, d\theta$




0 0

$$\oint_c y\,dx = \int_{\theta=0}^{2\pi} -\sin^2\theta\, d\theta = -\int_0^{2\pi} \frac{1-\cancel{\cos 2\theta}}{2}\, d\theta$$

0

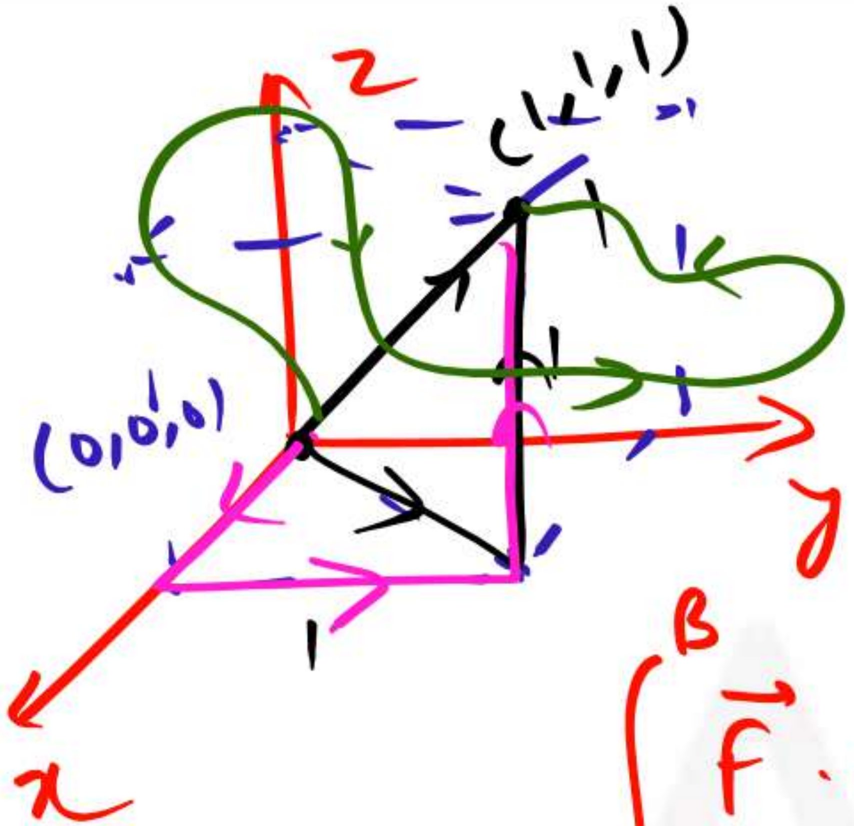
$= -\pi$

Q:16

The line integral of the vector field $\vec{F} = 5xz\hat{i} + (3x^2 + 2y)\hat{j} + x^2z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by $(t, t^2, t)$ is ________.

EE

[2016 : 2 Marks, Set-2]




$x = t \longrightarrow dx = dt$

$y = t^2 \longrightarrow dy = 2t\,dt$

$z = t \longrightarrow dz = dz$

$$\int_A^B \vec{F}\cdot\vec{dl} = \int \left(5xz\hat{i} + (3x^2+2y)\hat{j} + x^2z\hat{k}\right)\cdot\left(dx\hat{i}+dy\hat{j}+dz\hat{k}\right)$$

$$= \int_0^1 5xz\,dx + \int_0^1 (3x^2+2y)\,dy + \int_0^1 x^2z\,dz$$

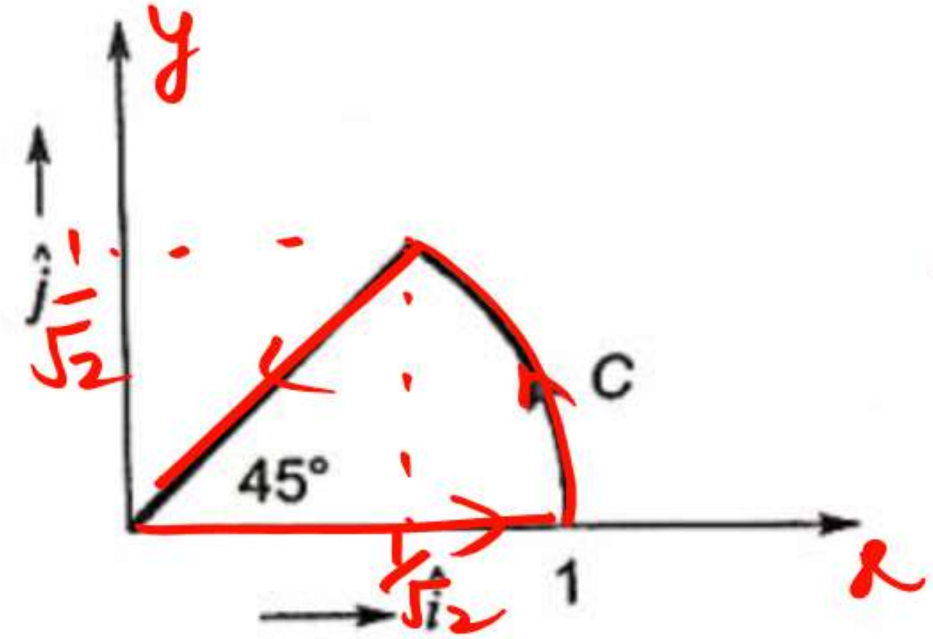
$$\int_0^1 5t^2\,dt + \int_0^1 10t^3\,dt + \int_0^1 t^3\,dt$$

$$\left(\frac{5t^3}{3}\right)_0^1 + \left(\frac{10t^4}{4}\right)_0^1 + \left(\frac{t^4}{4}\right)_0^1$$

$$\frac{5}{3} + \frac{10}{4} + \frac{1}{4}$$

$$= \frac{20+30+3}{12} = \frac{53}{12} \quad \underline{\text{Ans}}$$

Q.17 The vector function $\vec{F}(r) = -x\hat{i} + y\hat{j}$ is defined over a circular arc $C$ shown in the figure.




$\vec{F} = x\hat{i} + y\hat{j}$

$x^2 + y^2 = 1$

$\int \vec{F} \cdot \vec{dl} = \int (x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + \cancel{dz}\hat{k})$

$0$

$\int x\,dx + \int y\,dy$

$\left(\frac{x^2}{2}\right)_1^{\frac{1}{\sqrt{2}}} + \left(\frac{y^2}{2}\right)_0^{\frac{1}{\sqrt{2}}}$

$= -\frac{1}{4} + \frac{1}{4} = 0$

The line integral of $\int_C \vec{F}(r) \cdot d\vec{r}$ is

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{6}$

[2021 : 1 Mark]

method-2

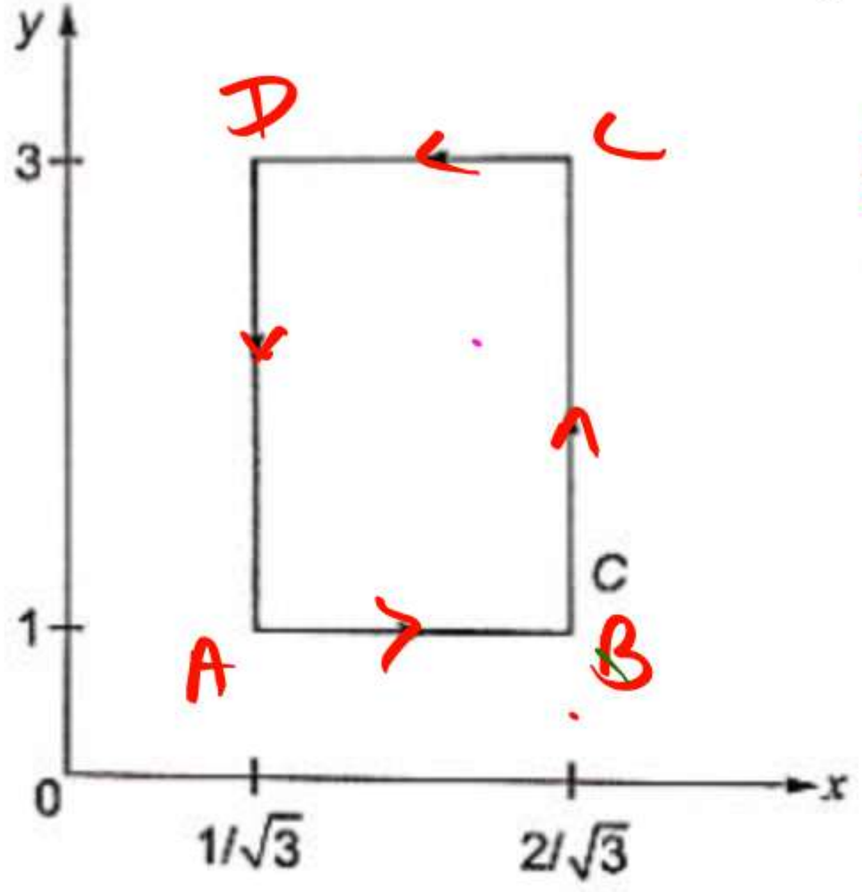
$x = \cos\theta \Rightarrow dx = -\sin\theta\, d\theta$

$y = \sin\theta \Rightarrow dy = \cos\theta\, d\theta$

$$= \int_{\theta=0}^{\pi/4} -\cancel{\cos\theta} \sin\theta\, d\theta + \int_{\theta=0}^{\pi/4} \cancel{\sin\theta} \cos\theta\, d\theta$$

$= 0$

Q.18 If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$, $\oint_C \vec{A}\cdot d\vec{l}$ over the path shown in the figure is




(a) 0

(b) $\frac{2}{\sqrt{3}}$

(c) 1

(d) $2\sqrt{3}$

[2010 : 2 Marks]

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$\vec{A} = xy\,\hat{a}_x + x^2\,\hat{a}_y$$

$$\oint \vec{A}\cdot\vec{dl} = \int_A^B \vec{A}\cdot\vec{dl} + \int_B^C \vec{A}\cdot\vec{dl} + \int_C^D \vec{A}\cdot\vec{dl} + \int_D^A \vec{A}\cdot\vec{dl}$$

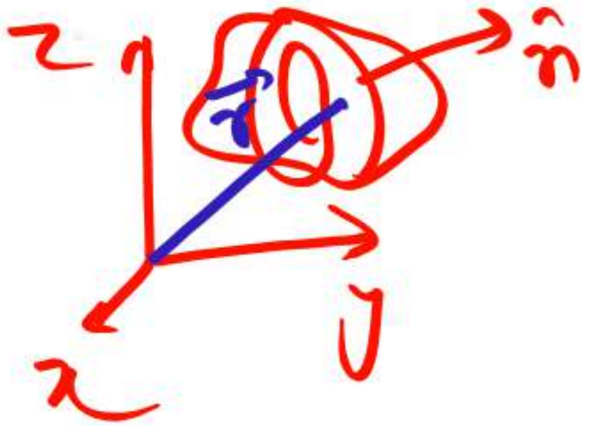
$$= \int_{1/\sqrt{3}}^{2/\sqrt{3}} xy\,dx\Big|_{y=1} + \int_1^3 x^2\,dy\Big|_{x=2/\sqrt{3}} + \int_{2/\sqrt{3}}^{1/\sqrt{3}} xy\,dx\Big|_{y=3} + \int_{3}^{1} x^2\,dy\Big|_{x=1/\sqrt{3}}$$

$$= \left(\frac{x^2}{2}\right)_{1/\sqrt{3}}^{2/\sqrt{3}} + \frac{4}{3}(y)_1^3 + 3\left(\frac{x^2}{2}\right)_{2/\sqrt{3}}^{1/\sqrt{3}} + \frac{1}{3}(y)_3^1$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

Q.19 Consider a closed surface $S$ surrounding a volume $V$. If $\vec{r}$ is the position vector of a point inside $S$, with $\hat{n}$ the unit normal on $S$, the value of the integral

$$\oiint_S 5\vec{r}\cdot\hat{n}\,dS \text{ is}$$




$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = r\hat{a}_r$

(a) 3 V
(b) 5 V
(c) 10 V
(d) 15 V

[2011 : 1 Mark]

$$\oint 5\vec{r}\cdot\hat{n}\,ds$$

$$5\int r\,\hat{a}_r\cdot r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$$

$$5\int r^3\sin\theta\,d\theta\,d\phi$$

$$5r^3\left(-\cos\theta\right)_0^{\pi}\left(\phi\right)_0^{2\pi}$$

$$10\pi r^3\cdot\left(-(-1)+1\right)$$

$$20\pi r^3 = \frac{3}{3}\times\frac{4}{4}\times 20\pi r^3 = 15\times\frac{4}{3}\pi r^3$$

$$= 15V$$

Q:20.

Consider the time-varying vector $I = \hat{x}15\cos(\omega t) + \hat{y}5\sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle $\theta$ that $I$ makes with the $x$ axis (in degrees, such that $0 \le \theta \le 180$) is 90°.

[2016 : 1 Mark, Set-2]

$\vec{I} = 15\cos\omega t\,\hat{i} + 5\sin\omega t\,\hat{j}$

$|\vec{I}| = \sqrt{225\cos^2\omega t + 25\sin^2\omega t}$

$|\vec{I}| = \sqrt{25 + 200\cos^2\omega t}$

for $|\vec{I}|$ to be minimum $\cos^2\omega t = 0$

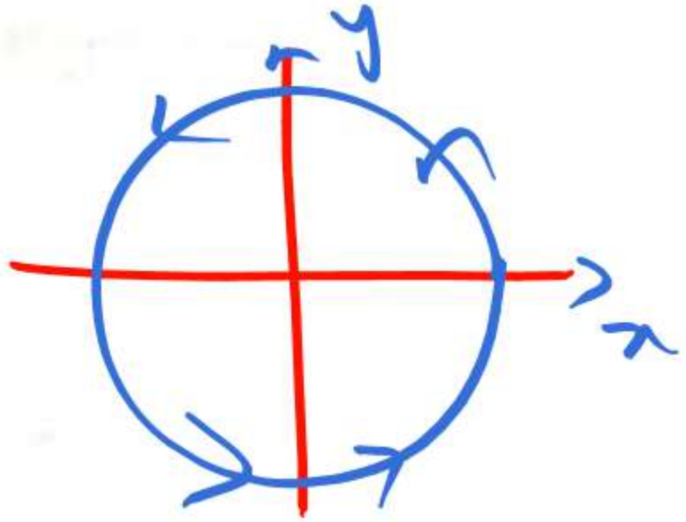
$\omega t = 90^\circ$

$\vec{I} = 15(0)\hat{i} + 5\hat{j} = 5\hat{j}$

angle of $\vec{I}$ with x-axis $= \underline{90^\circ}$

Q:21

Evaluate $\int_c \vec{F}\cdot d\vec{r}$ where $\overline{F} = \dfrac{y\hat{i} - x\hat{j}}{x^2+y^2}$ & C is:




(i) Circular path $x^2+y^2=1$ described counter clockwise.

(ii) The square formed by the lines $x=\pm 1,\ y=\pm 1$, counter clockwise.

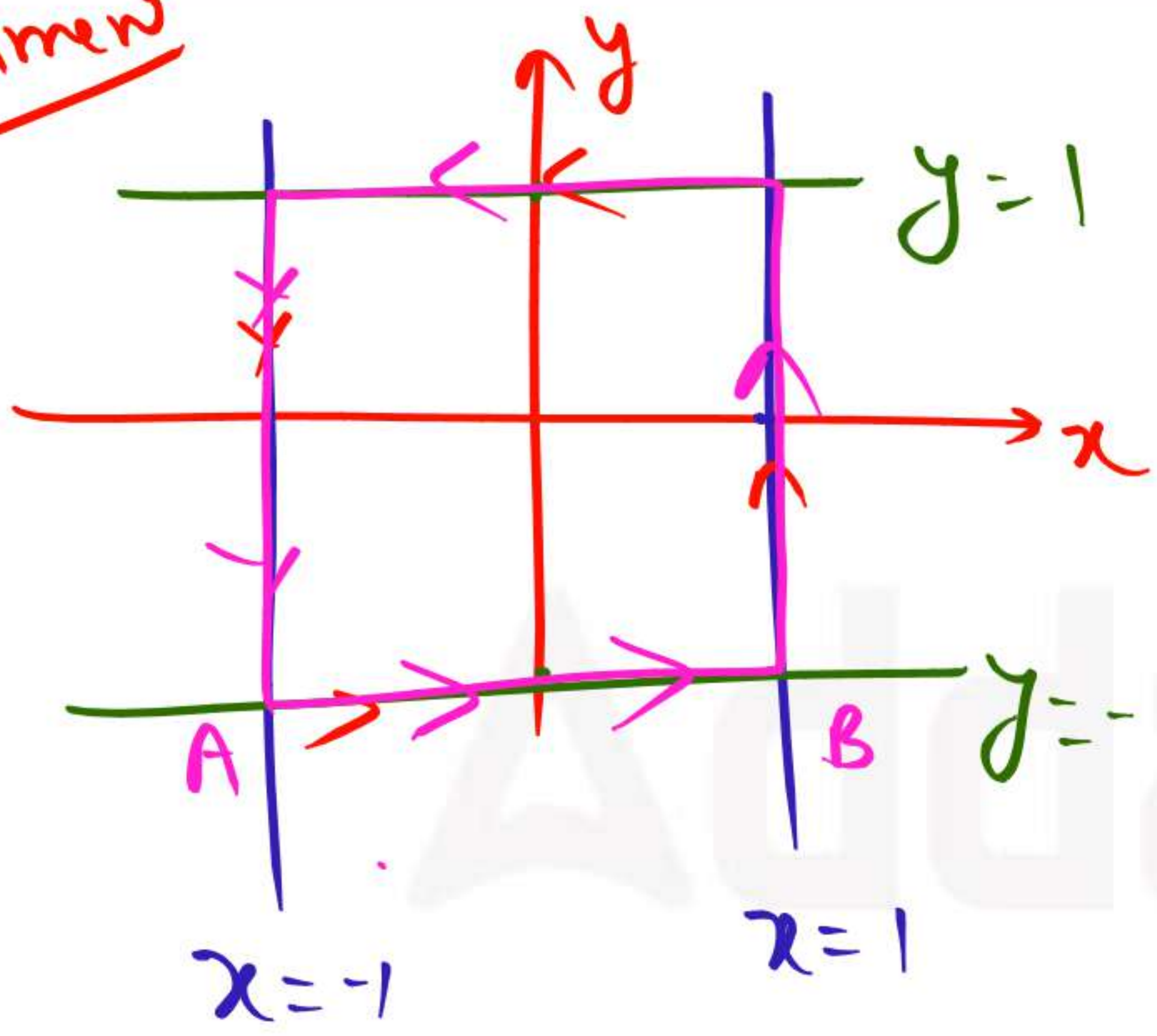
(i) $\oint_c \dfrac{y\,dx - x\,dy}{x^2+y^2}$

$dx = -\sin\theta\, d\theta \Leftarrow x = \cos\theta$

$dy = \cos\theta\, d\theta \Leftarrow y = \sin\theta$

$= \displaystyle\int_{\theta=0}^{2\pi} \frac{-\sin^2\theta\, d\theta - \cos^2\theta\, d\theta}{} = -\int_0^{2\pi} d\theta = -(\theta)_0^{2\pi} = -2\pi$

Assignment



Q:22

Find the work done in moving a particle once round the circle $x^2 + y^2 = a^2, z = 0$ under the force field given by $F = (2x - y + z)\hat{i} + (x + y + z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$.

Sol:

$$\oint \vec{F} \cdot \vec{dr}$$

$x = a\cos\theta \Rightarrow dx = -a\sin\theta d\theta$

$y = a\sin\theta \Rightarrow dy = a\cos\theta d\theta$

$$\int (2x - y + \cancel{z})dx + \int (x + y + \cancel{z^2})dy$$

$0 \qquad z = 0$

$$\int (2x - y)dx + \int (x + y)dy$$

$$\int_0^{2\pi} (2a\cos\theta - a\sin\theta)a\sin\theta d\theta + \int_{\theta=0}^{2\pi} (a\cos\theta + a\sin\theta)a\cos\theta d\theta$$

# GATE 2024

Th, Fr. Sat → 9pm

EMFT → ECE & EE.

प्रचण्ड Batch

Sat & Sund

3p.m

## Engineering Mathematics

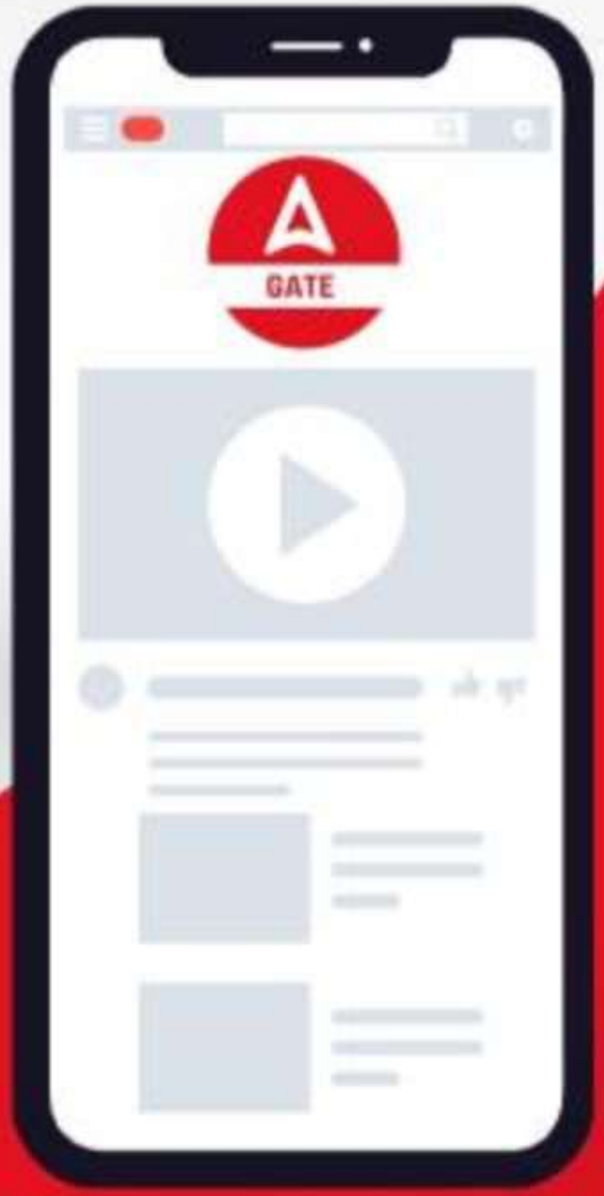
Rank

# LINEAR ALGEBRA

**Question practice on basics of matrices**

18-March

11 A.M.

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