Del Operator

Gradient and its Applications

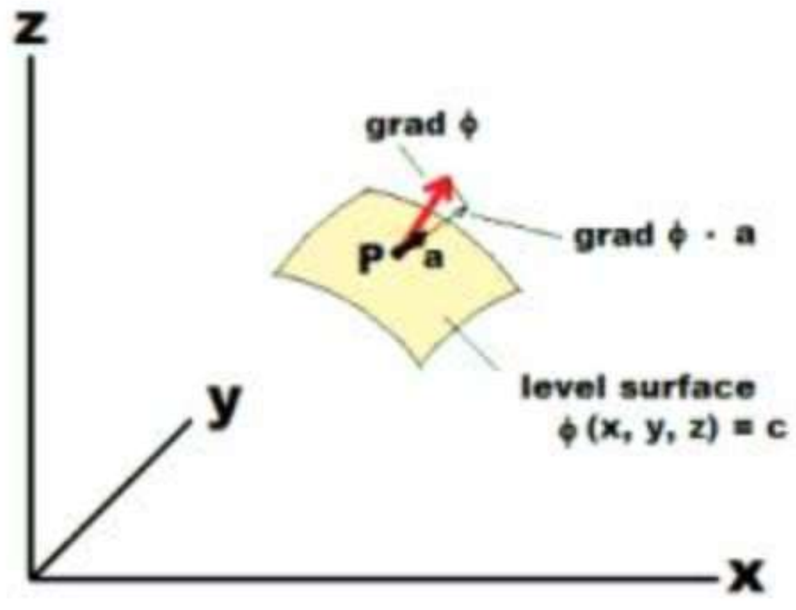
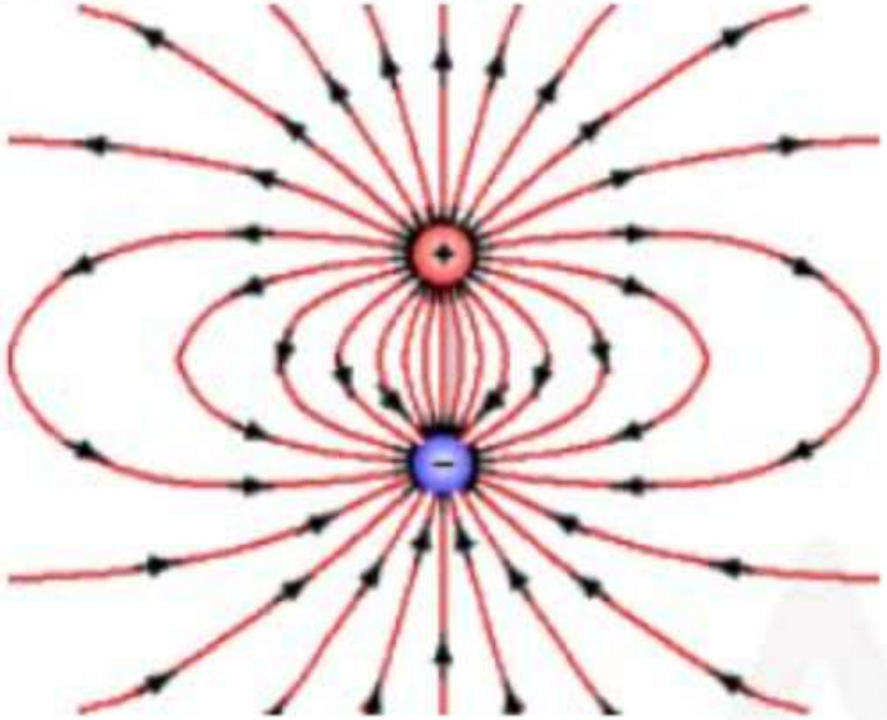
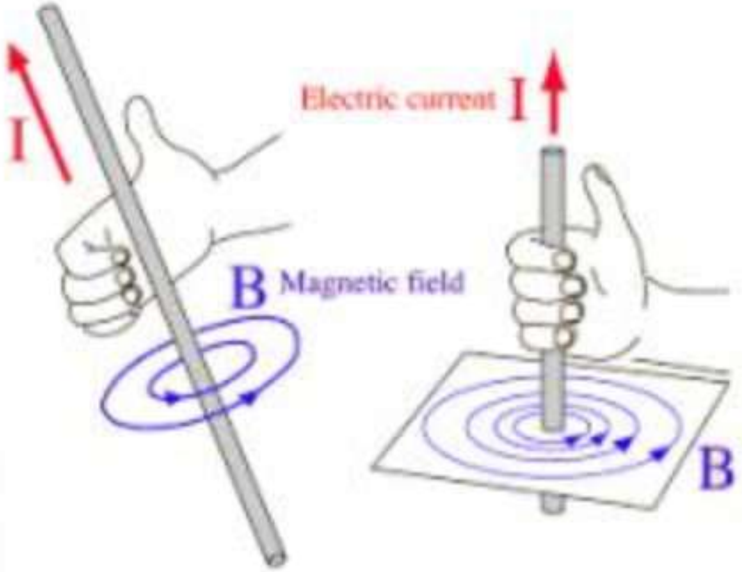
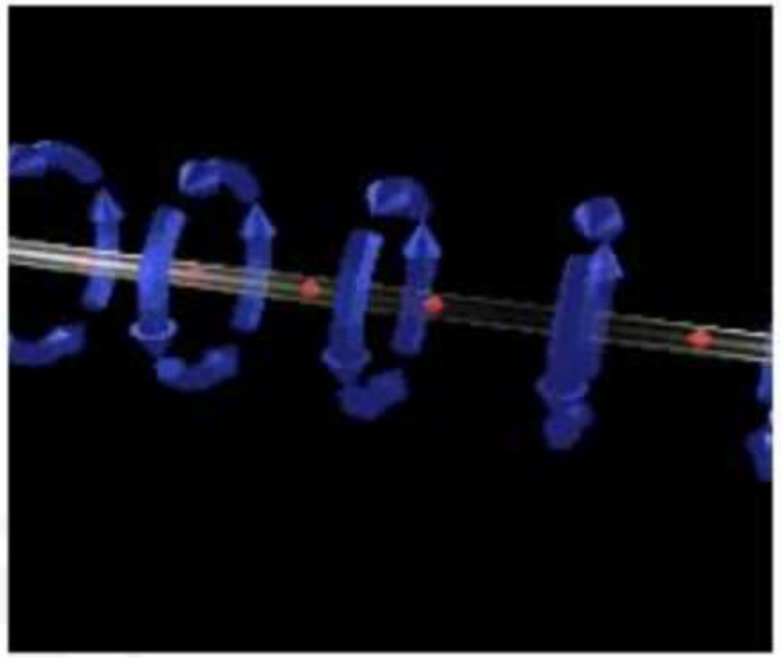


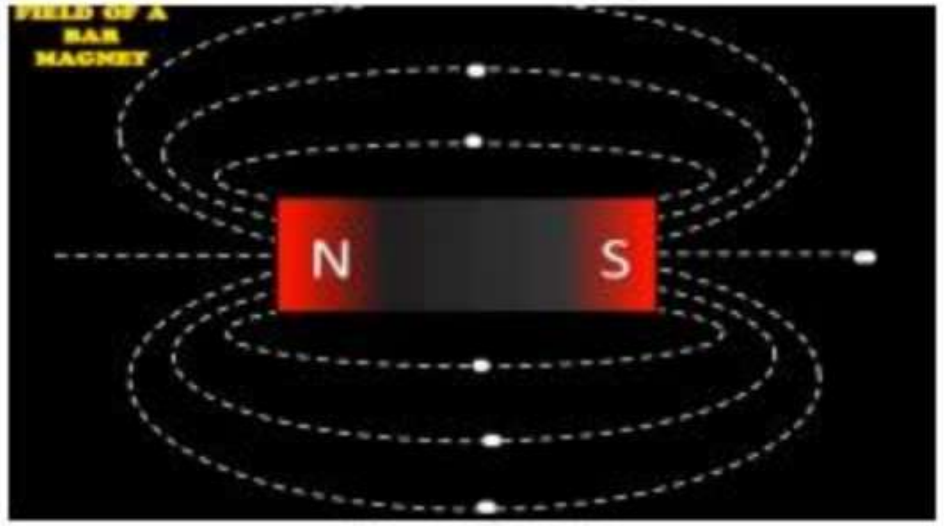
Fig. 1

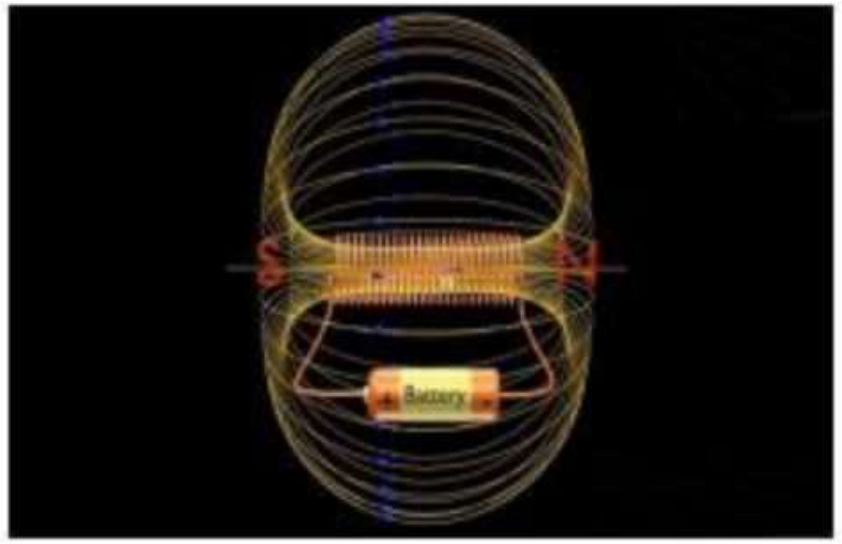
Recap

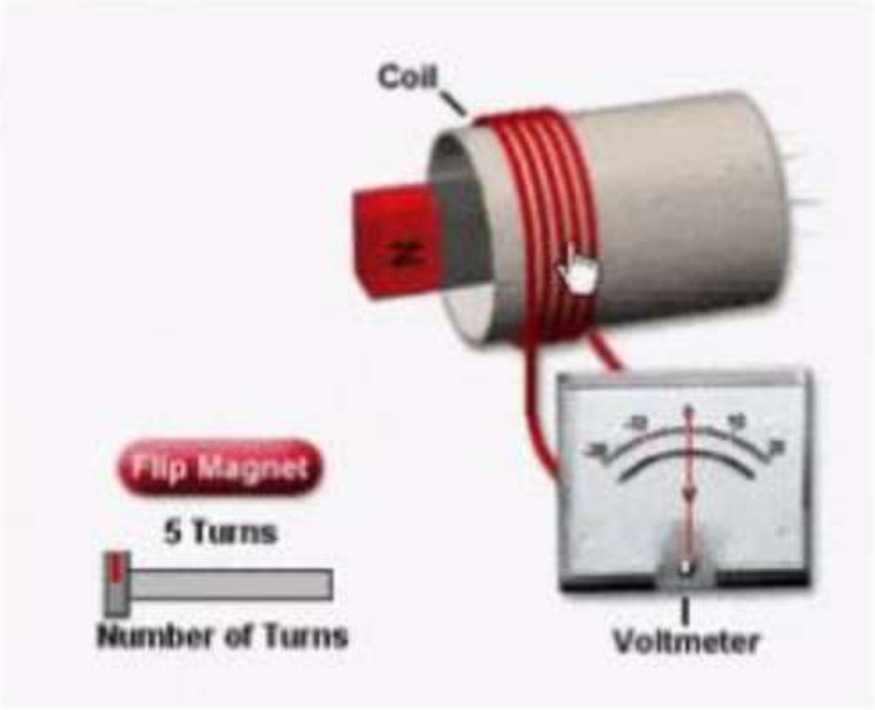



I
Electric current I
B Magnetic field
B

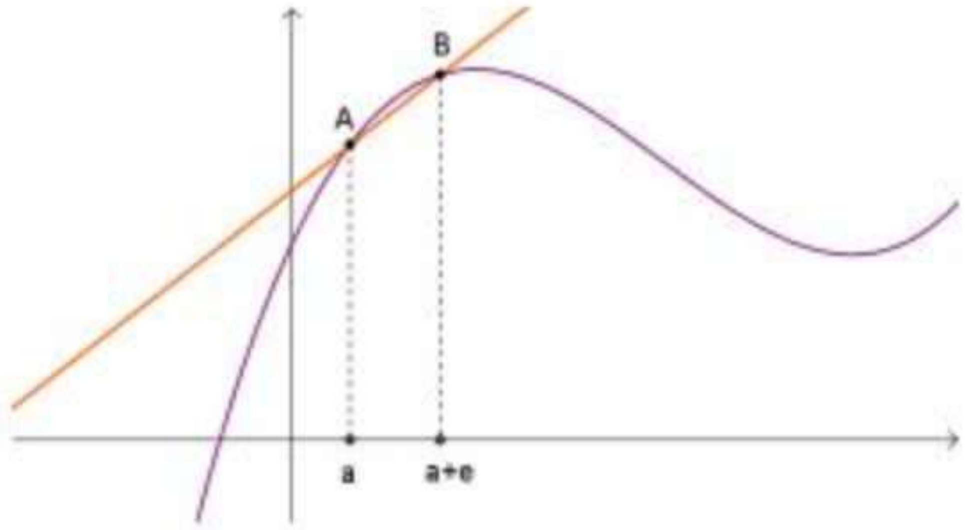


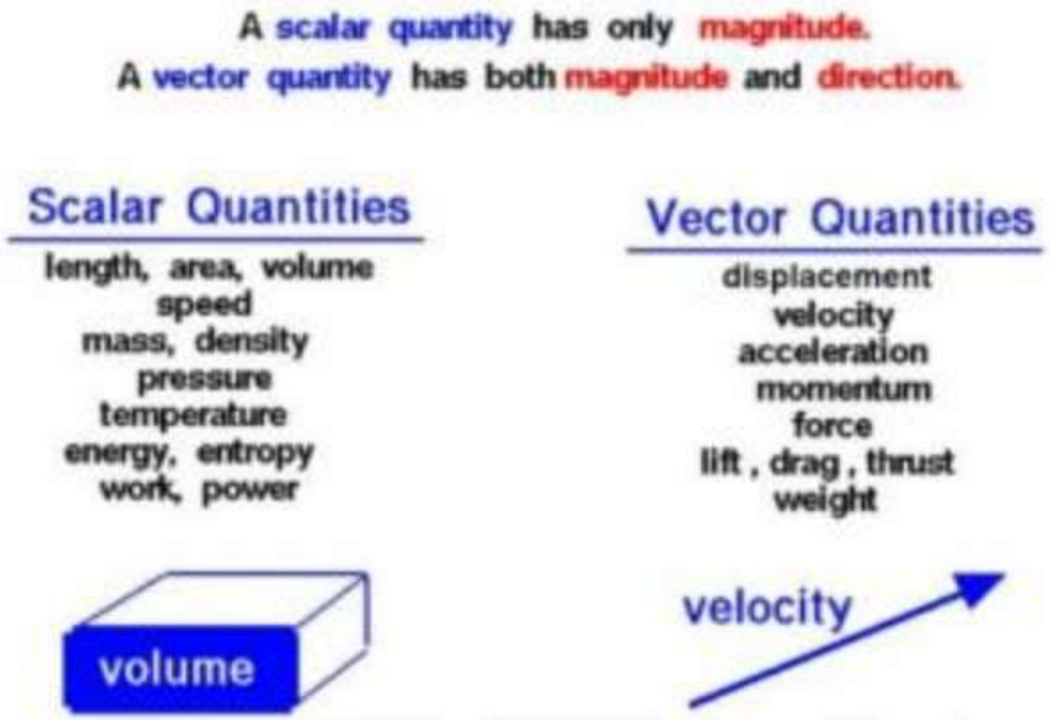

FIELD OF A BAR MAGNET
N
S

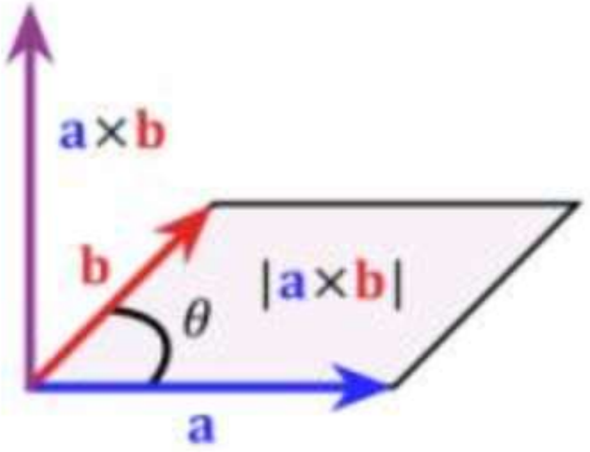


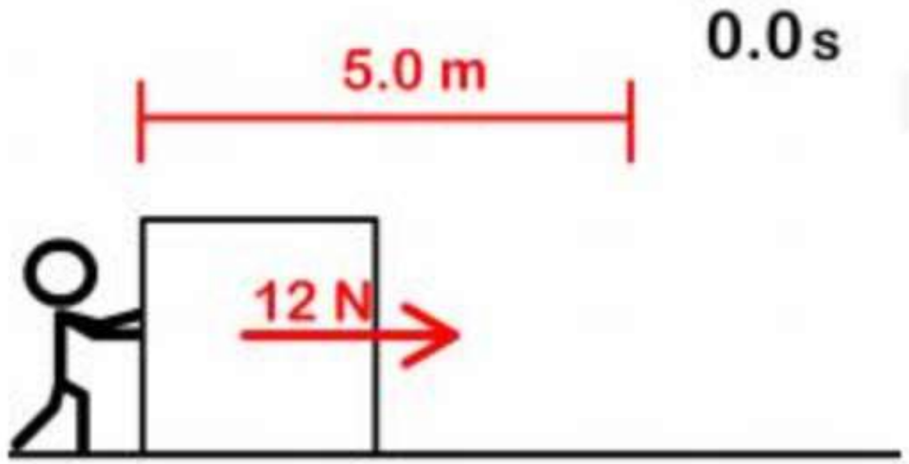

Coil
Flip Magnet
5 Turns
Number of Turns
Voltmeter

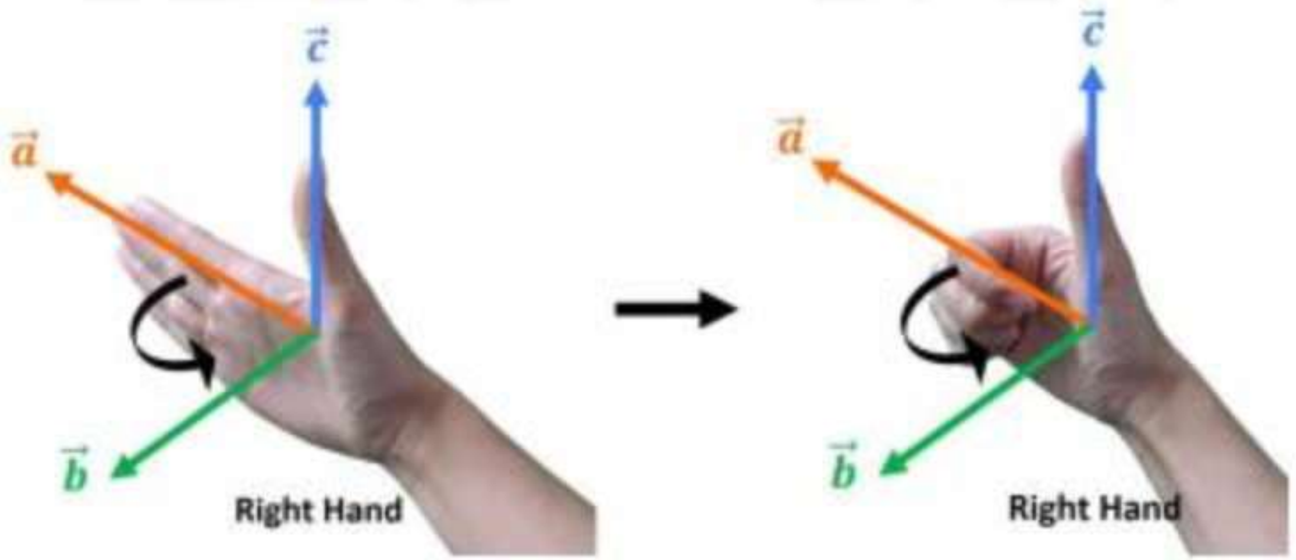
Recap

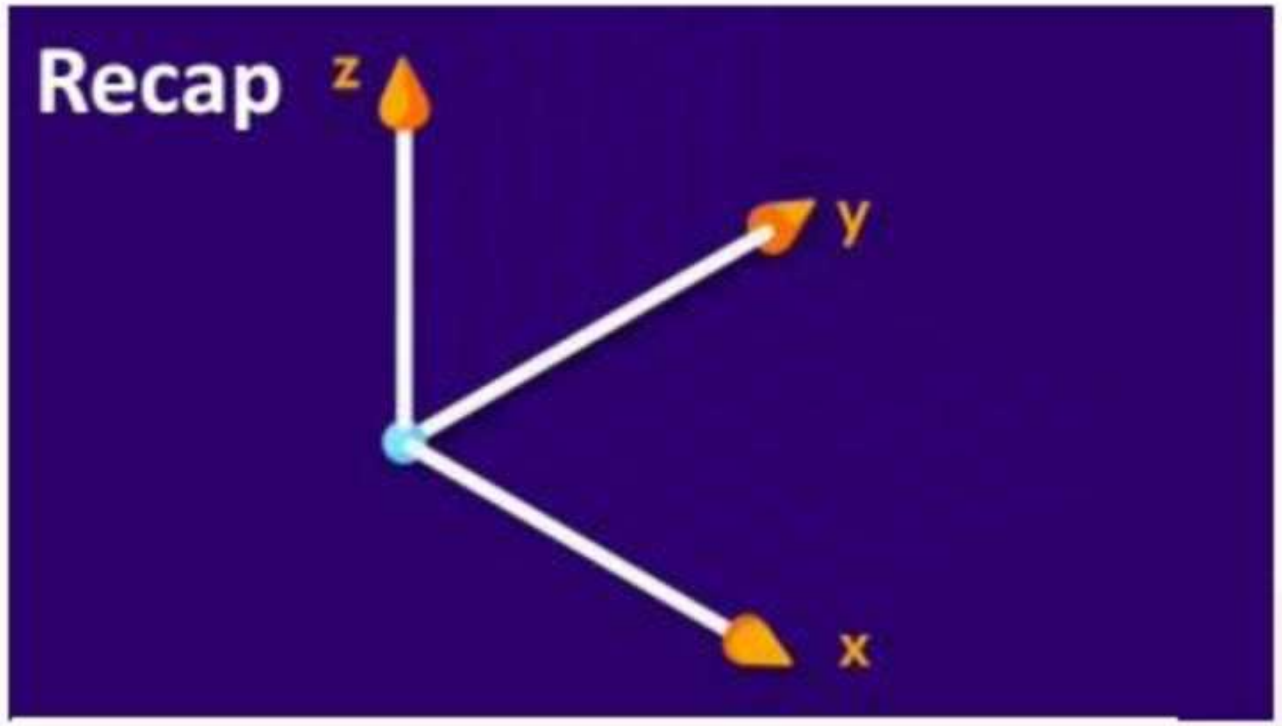

B
A
a
a+e

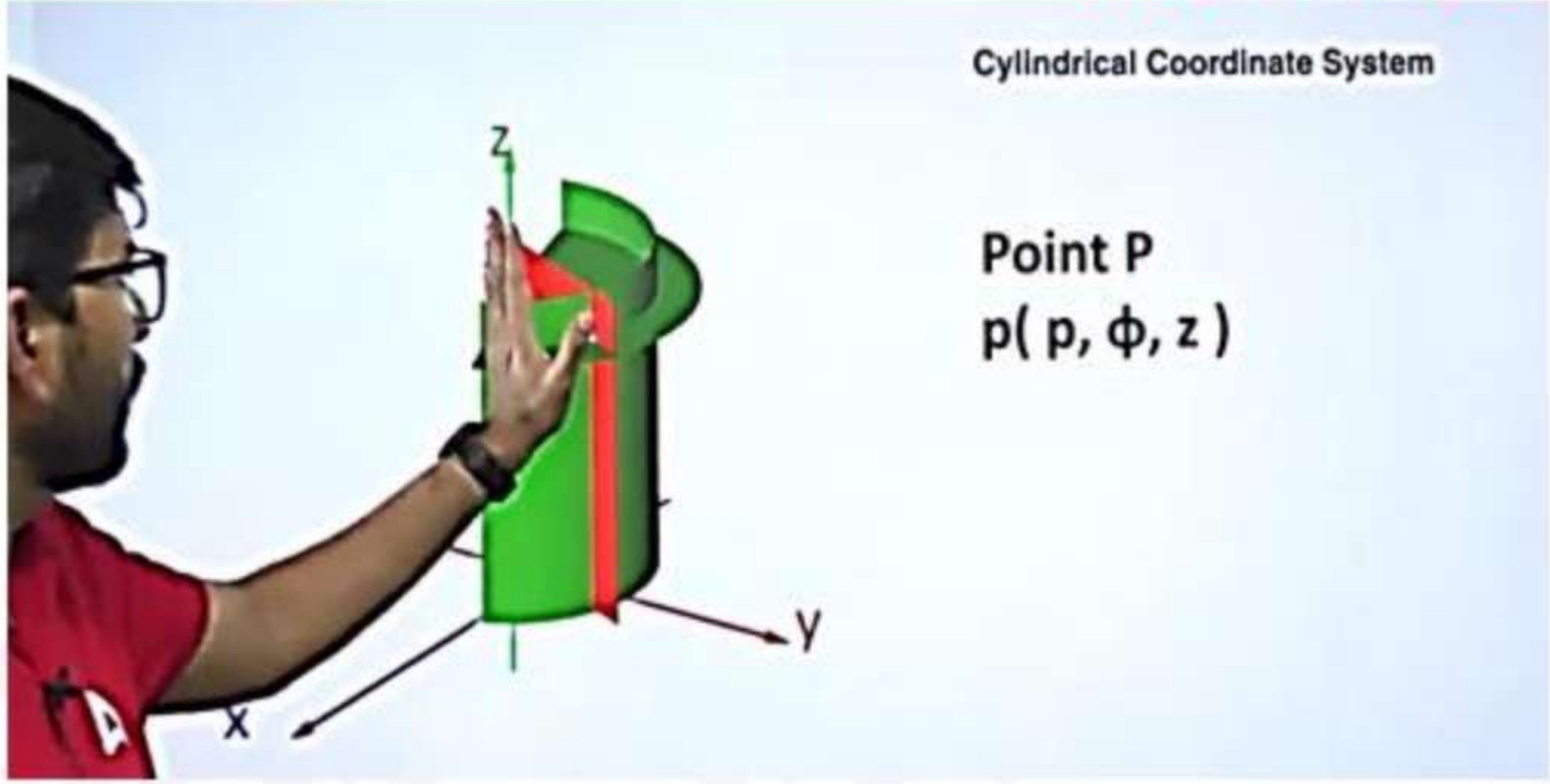

A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.
Scalar Quantities
length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power
Vector Quantities
displacement
velocity
acceleration
momentum
force
lift, drag, thrust
weight
volume
velocity

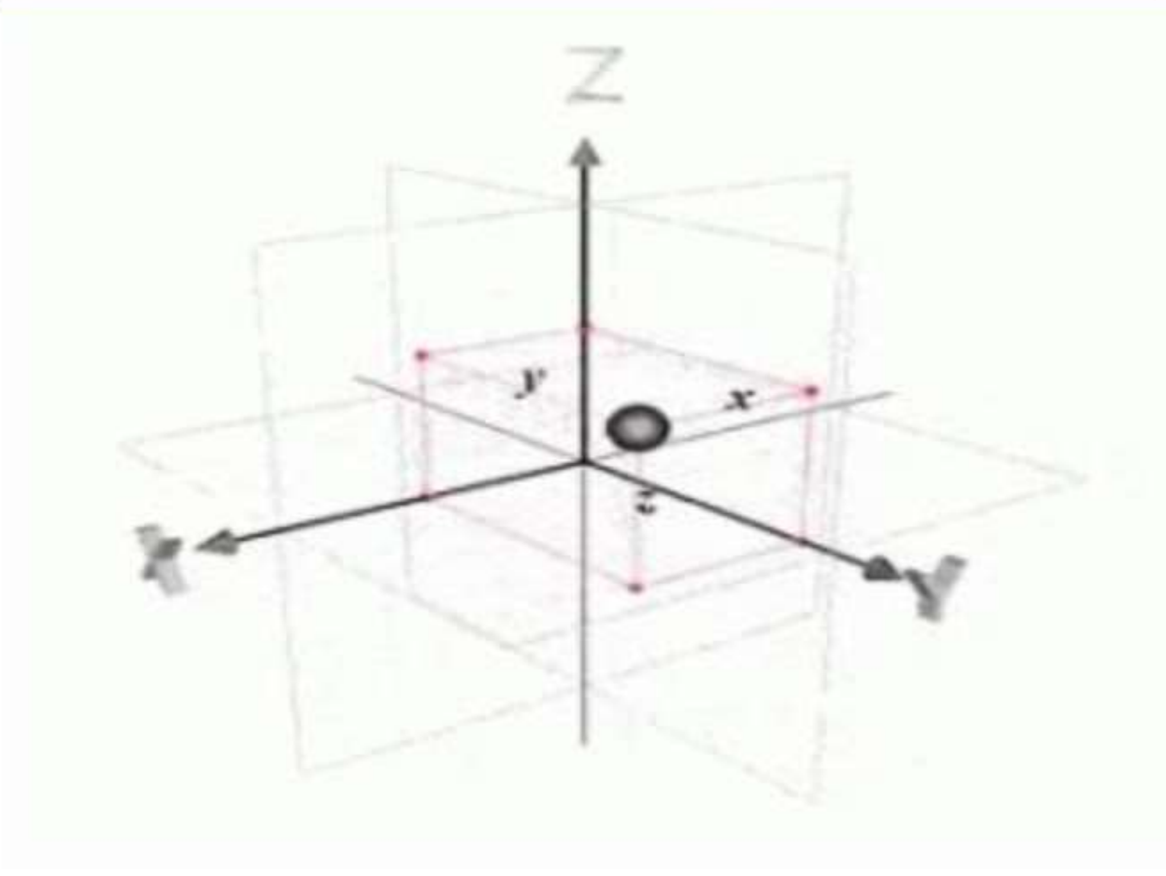

a × b
b
θ
|a × b|
a

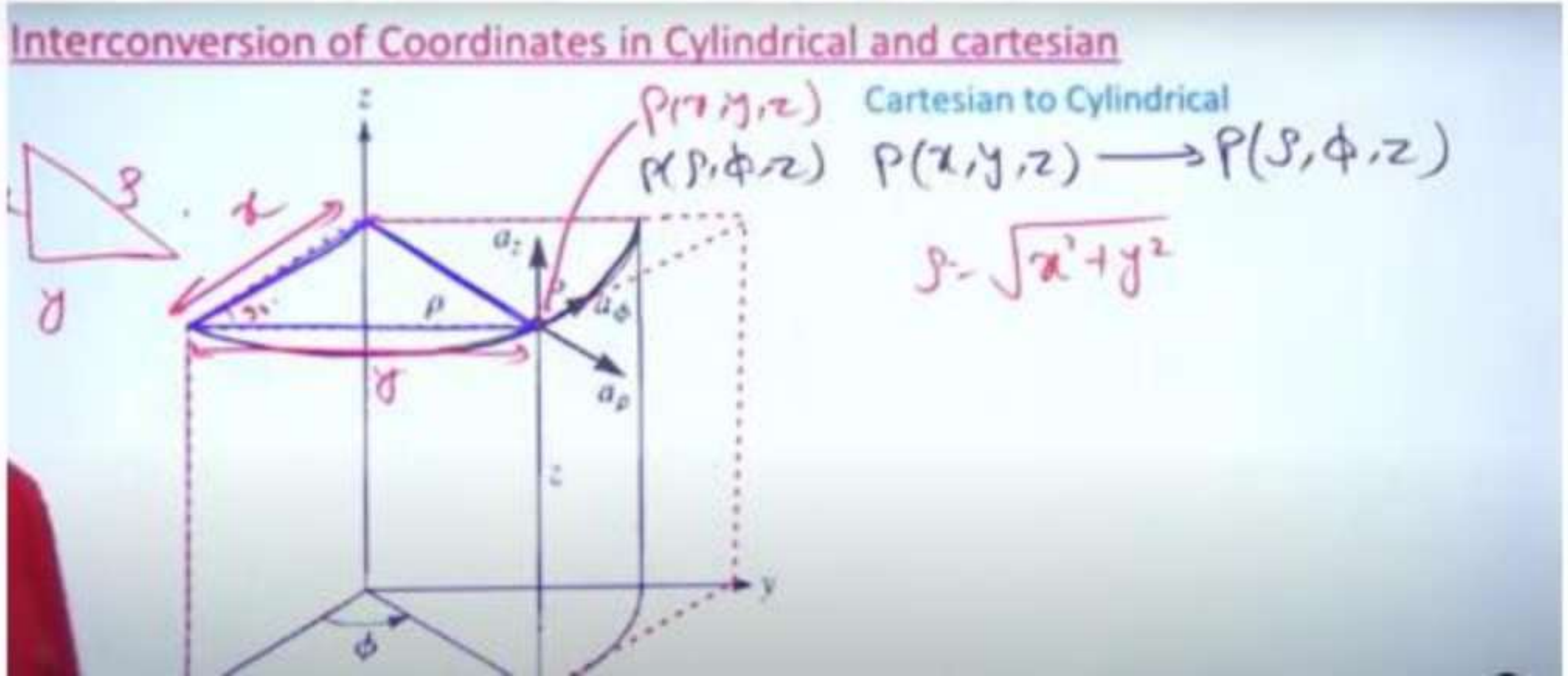

0.0 s
5.0 m
12 N


c
a
b
Right Hand
c
a
b
Right Hand


Recap
z
y
x


Cylindrical Coordinate System
Z
Point P
p(p, ϕ, z)
y
x


Z
y
x
z
X
Y


Interconversion of Coordinates in Cylindrical and cartesian
Cartesian to Cylindrical
P(x,y,z) ⟶ P(ρ,ϕ,z)
ρ = √(x² + y²)
x
y
ρ
ϕ
z

Cylindrical Coordinate System

Cylindrical Coordinate System

z

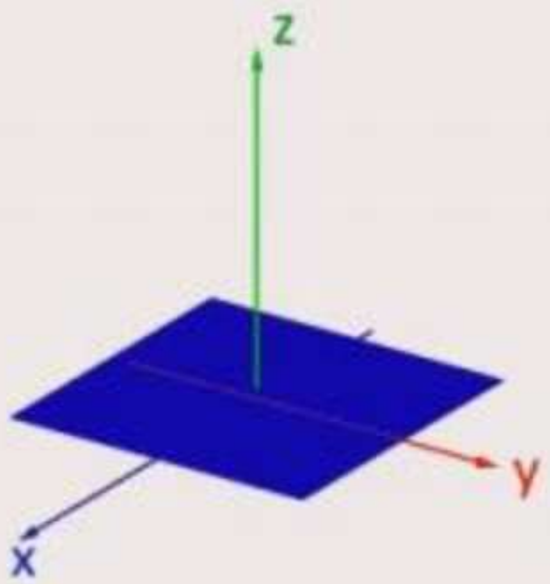
y

x

Constant r surface

$R \geq 0^{o}$

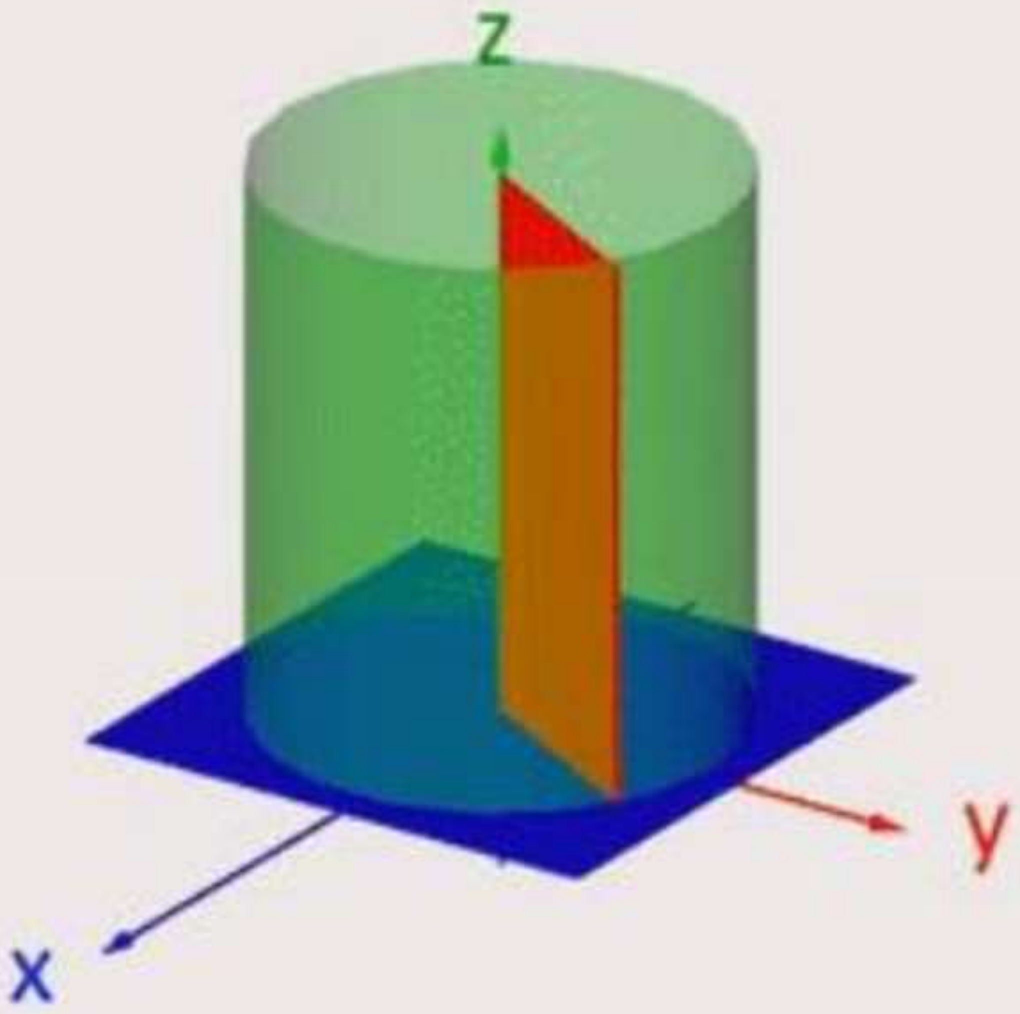
Cylindrical Coordinate System



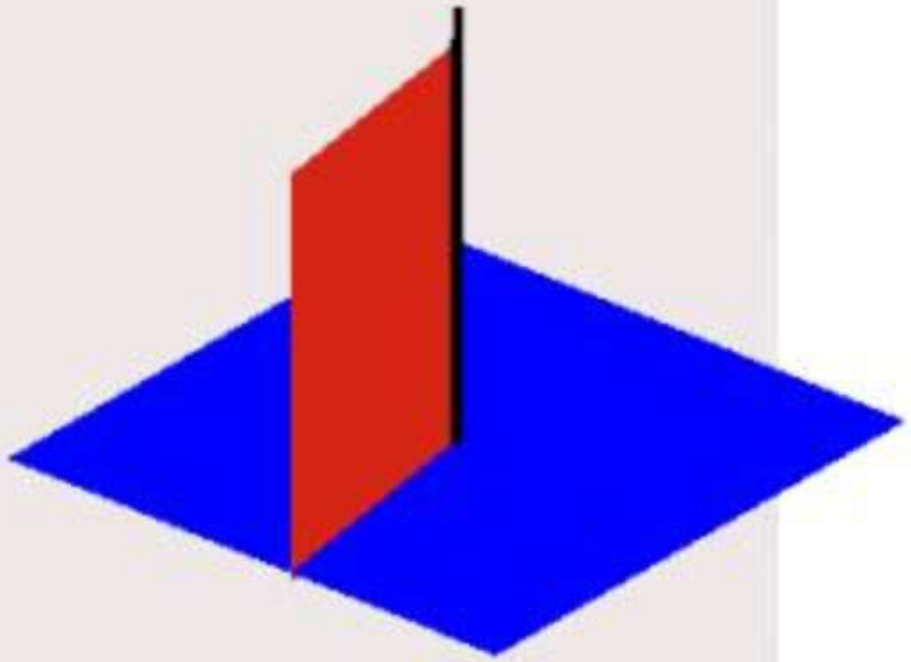
Constant z surface

$-\infty \leq Z \leq +\infty$

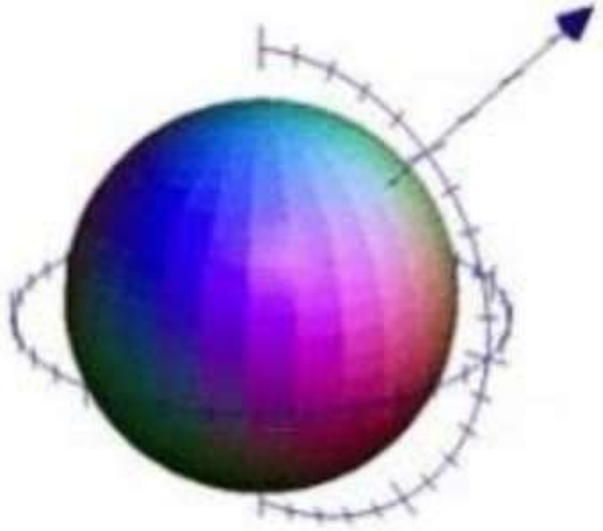
Cylindrical Coordinate System

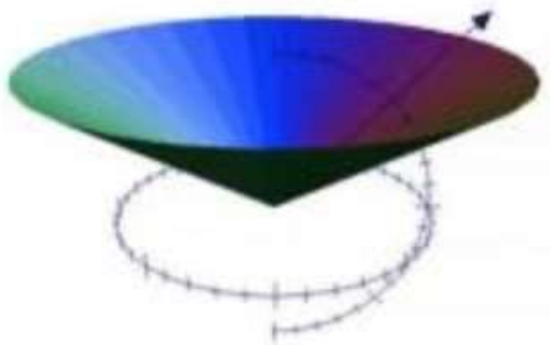


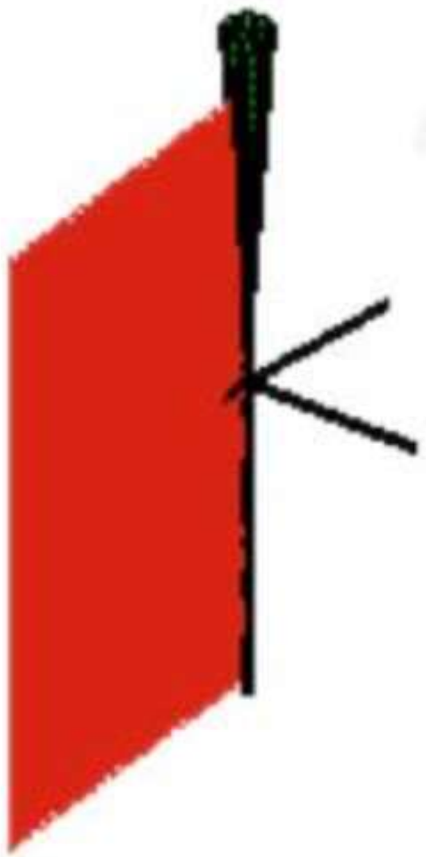
Cylindrical coordinate system planes

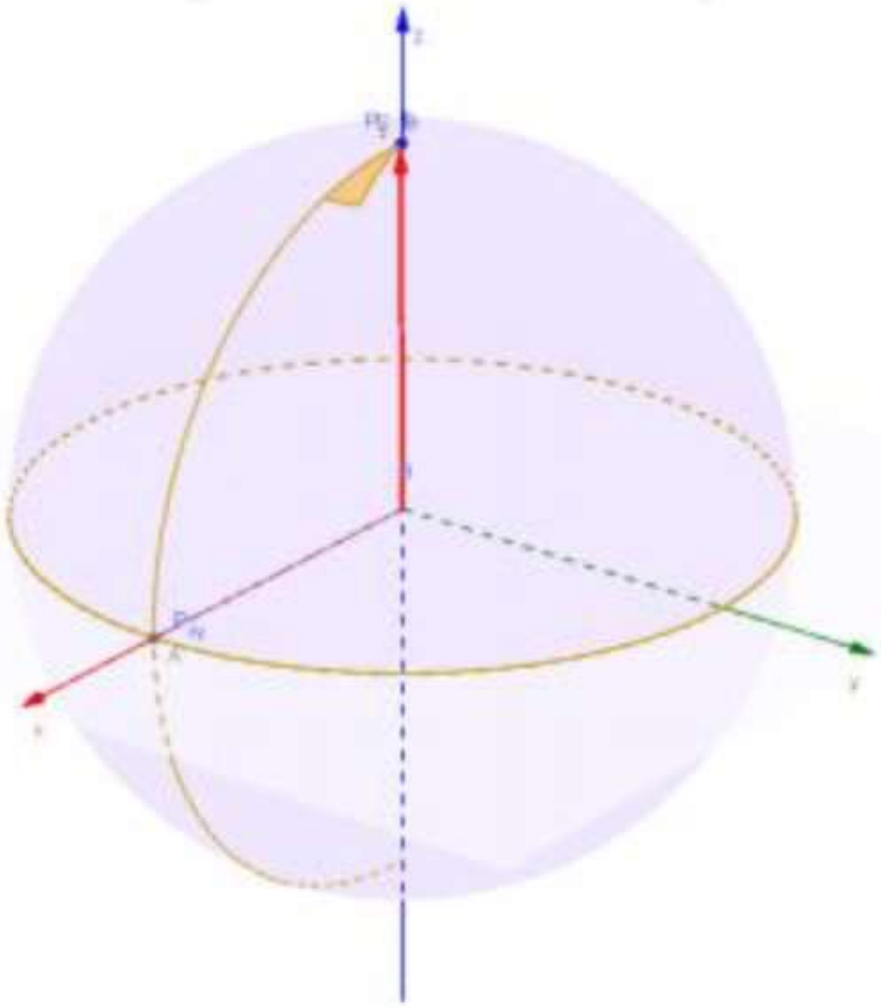


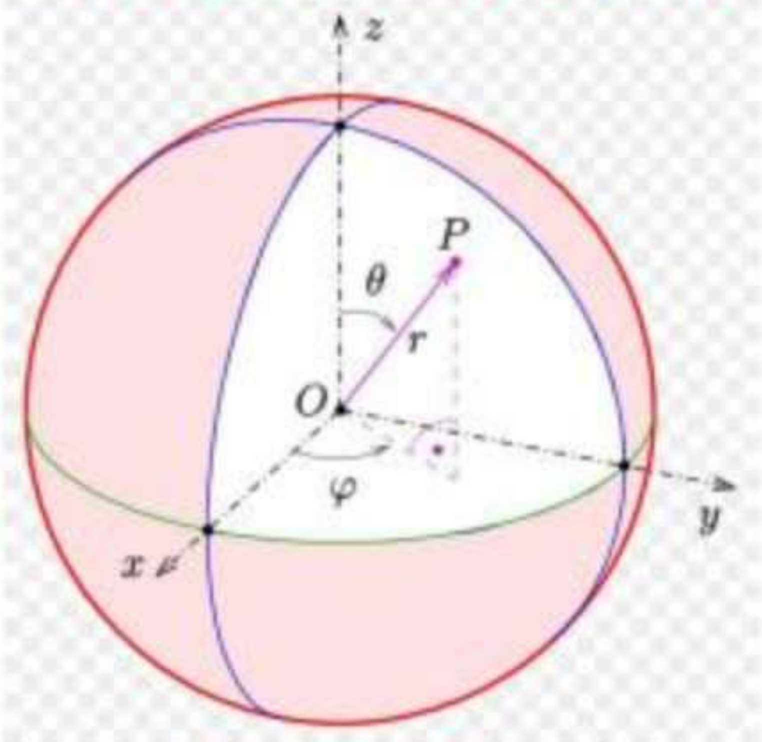
Spherical Coordinates P(r, Θ, ϕ)











Recap

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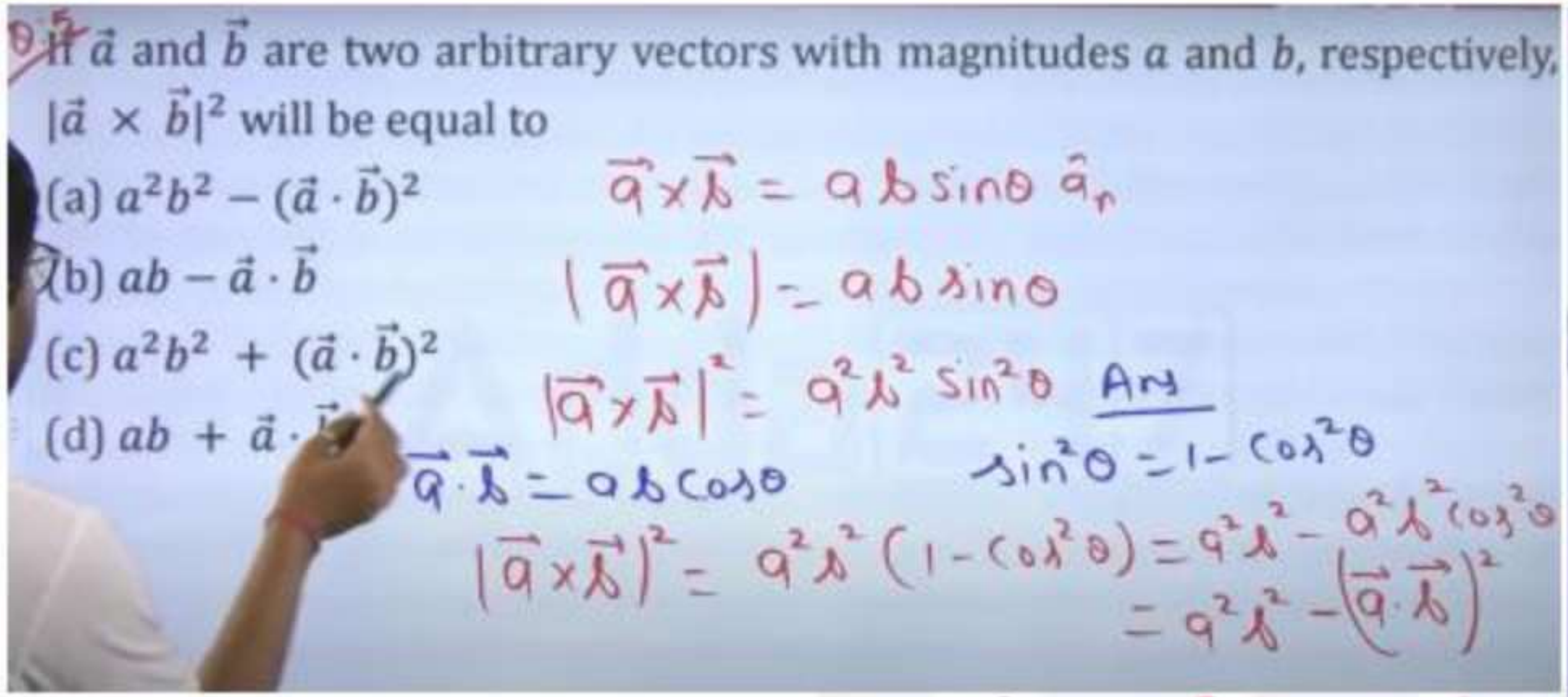
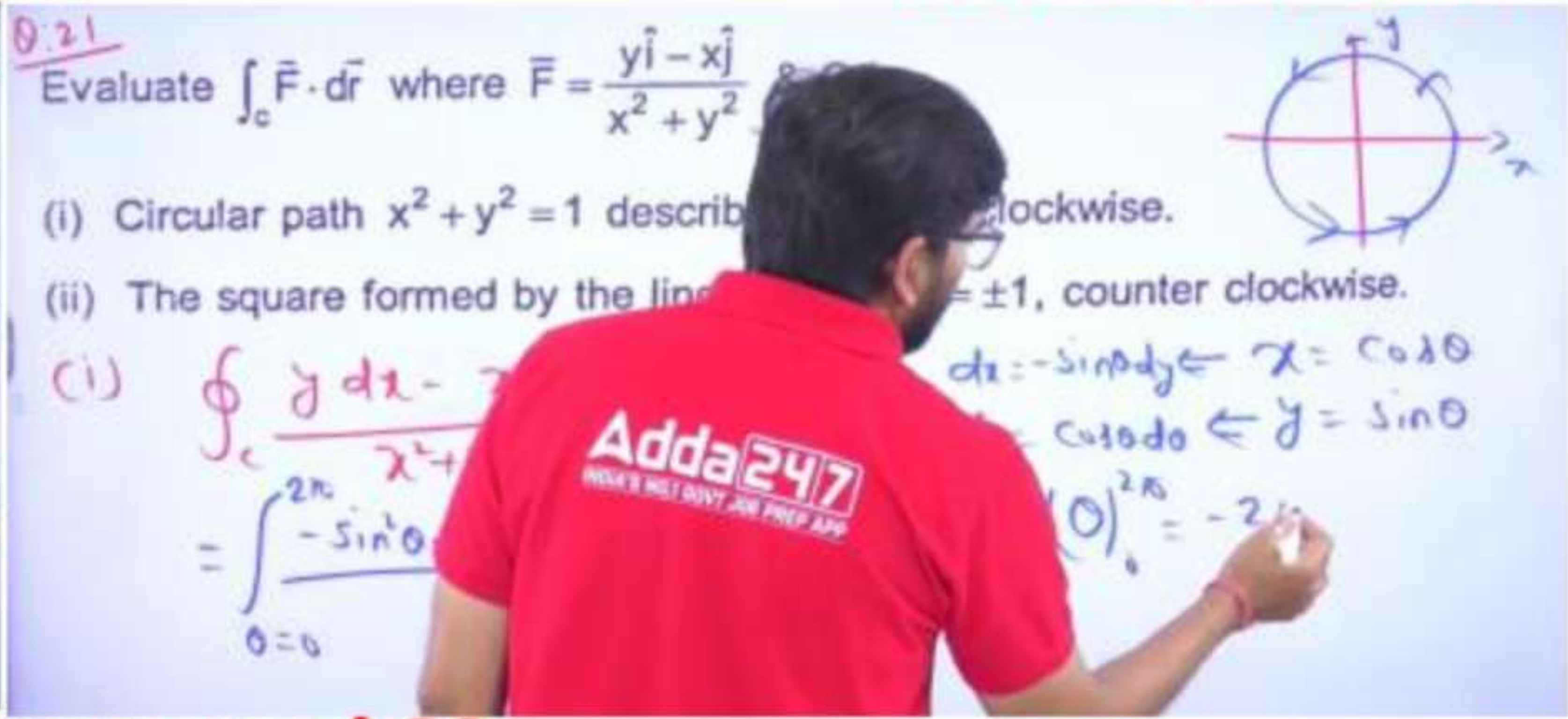
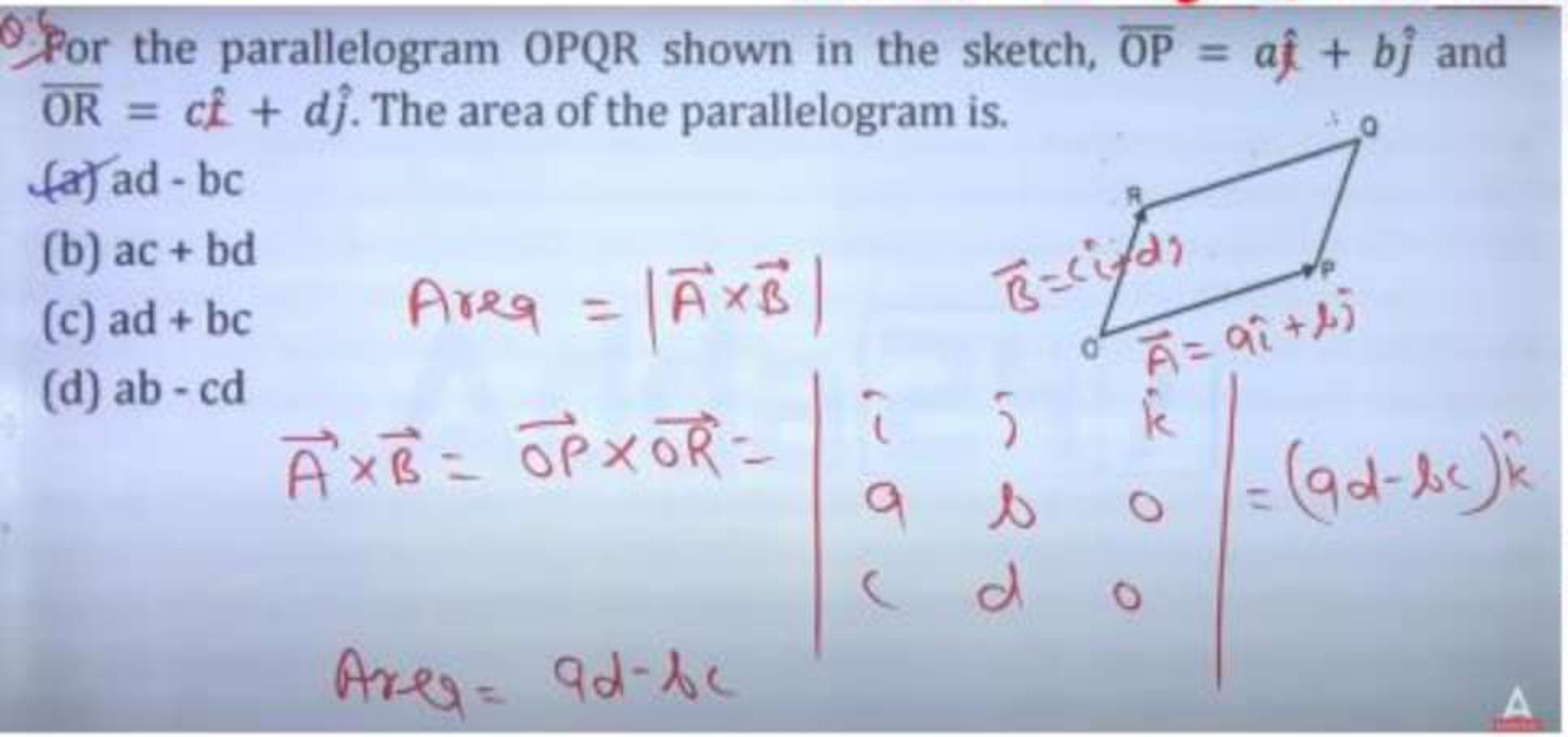
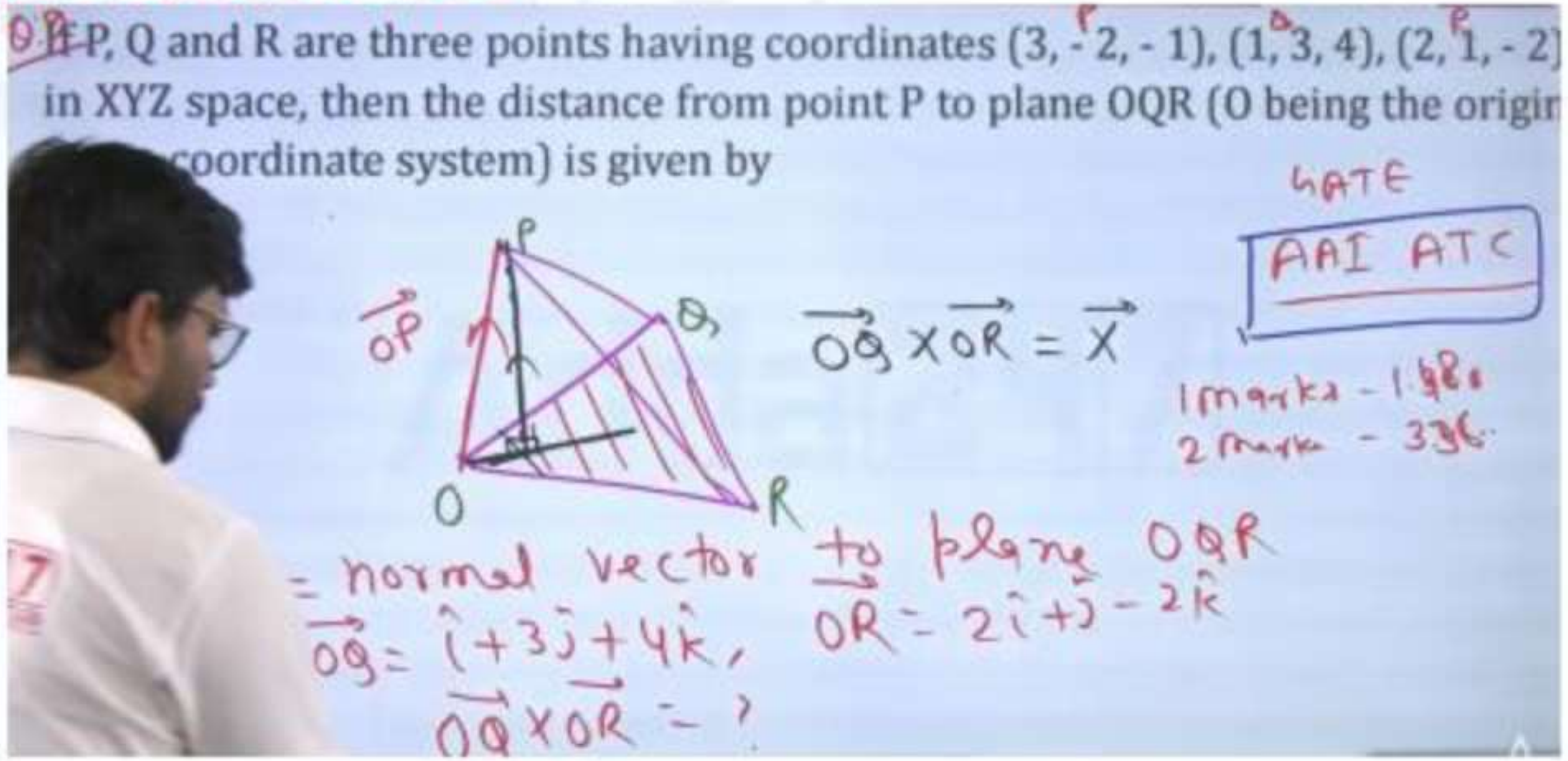




Number of Questions covered-22





Spherical Coordinate systems

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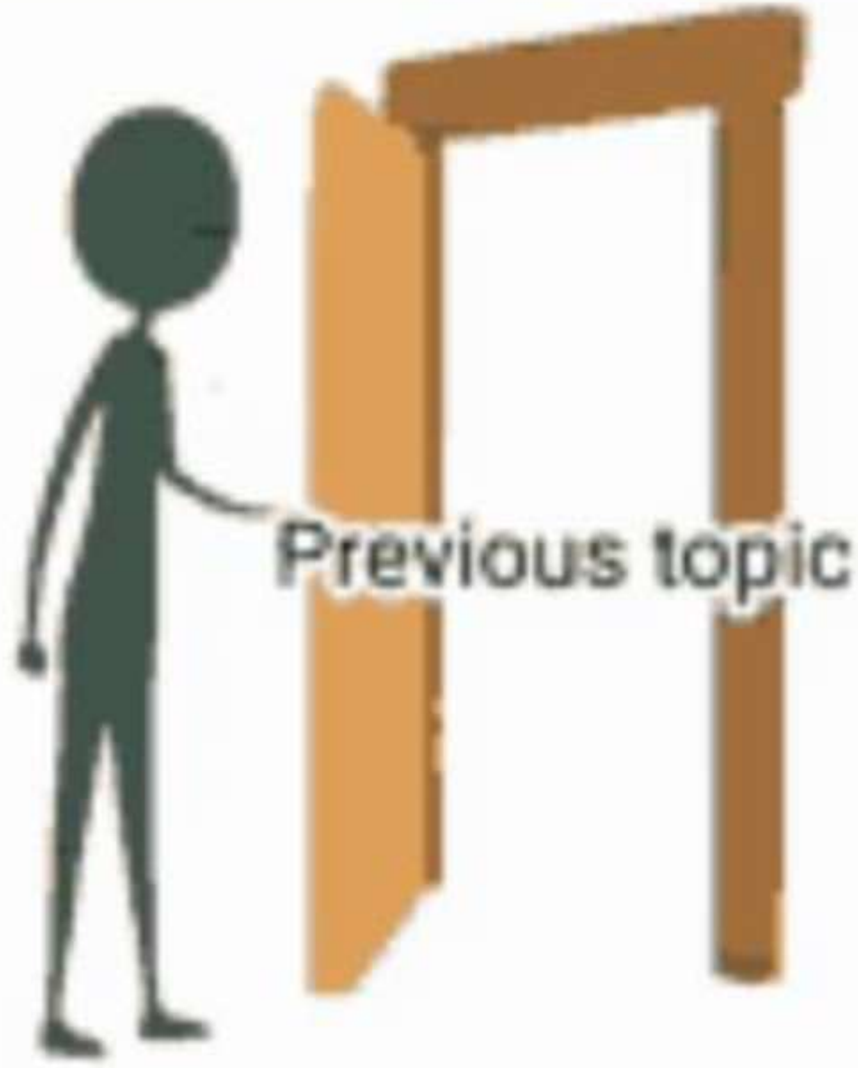


1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical and Spherical Coordinate systems
7. Vector integrals(Line and closed line)

Vector Differentials

Del Operator

Gradient and its Applications

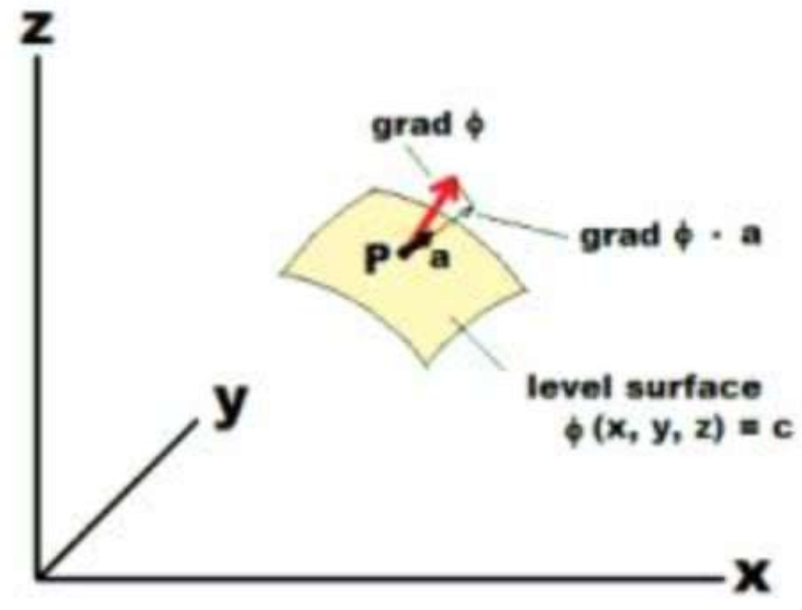


Fig. 1

Del Operator:- It is differential operator in Vector calculus.

- Del operator is first order differential operator.
- Line integral is first order integral.
- Del Operator is a vector.
- Del operator symbol is named as Nebla.

$\frac{d}{dx}(f(x))$

$\frac{\partial}{\partial y} f(x,y)$

∇

$\int \vec{A}\cdot\vec{dl} = \int (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(dx\hat{i}+dy\hat{j}+dz\hat{k}) = \int A_x dx + \int A_y dy + \int A_z dz$

$\int \vec{A}\cdot\vec{ds} = \int A_x \, dy\,dz$

$\int \rho_v \, dV = \int \rho_v \, dx\,dy\,dz$

line integral → single order
surface → double integral
Volume → triple integral.

Del Operator is Cartesian Coordinate Systems

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

$\int \cdots dx \rightarrow \frac{d}{dx}$

$\int f(x,y)\,dy \rightarrow \frac{\partial}{\partial y}$

Del Operator is Cylindrical Coordinate Systems

$$\vec{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

$$\nabla = \frac{\partial}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} \hat{a}_\phi + \frac{\partial}{\partial z} \hat{a}_z$$

Del Operator is Spherical Coordinate Systems

$$\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

$\nabla \rightarrow$ It is vector.

First Order Differential Operations using Del operator

① $\nabla V \rightarrow$ Gradient

② $\nabla \cdot \vec{A} \rightarrow$ Divergence

③ $\nabla \times \vec{A} \rightarrow$ Curl

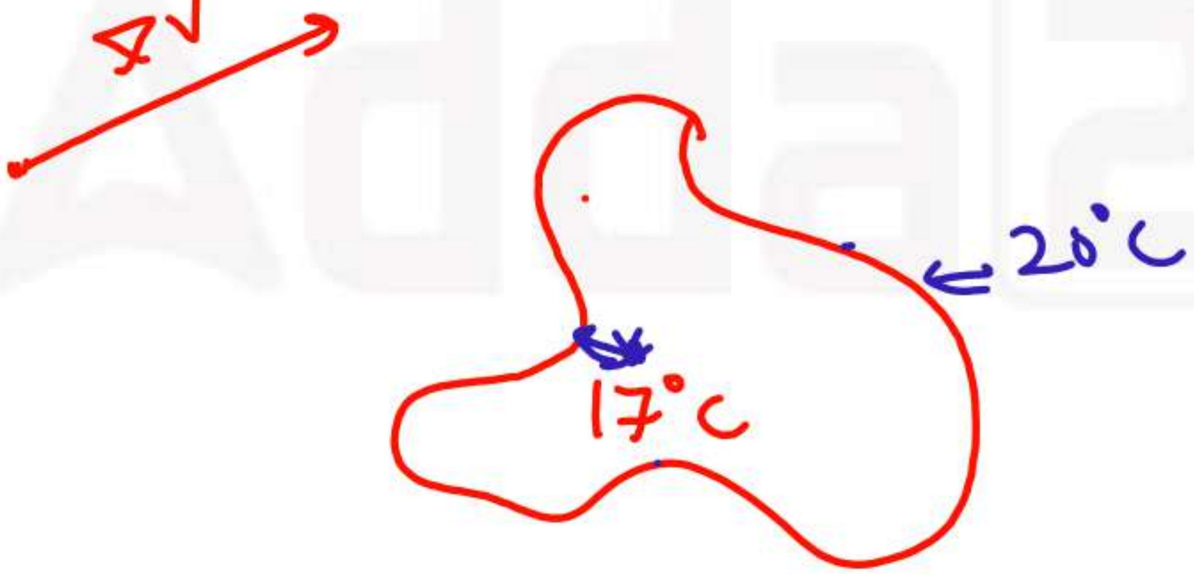
* Gradient operation is performed on Scalars.

* Gradient results a vector.

Gradient:- Gradient of a non uniform scalar field at a point is a vector, of which magnitude is maximum space rate of change at the point and its direction is in the direction in which maximum space rate of change occurs.

Uniform field → $T = 25^\circ C$

Non-uniform field → $T = (5xy^2 - 3z)^\circ C$

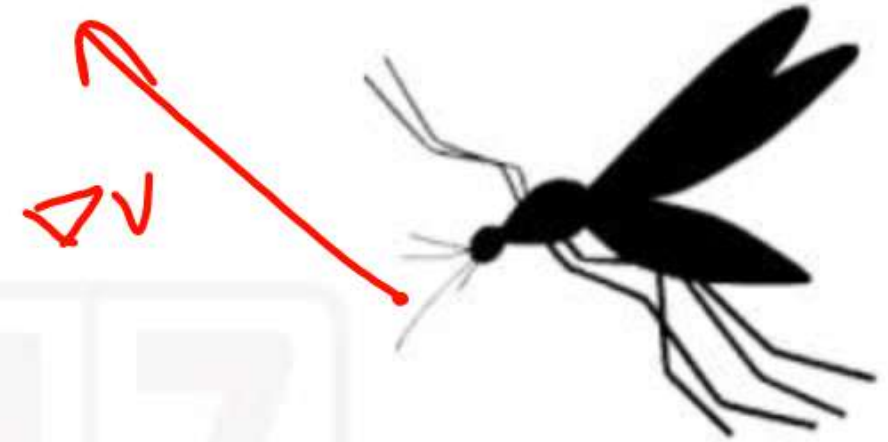


Q:23 Temperature in an auditorium is given by T=15x^2yz^3. A mosquito located At point (-1,2,4) feels cold , in which direction it must fly to get relax?

Sol:→ direction of gradient

= unit vector of gradient

$$= \frac{\nabla T}{|\nabla T|}$$



$$T = 15x^2yz^3$$

$$\nabla T = 30xyz^3\,\hat{i} + 15x^2z^3\,\hat{j} + 45x^2yz^2\,\hat{k}$$

$$\nabla T\big|_{(-1,2,4)} = -60\times 64\,\hat{i} + 15\times 64\,\hat{j} + 90\times 16\,\hat{k}$$

$$\text{direction} = \frac{-60\times 64\,\hat{i} + 15\times 64\,\hat{j} + 90\times 16\,\hat{k}}{\sqrt{(\ )^2 + (\ )^2 + (\ )^2}}$$

Calculation of Gradient in Cartesian Coordinate systems

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

$$\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{B} = 2x^2\hat{i} - 3x\hat{j} + 2y^2 z\hat{k}$$

$V(x,y,z)$

$$\nabla V = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right) V(x,y,z)$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$\nabla V\big|_{P(x_1,y_1,z_1)}$ → gradient at the point 'P'.

Calculation of Gradient in Cylindrical Coordinate systems

$V(\rho, \phi, z)$

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

Calculation of Gradient in Spherical Coordinate systems

$V(r, \theta, \phi)$

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}V\,\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$$

Applications of Gradient

1. To find maximum rate of change and its direction at any point
2. To find Directional Derivative ??
3. To find vector normal to a curve or surface at any point ??

DIRECTIONAL DERIVATIVE

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$$

DIRECTIONAL DERIVATIVE

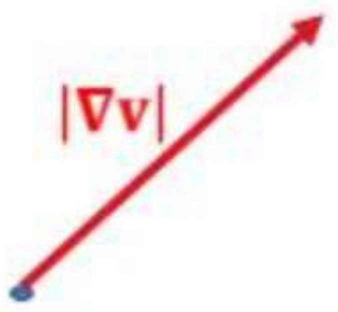
Application of gradient

DIRECTIONAL DERIVATIVE

Application of gradient

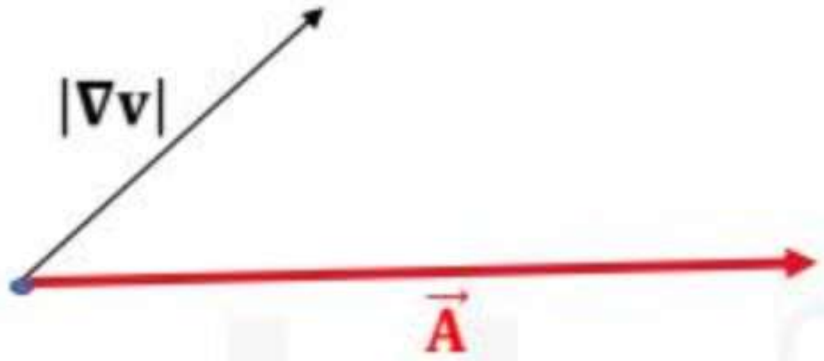
DIRECTIONAL DERIVATIVE

Application of gradient



DIRECTIONAL DERIVATIVE

Application of gradient

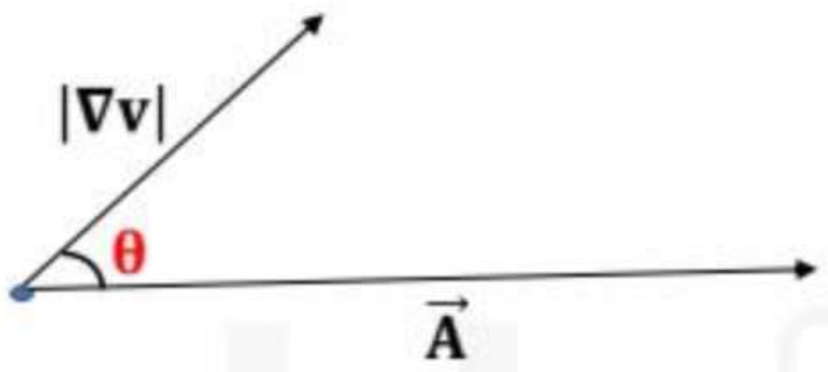


directional derivative of v in the direction of $\vec{A}$

—

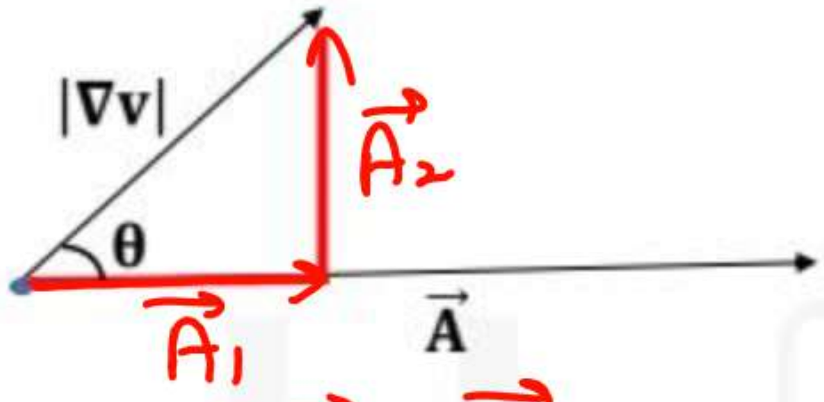
DIRECTIONAL DERIVATIVE

Application of gradient



DIRECTIONAL DERIVATIVE

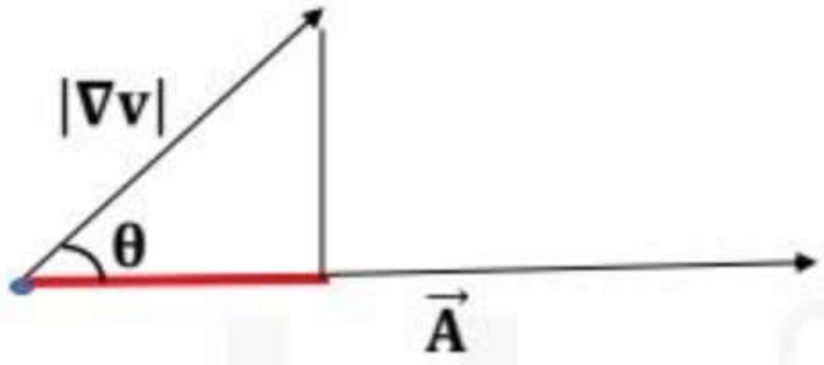
Application of gradient



$\nabla V = \vec{A_1} + \vec{A_2}$

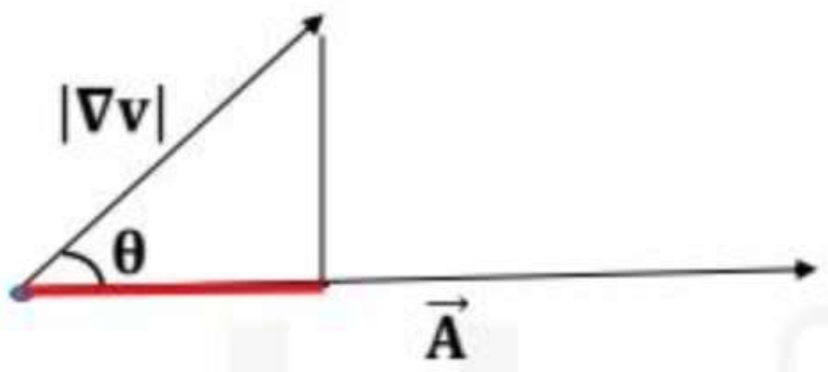
DIRECTIONAL DERIVATIVE

Application of gradient



DIRECTIONAL DERIVATIVE

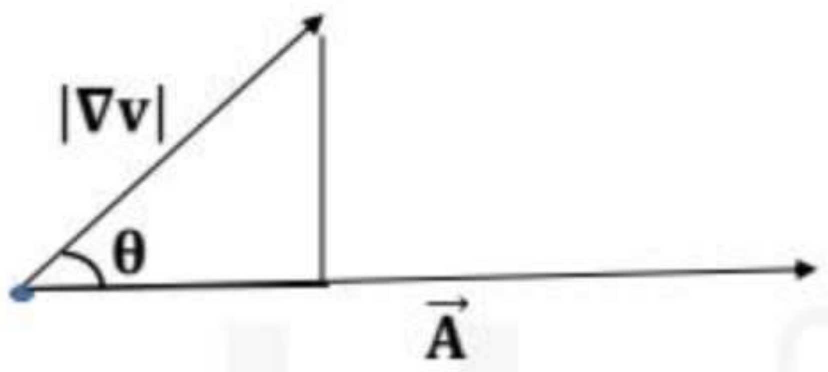
Application of gradient



$|\nabla v| \cos\theta$

DIRECTIONAL DERIVATIVE

Application of gradient

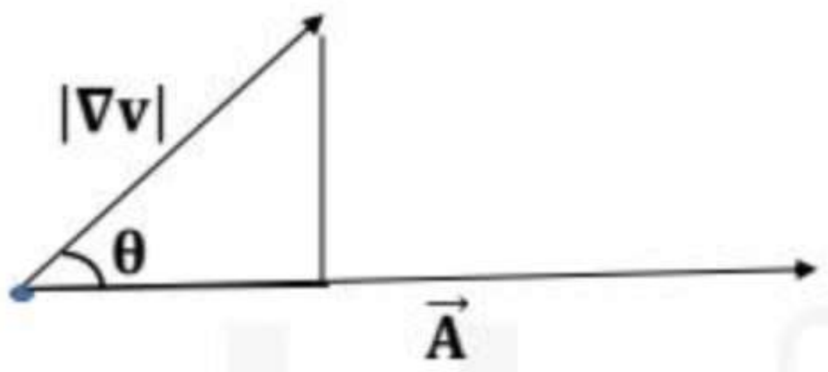


$|\nabla v| \cos\theta$

$\vec{A}.\vec{B} = AB\cos\theta$

DIRECTIONAL DERIVATIVE

Application of gradient



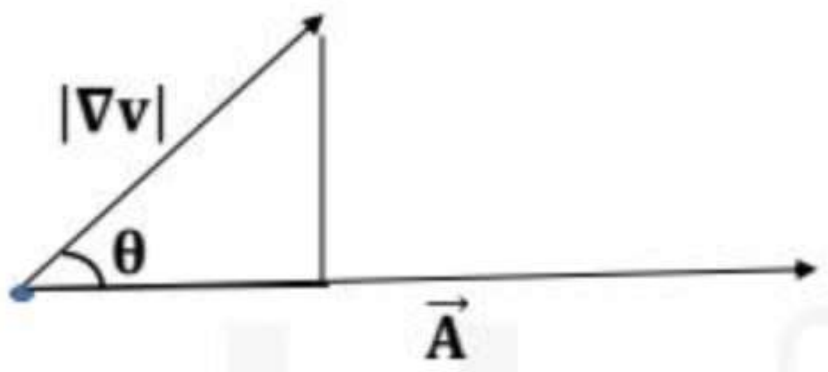
$|\nabla v| \cos\theta$

$$\vec{A}.\vec{B} = AB\cos\theta$$

$$\vec{A}.\hat{a}_B = A\cos\theta$$

DIRECTIONAL DERIVATIVE

Application of gradient



$|\nabla \mathbf{v}| \cos\theta$

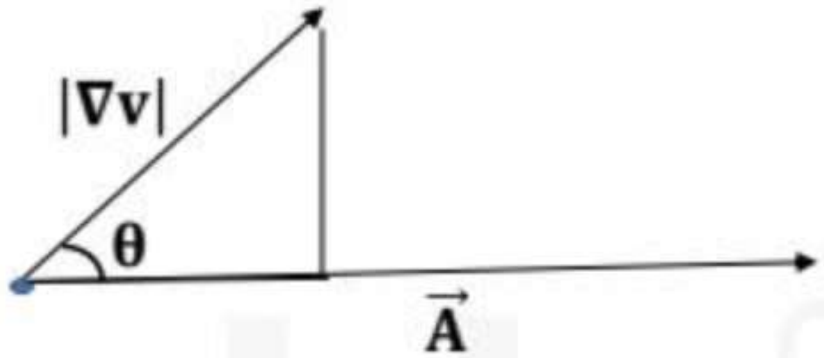
$\vec{\nabla \mathbf{v}} . \hat{a}_A$

$\vec{A} . \vec{B} = AB \cos\theta$

$\vec{A} . \hat{a}_B = A \cos\theta$

DIRECTIONAL DERIVATIVE

Application of gradient



$|\nabla v| \cos\theta$

$\vec{\nabla v} \cdot \hat{a}_A$

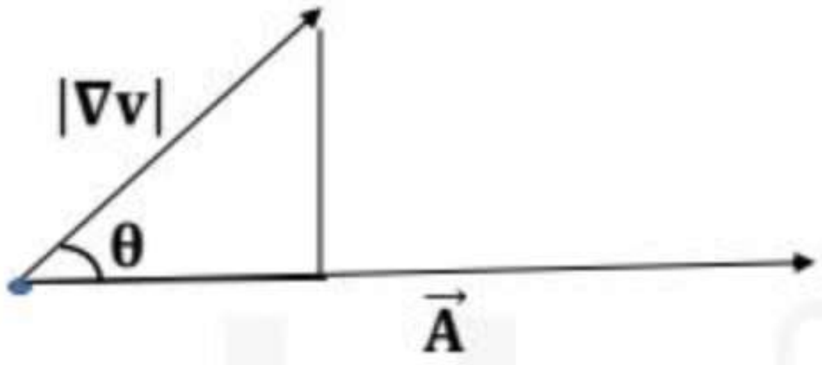
$\vec{A} \cdot \vec{B} = AB \cos\theta$

$\vec{A} \cdot \hat{a}_B = A \cos\theta$

Directional derivative of scalar v at a point in the direction of $\vec{A}$

DIRECTIONAL DERIVATIVE

Application of gradient



$|\nabla v| \cos\theta$

$\vec{\nabla v} \cdot \hat{a}_A$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \hat{a}_B = A\cos\theta$

Directional derivative of scalar v at a point in the direction of $\vec{A}$ = $\vec{\nabla v} \cdot \hat{a}_A$

Q:24

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol:

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

let $V = 2x^2 + 3y - z$

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

let $V = 2x^2 + 3y - z$

$\nabla V = 4xi + 3\hat{\imath} - \hat{k}$

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

$$\text{let } V = 2x^2 + 3y - z$$

$$\nabla V = 4xi + 3\hat{\imath} - \hat{k}$$

$$\nabla_V|_{(3,1,-1)} = 12\hat{I} + 3\hat{\imath} - \hat{k}$$

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} =$

$5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

$$\text{let } V = 2x^2 + 3y - z$$

$$\nabla V = 4xi + 3\hat{\imath} - \hat{k}$$

$$\nabla_V\big|_{(3,1,-1)} = 12\hat{I} + 3\hat{\imath} - \hat{k}$$

$$\vec{A}\big|_{(3,1,-1)} = 15\hat{I} + 6\hat{\imath} + 4\hat{k}$$

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

$$\text{let } V = 2x^2 + 3y - z$$

$$\nabla V = 4xi + 3\hat{\imath} - \hat{k}$$

$$\nabla_V|_{(3,1,-1)} = 12\hat{I} + 3\hat{\imath} - \hat{k}$$

$$\vec{A}|_{(3,1,-1)} = 15\hat{I} + 6\hat{\imath} + 4\hat{k}$$

$$\hat{a}_A = \frac{15\hat{I}+6\hat{\imath}+4\hat{k}}{\sqrt{225+36+16}}$$

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} =$

$5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

$$\text{let } V = 2x^2 + 3y - z$$

Directional derivative $= \vec{\nabla}\vec{v}.\hat{a}_A$

$$\nabla V = 4xi + 3\hat{\imath} - \hat{k}$$

$$\nabla_V|_{(3,1,-1)} = 12\hat{I} + 3\hat{\imath} - \hat{k}$$

$$\vec{A}|_{(3,1,-1)} = 15\hat{I} + 6\hat{\imath} + 4\hat{k}$$

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Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{I} - 2xz\hat{\imath} + 4\hat{k}$ at point (3, 1, -1)

Sol.

$$\text{let } V = 2x^2 + 3y - z$$

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$$\vec{A}|_{(3,1,-1)} = 15\hat{I} + 6\hat{\imath} + 4\hat{k}$$

$$\hat{a}_A = \frac{15\hat{I}+6\hat{\imath}+4\hat{k}}{\sqrt{225+36+16}}$$

$$\text{Directional derivative} = \vec{\nabla v}.\hat{a}_A$$

$$\vec{\nabla v}.\hat{a}_A = \frac{15 \times 12 + 6 \times 3 - 4}{\sqrt{277}}$$

Q:24

Find directional derivative of $2x^2 + 3y - z$ in the direction of $\vec{A} = 5x\hat{i} - 2xz\hat{i} + 4\hat{k}$ at point (3, 1, -1)

Sol.

let $V = 2x^2 + 3y - z$

$\nabla V = 4xi + 3\hat{j} - \hat{k}$

$\nabla_V|_{(3,1,-1)} = 12\hat{i} + 3\hat{j} - \hat{k}$

$\vec{A}|_{(3,1,-1)} = 15\hat{i} + 6\hat{j} + 4\hat{k}$

$\hat{a}_A = \frac{15\hat{i}+6\hat{j}+4\hat{k}}{\sqrt{225+36+16}}$

Directional derivative $= \vec{\nabla v}.\hat{a}_A$

$\vec{\nabla v}.\hat{a}_A = \frac{15 \times 12 + 6 \times 3 - 4}{\sqrt{277}}$

$= 11.656$

Home Work

Adda247

Q:25

The directional derivative of $f(x, y, z) = x(x^2 - y^2) - z$ at A(1, -1, 0) in the direction of $\bar{p} = (2\hat{i} - 3\hat{j} + 6\hat{k})$ is:

1. -8/49
2. 8/7
3. -8/7 ✓
4. 0

$$\nabla f = (3x^2 - y^2)\hat{i} - 2xy\hat{j} - \hat{k}$$

$$\nabla f \big|_{(1,-1,0)} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{P} \big|_{(1,-1,0)} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\hat{a}_p = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4+9+36}}$$

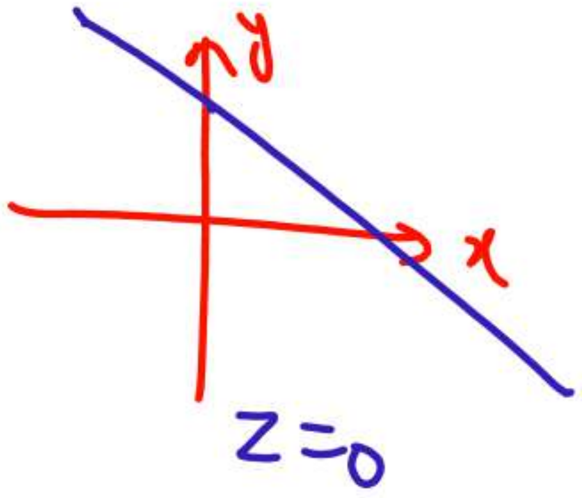
directional der. $= \nabla f \cdot \hat{a}_p = \dfrac{4-6-6}{\sqrt{49}} = \dfrac{-8}{7}$

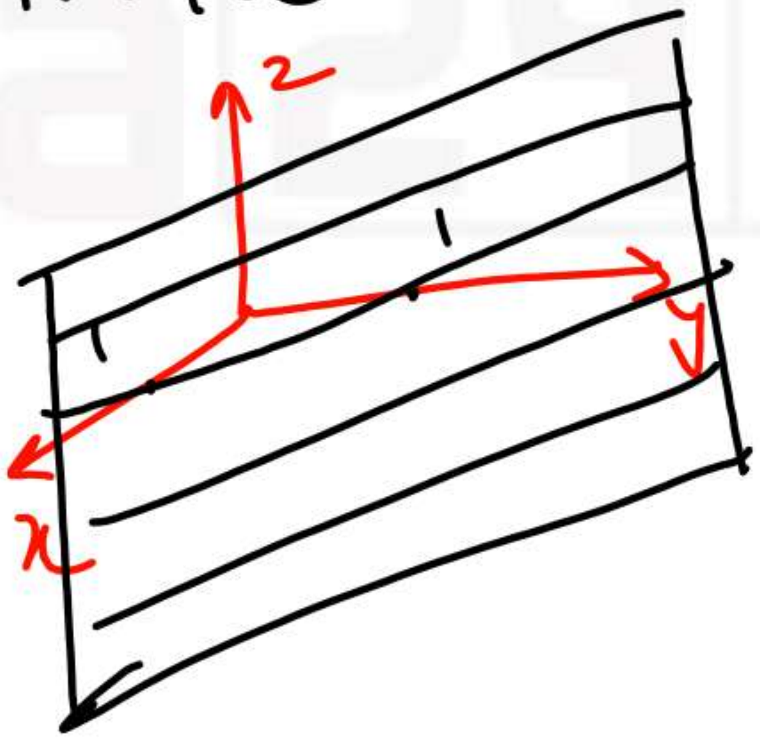
Vector Normal to a Surface/Curve

$y = mx + c$ → Curve which straight line

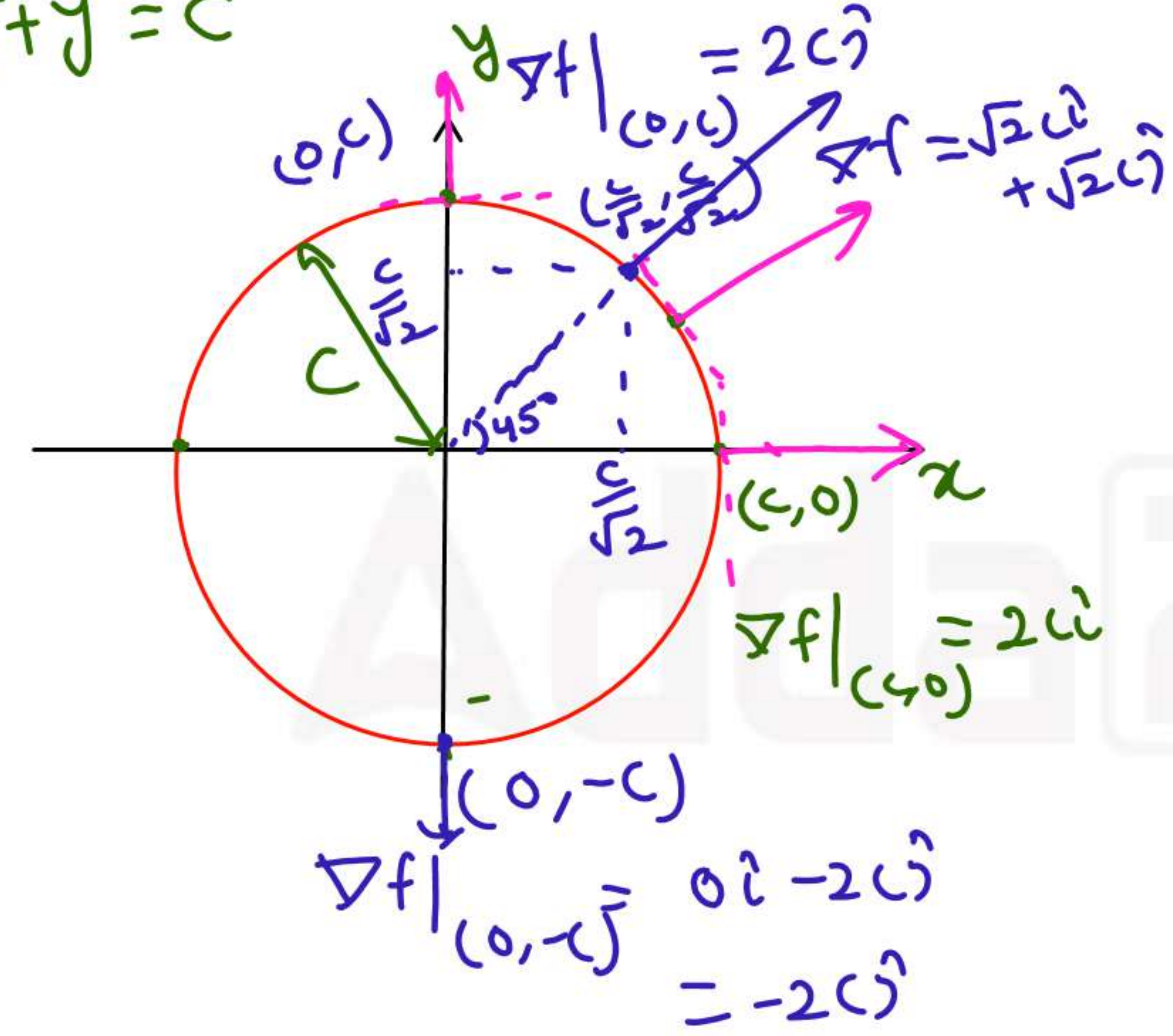
$x^2 + y^2 = c^2$ → Circular Curve

$x + y = 1 \quad \forall z \in (-\infty, \infty)$ → Surface





$x^2+y^2=c^2$



$x^2+y^2=c^2$

$x^2+y^2-c^2=0$

$f(x,y,z)=0$

$\nabla f = 2x\hat{i} + 2y\hat{j} + 0\hat{k}$

In general a curve or surface given by

$$f(x,y,z)=0$$

$$f(\rho,\phi,z)=0$$

$$f(r,\theta,\phi)=0$$

then ∇f at a point gives normal vector to that curve/surface at that point.

GATE 2024

GATE

Sat & Sunday

3 P.M.

6PM → PSA → EE
th, fri, Sat

EMFT
9 P.M.

Mon, tu, wed → Comm
9PM
→ 6P.M
N/W

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Engineering Mathematics

LINEAR ALGEBRA

Question practice on basics of matrices


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