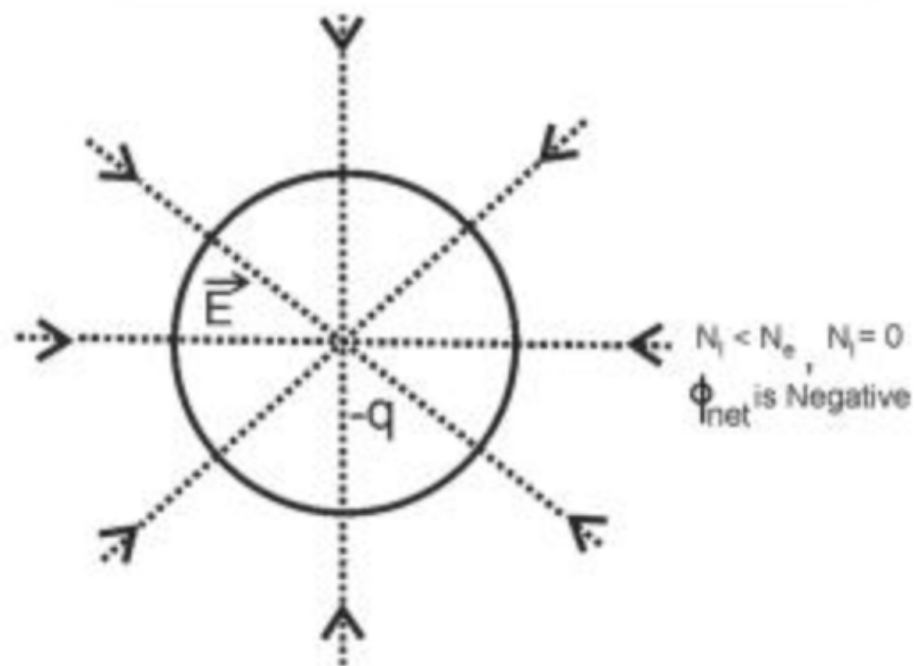
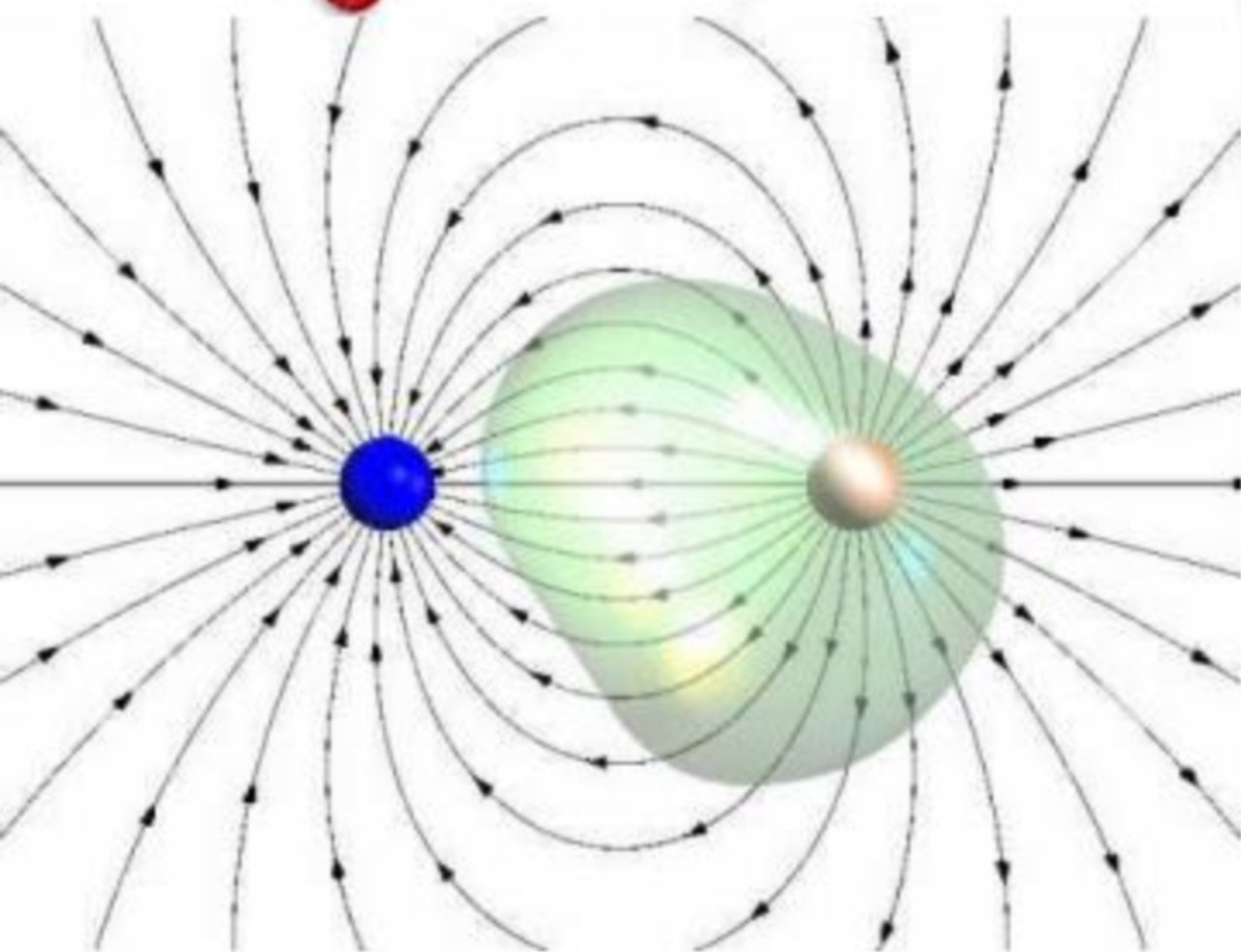


today's
topics

Electric Potential



Q:25 The directional derivative of $f(x, y, z) = x(x^2 - y^2) - z$ at $A(1, -1, 0)$ in the direction of $\vec{p} = (2\hat{i} - 3\hat{j} + 6\hat{k})$ is:

- 8/49
- 8/7
- 8/7
- 0

Handwritten solution:

$$\nabla f = (2xy^2)\hat{i} - 2xy\hat{j} - \hat{k}$$

$$\nabla f|_A = (2(-1)^2)\hat{i} + 2\hat{j} - \hat{k} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{p} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$|\vec{p}| = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

$$\vec{p} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

$$\text{Directional derivative} = \nabla f \cdot \vec{p} = (2\hat{i} + 2\hat{j} - \hat{k}) \cdot \left(\frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}\right) = \frac{4 - 6 - 6}{7} = \frac{-8}{7}$$

Q:21 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$

(i) Circular path $x^2 + y^2 = 1$ described clockwise.

(ii) The square formed by the lines $x = \pm 1, y = \pm 1$, counter clockwise.

Handwritten solution for (i):

$$\int_C \frac{y dx - x dy}{x^2 + y^2}$$

$$= \int_0^{2\pi} \frac{-\sin^2 \theta}{1} d\theta$$

$$= \int_0^{2\pi} -\sin^2 \theta d\theta = -\pi$$

Handwritten solution for (ii):

$$d\vec{r} = -\sin \theta d\theta \hat{i} + \cos \theta d\theta \hat{j}$$

$$x = \cos \theta, y = \sin \theta$$

$$\int_C \frac{y dx - x dy}{x^2 + y^2} = \int_0^{2\pi} \frac{\sin \theta (-\sin \theta d\theta) - \cos \theta (\cos \theta d\theta)}{1} = \int_0^{2\pi} (-\sin^2 \theta - \cos^2 \theta) d\theta = -2\pi$$

Number of Questions covered-56

Q:54 Which one of the following describes the relationship among the three vectors, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 4\hat{k}$

(a) The vectors are mutually perpendicular

(b) The vectors are linearly independent

(c) The vectors are linearly independent

(d) The vectors are unit vectors

Handwritten solution:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{bmatrix}$$

$$R_3 = 3R_1 + R_2$$

$\vec{A} \cdot \vec{B} = 2 + 3 + 4 = 9$

Q:56 If two charges of $10\mu\text{C}$ & $-10\mu\text{C}$ are located at $(-1, 1, 0)$ & $(2, 2, 0)$ then find force on a charge $1\mu\text{C}$ placed at $(0, 0, 2)$.

Handwritten solution:

Charge 1: $10\mu\text{C}$ at $(-1, 1, 0)$

Charge 2: $-10\mu\text{C}$ at $(2, 2, 0)$

Charge 3: $1\mu\text{C}$ at $(0, 0, 2)$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$= \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 12} \frac{(\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{6}} + \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 12} \frac{(-2\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{12}}$$

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प्रचण्ड Batch

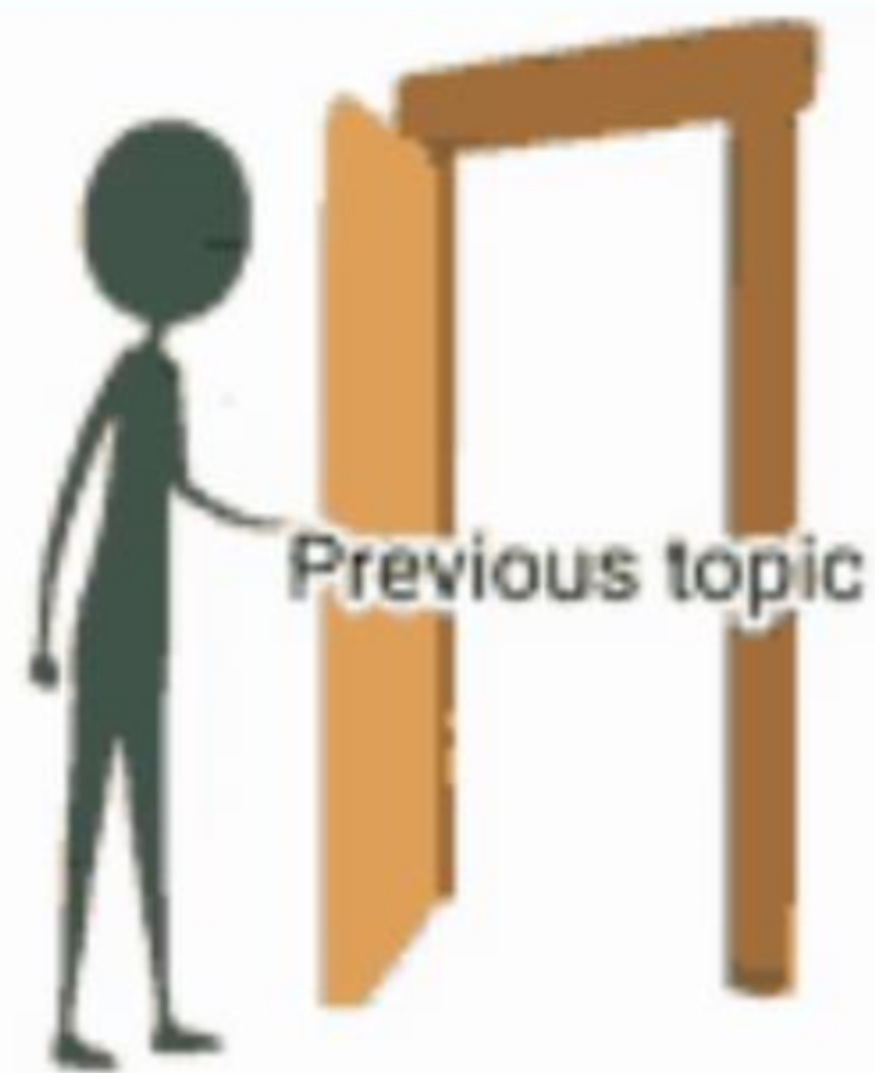
Electromagnetic Field Theory

ELECTRIC POTENTIAL

LEC-12

EE & ECE

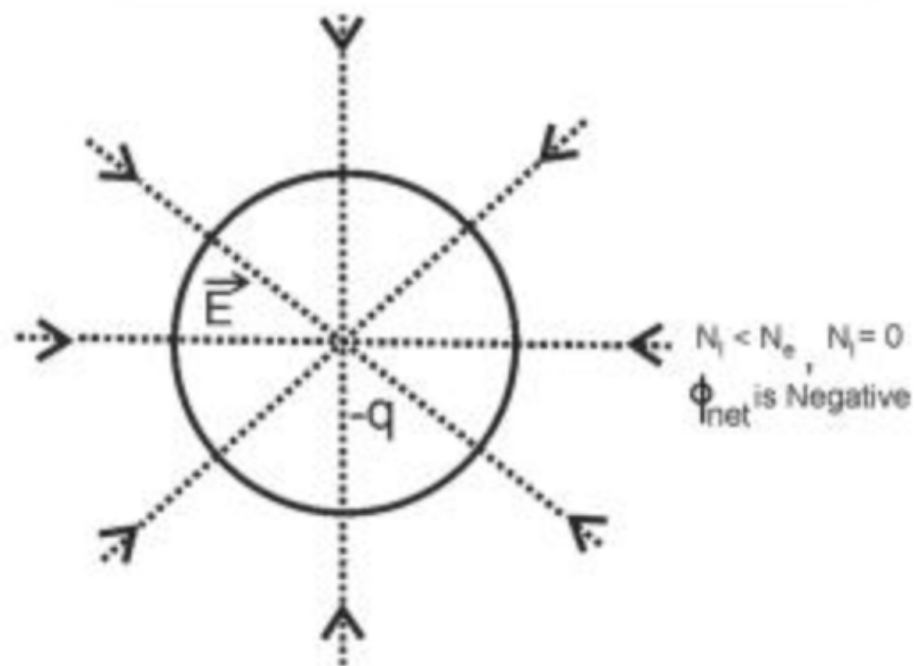
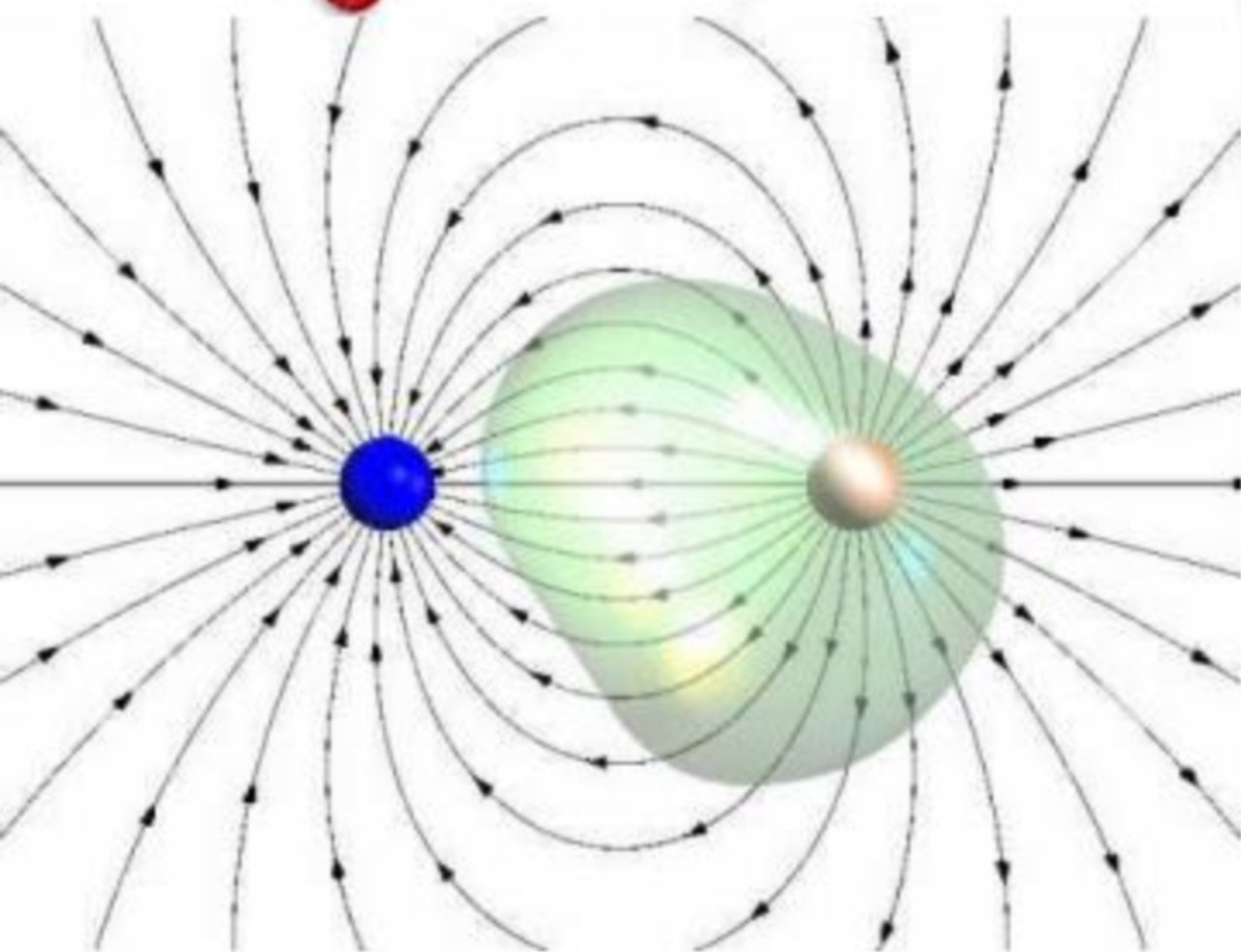




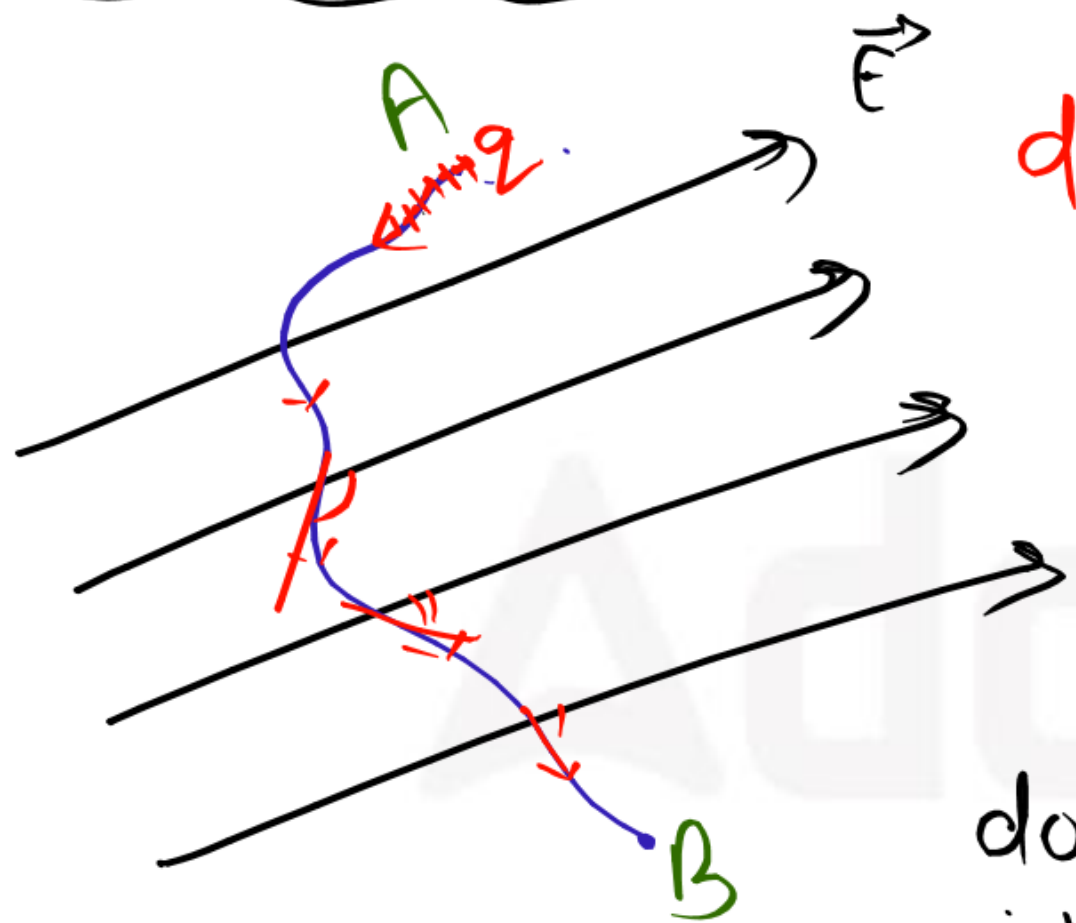
- 1. Basic introduction of Fields**
- 2. Basics of Vectors**
- 3. Coordinate Systems**
- 4. Vector Integrals**
- 5. Vector differentials**
- 6. Coulomb's law and Gauss law**
- 7. Field due to line, Surface and Spherical Volume Charge**

today's
topics

Electric Potential



Electric Potential :-



+Q

$$dw = -\vec{F} \cdot d\vec{l}$$

$$\vec{F} = q\vec{E}$$

- $\vec{F}_e \rightarrow$ Newton
- $\vec{E} \rightarrow$ N/C
- $\vec{D} \rightarrow$ C/m^2

Here -ve sign

represents that work is done by external agent in opposition of field.

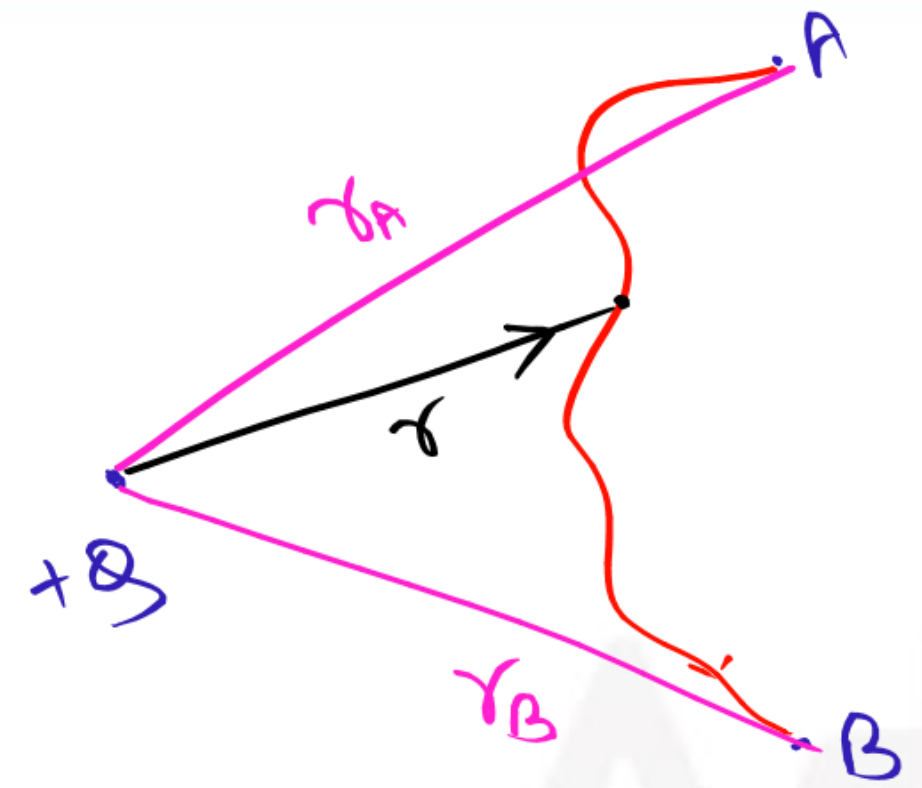
$$W_{AB} = \int_A^B dw = -q \int_A^B \vec{E} \cdot d\vec{l}$$

$$W_{AB} = -q \int_A^B \vec{E} \cdot d\vec{l}$$

Here W_{AB} is the work done to displace the charge 'q' from 'A' to 'B'.

Potential difference \rightarrow It is work done to displace a test charge from point 'A' to 'B' in electric field space

$$= \frac{W_{AB}}{q} = - \int_A^B \vec{E} \cdot d\vec{l}$$



$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

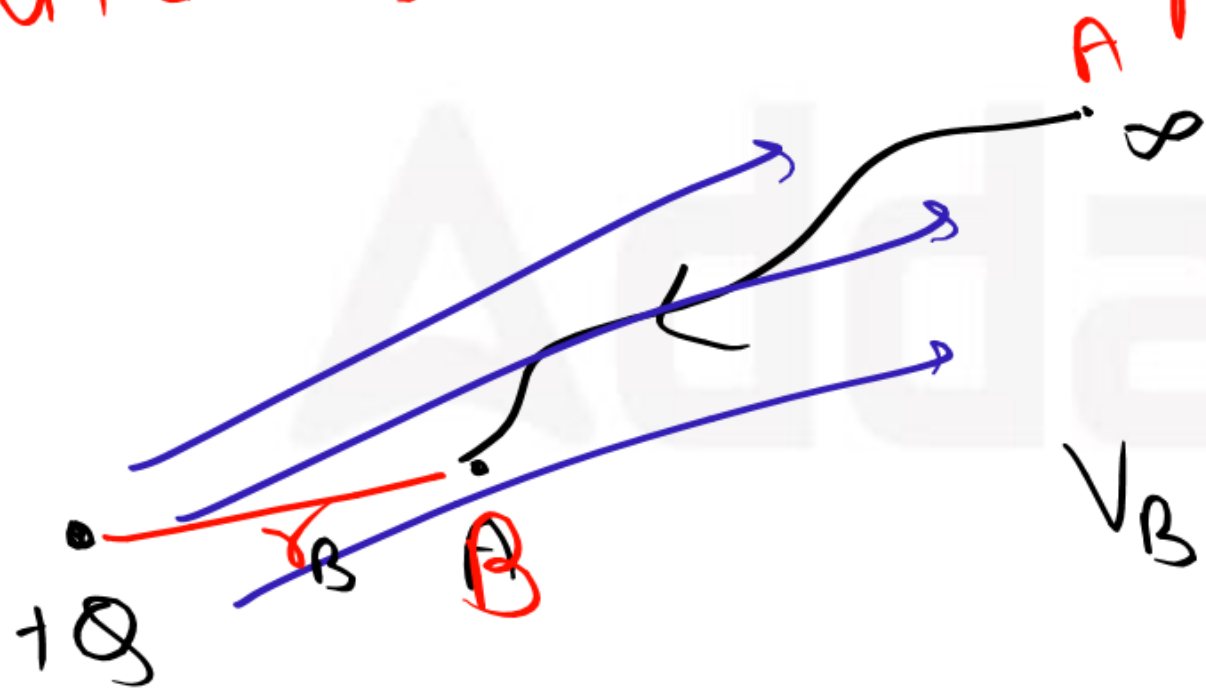
$$V_{BA} = - \int_A^B \vec{E} \cdot d\vec{u}$$

$$V_{BA} = - \int_A^B \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \cdot (dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi)$$

$$V_{BA} = - \int_A^B \frac{Q}{4\pi\epsilon_0 r^2} dr = + \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} \right)_A^B$$

$$V_{BA} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right) = \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A} = V_B - V_A$$

Potential at a point \rightarrow potential difference with respect to a reference point at which potential is zero is potential at this point.

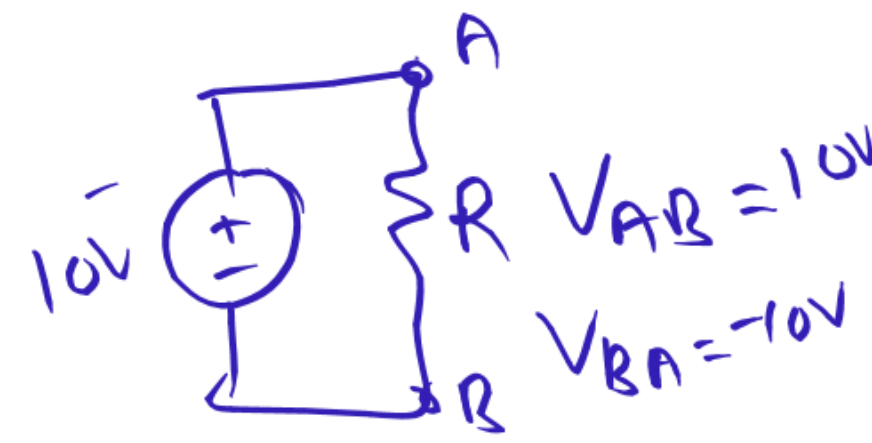


$$V_B = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} \right)_B - \frac{Q}{4\pi\epsilon_0 r_A} \Big|_{r_A \rightarrow \infty}$$

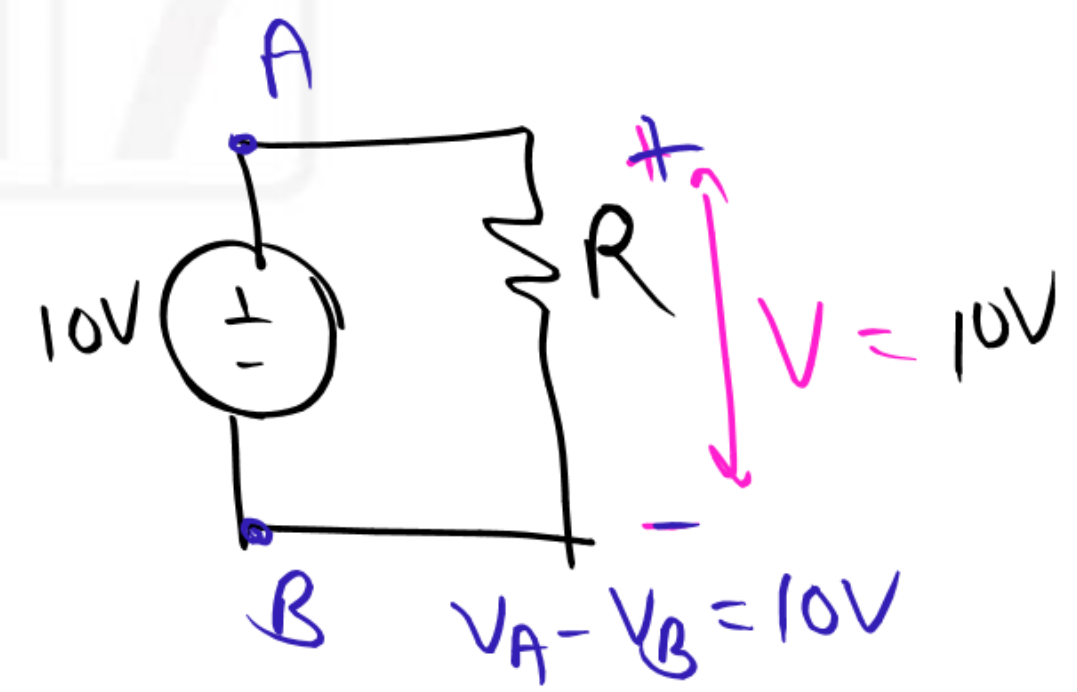
$$= \frac{Q}{4\pi\epsilon_0 r_B}$$

$\perp Q$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$



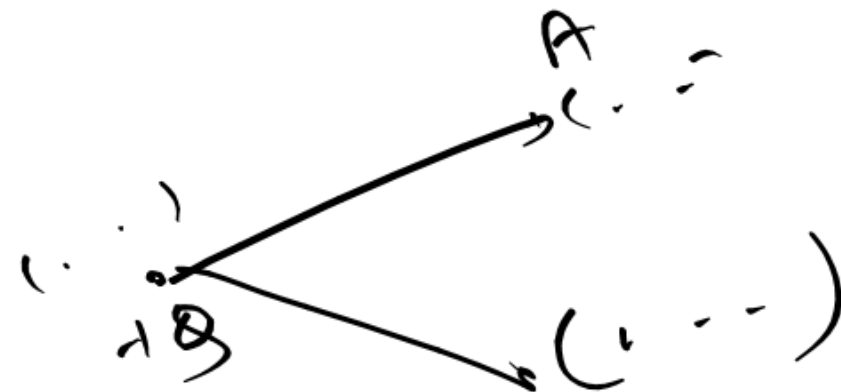
$V_{BA} = - \int_A^B \vec{E} \cdot d\vec{u} = V_B - V_A$



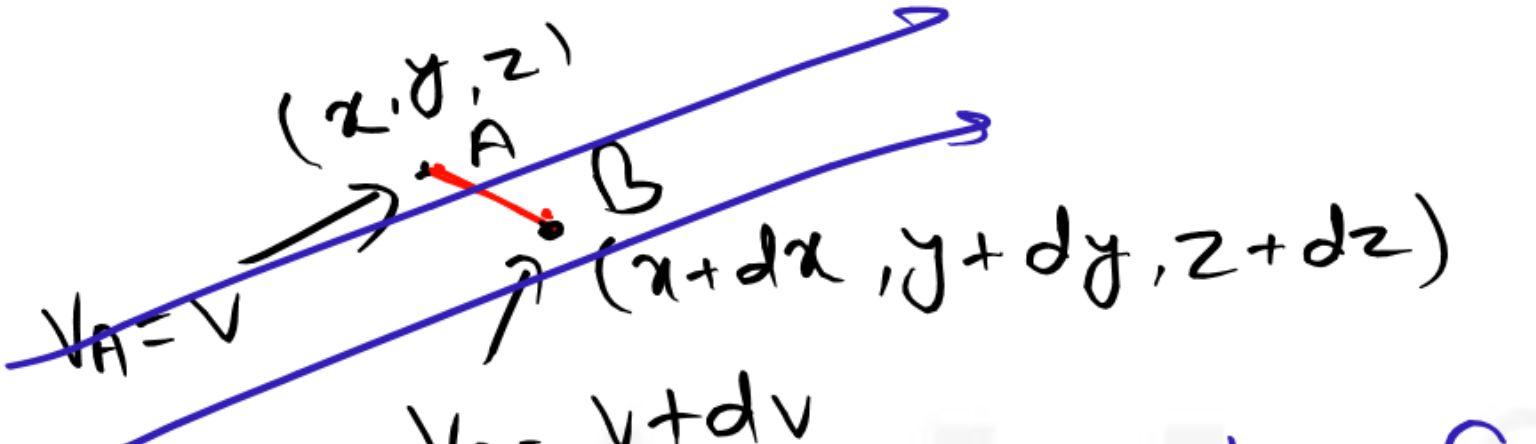
$$\textcircled{1} \quad V_{BA} = - \int_A^B \vec{E} \cdot d\vec{l}$$

$$\textcircled{2} \quad V_{BA} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

$$\textcircled{3} \quad V = \frac{Q}{4\pi\epsilon_0 r}$$



Relation Between Electric Field Intensity and Electric Potential



$V_A = V$
 $V_B = V + dv$

$$\vec{F} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$dw = -\vec{F} \cdot d\vec{l} = -\left(E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z\right) \cdot (dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z)$$

$$dv = \frac{dw}{q} = -\left(E_x dx + E_y dy + E_z dz\right) \quad \text{--- (1)}$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz \quad \text{--- (2)}$$

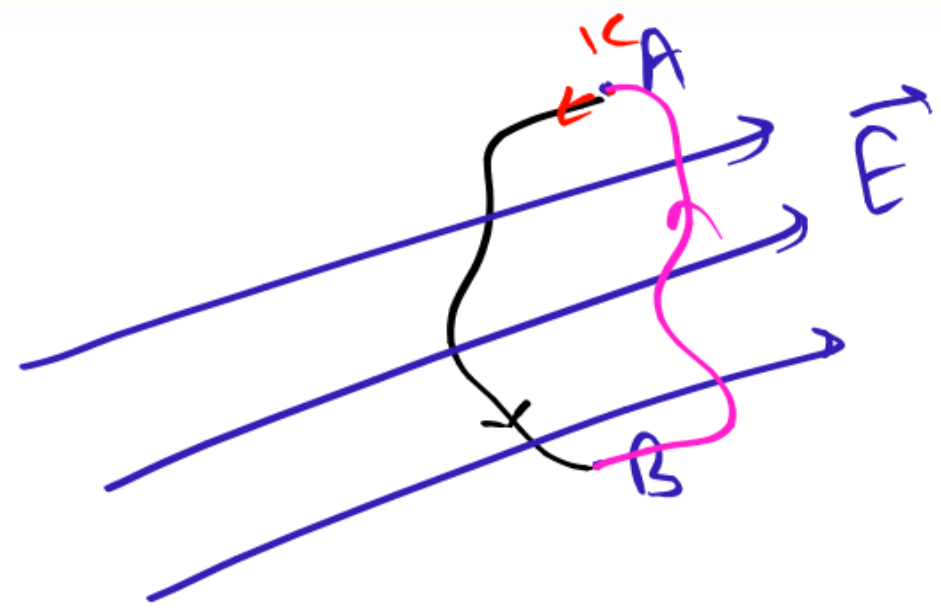
$$E_x = -\frac{\partial v}{\partial x}, \quad E_y = -\frac{\partial v}{\partial y}, \quad E_z = -\frac{\partial v}{\partial z}$$

$$\begin{aligned} \vec{E} &= E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z \\ &= -\left(\frac{\partial v}{\partial x} \hat{a}_x + \frac{\partial v}{\partial y} \hat{a}_y + \frac{\partial v}{\partial z} \hat{a}_z \right) \end{aligned} \quad v(x, y, z)$$

$$\boxed{\vec{E} = -\nabla v}$$

* Electric field vector will be oriented in the direction in which electric potential is decreasing.

* Electric field is oriented from high potential to low potential.



$$V_{BA} = - \int_A^B \vec{E} \cdot d\vec{l}$$

$$V_{AB} = - \int_B^A \vec{E} \cdot d\vec{l} \Rightarrow V_{AB} = - V_{BA}$$

$$V_{AB} + V_{BA} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Applying

Stoke's theorem

$$\int (\nabla \times \vec{E}) \cdot d\vec{s} = \int 0 \cdot d\vec{s}$$

integral form

$$\Rightarrow \nabla \times \vec{E} = 0$$

Maxwell's 2nd eqn
in differential form

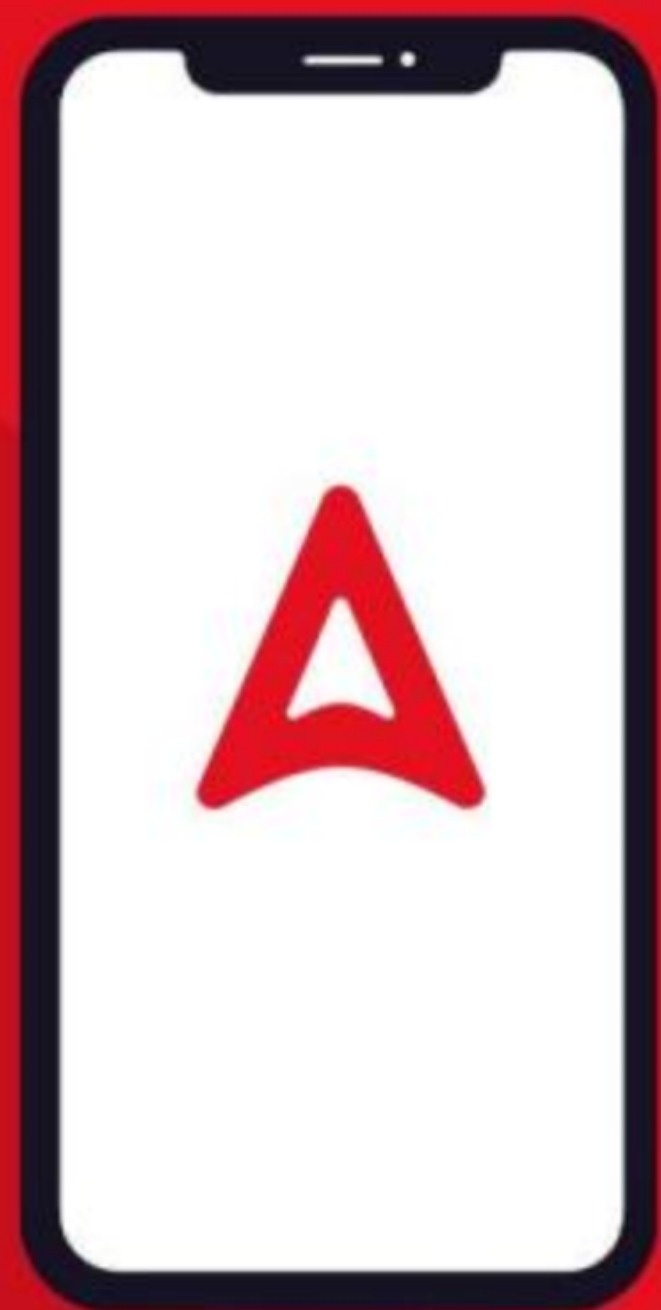
$$\nabla \cdot \vec{D} = \rho_v \quad \leftarrow \text{Gauss law}$$

$$\nabla \times \vec{E} = 0$$

\Rightarrow Static electric field is having curl zero so it is irrotational vector i.e. it is a straight vector.

* As electric field is conservative so its line integral is independent of path.

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