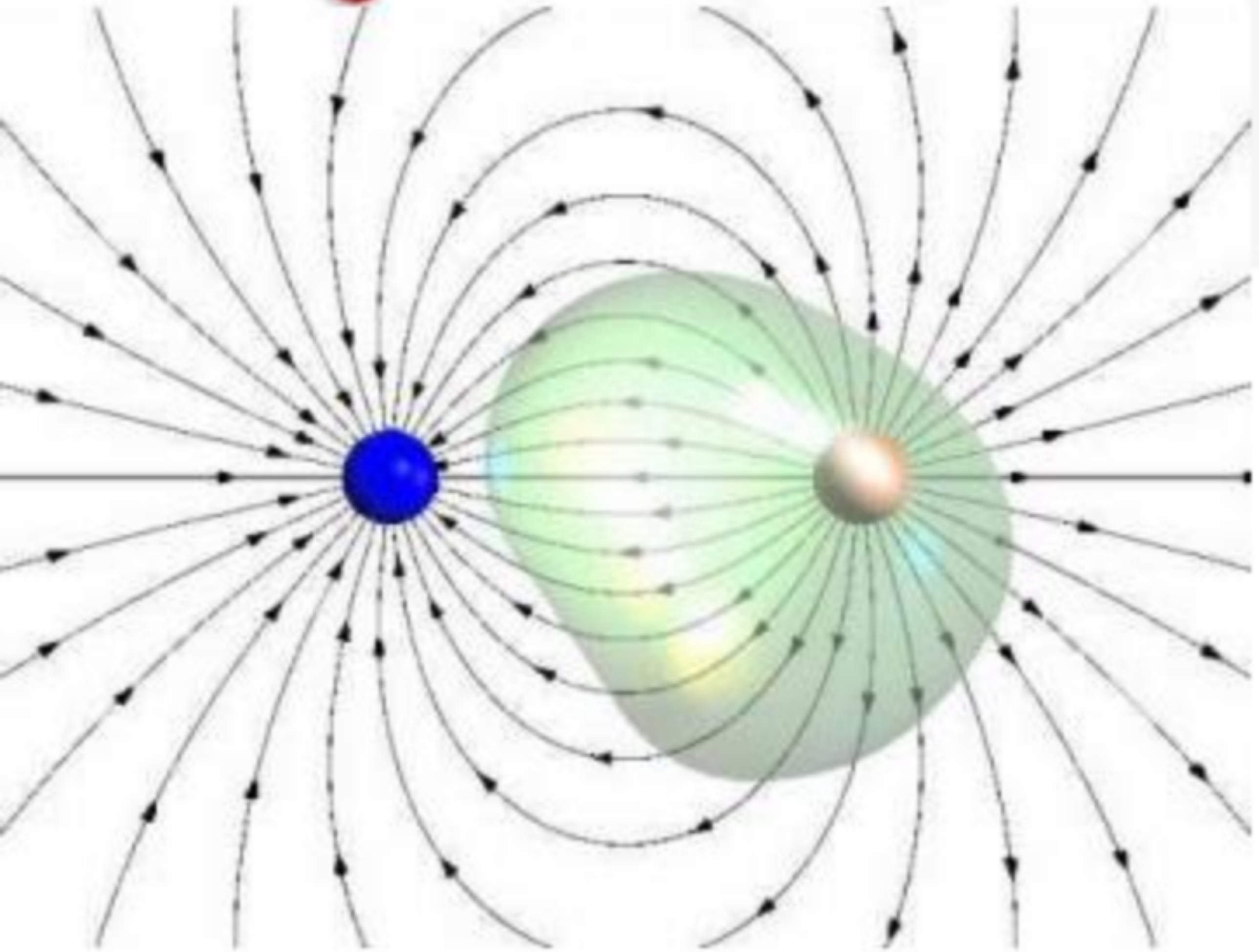
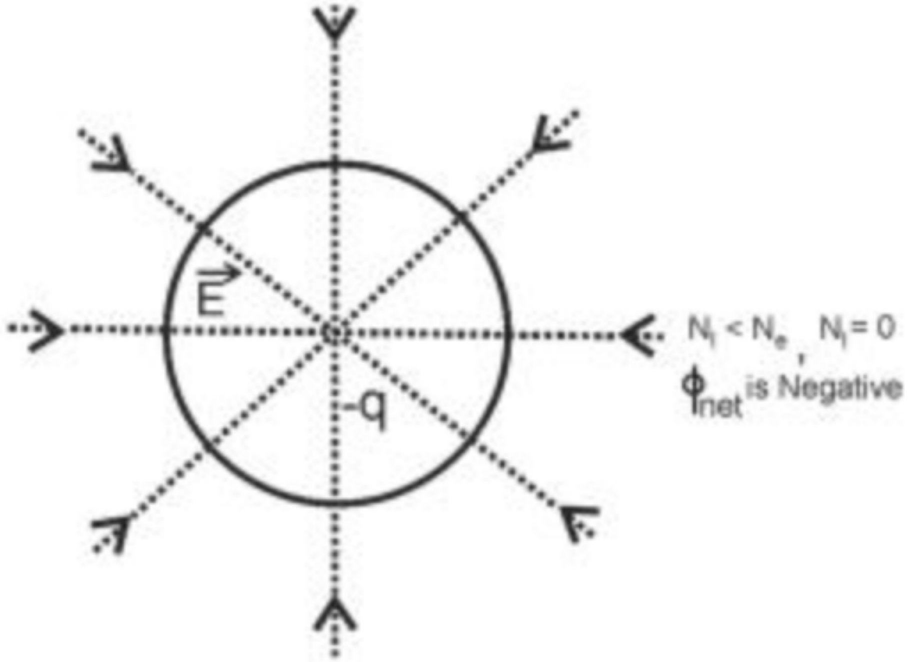
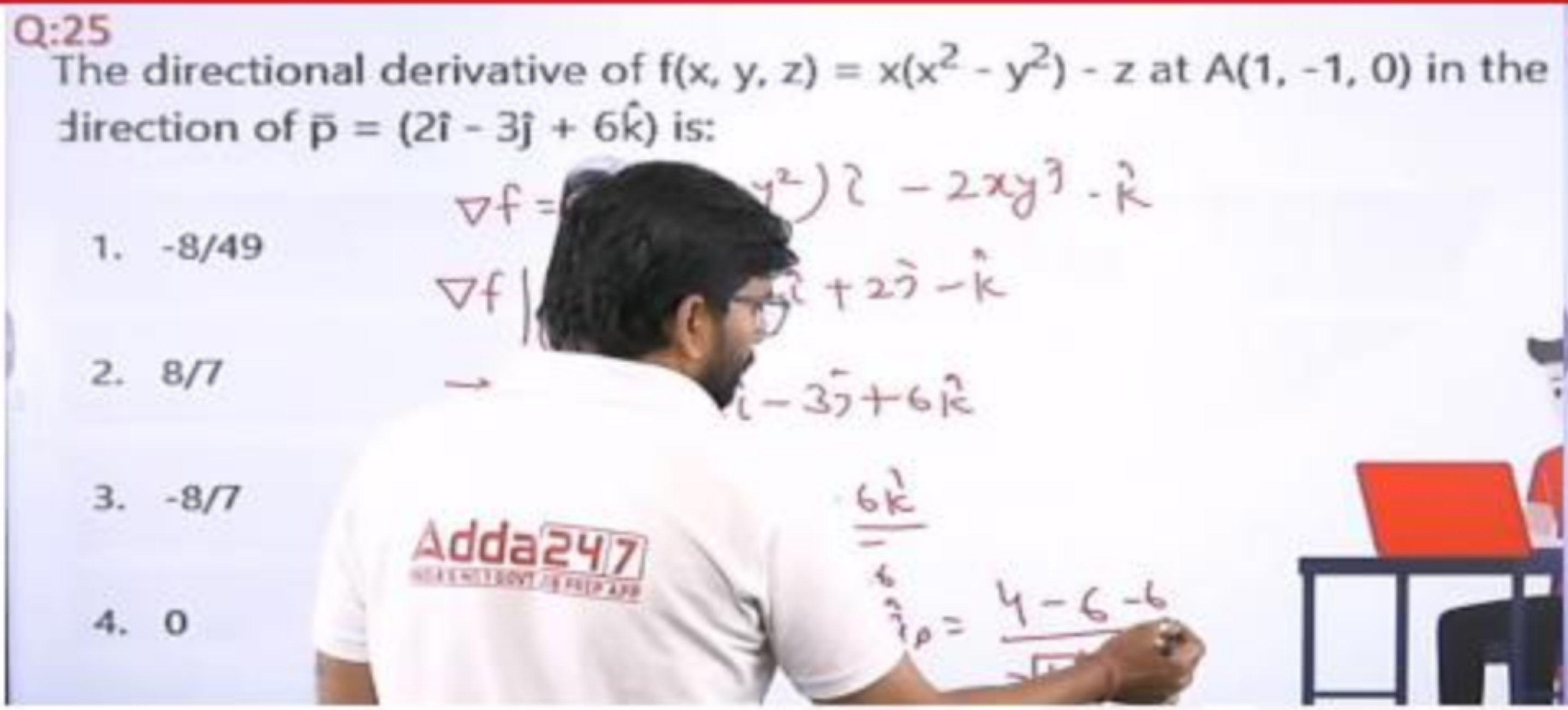
Vector Differentials

Electric Potential

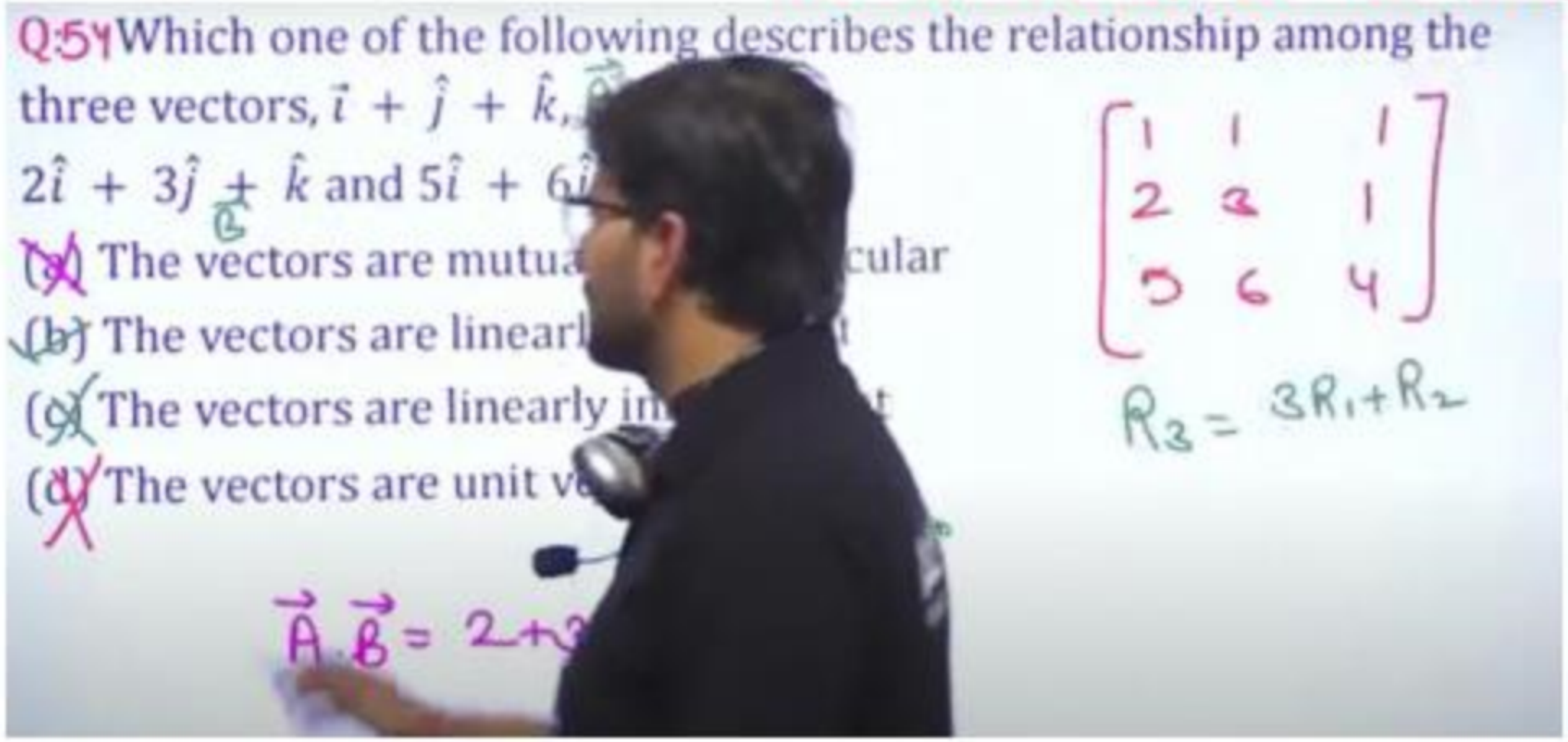


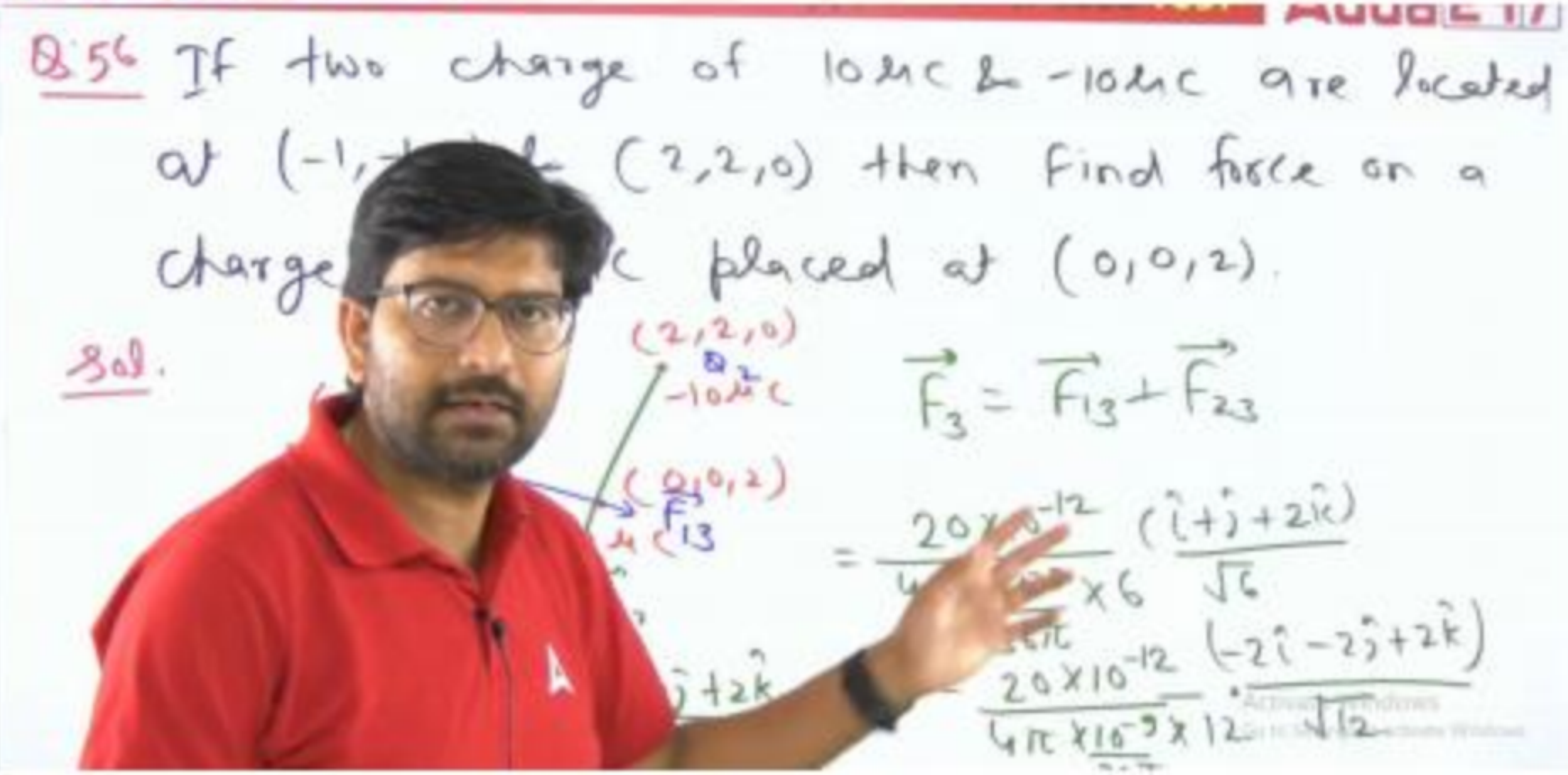


Recap



Number of Questions covered-56





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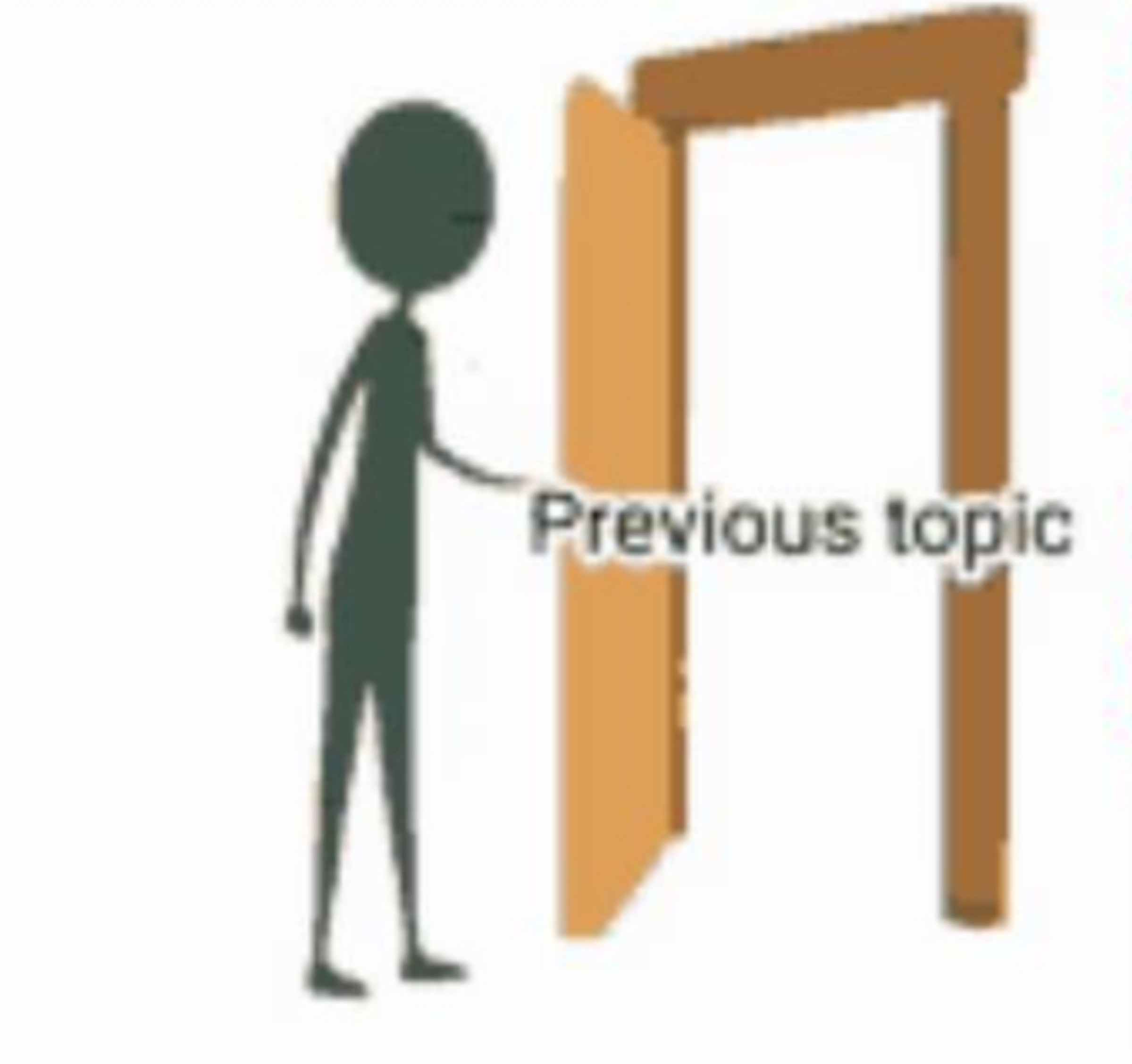
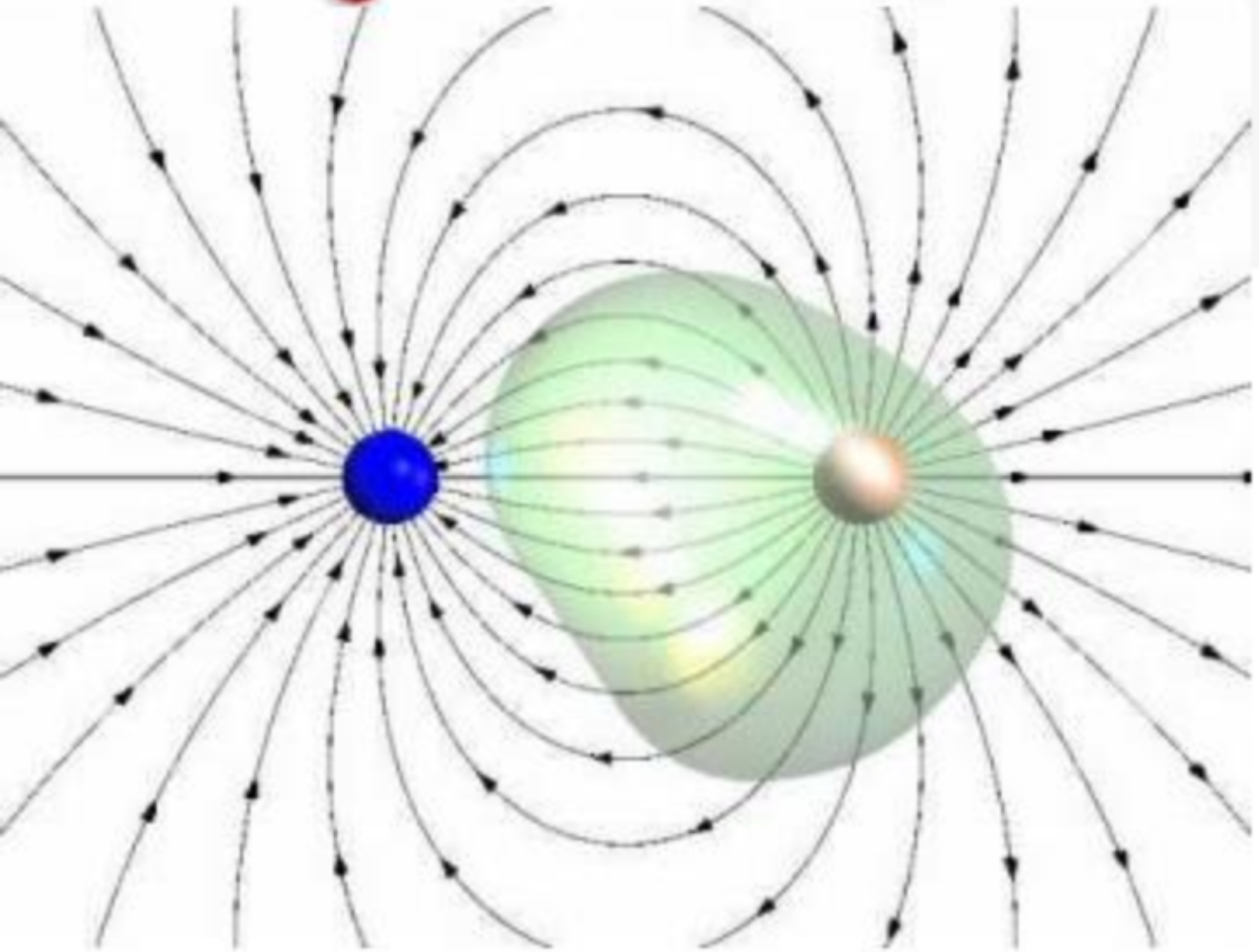
GATE 2024
GATE
प्रचण्ड Batch
Electromagnetic Field Theory
ELECTRIC POTENTIAL
LEC-12
EE &ECE

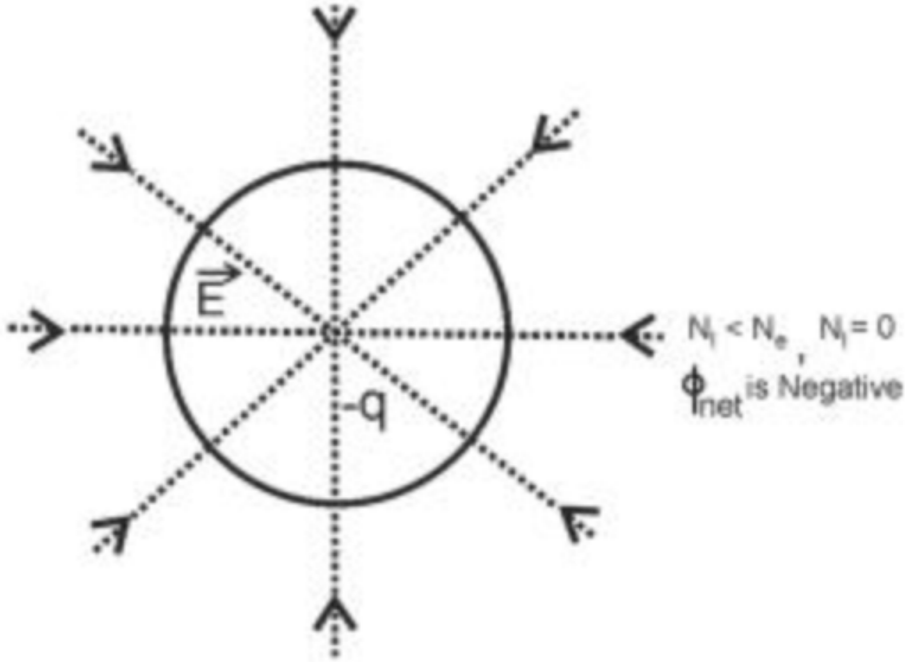


1. Basic introduction of Fields
2. Basics of Vectors
3. Coordinate Systems
4. Vector Integrals
5. Vector differentials
6. Coulomb’s law and Gauss law
7. Field due to line, Surface and Spherical Volume Charge

Vector Differentials

Electric Potential





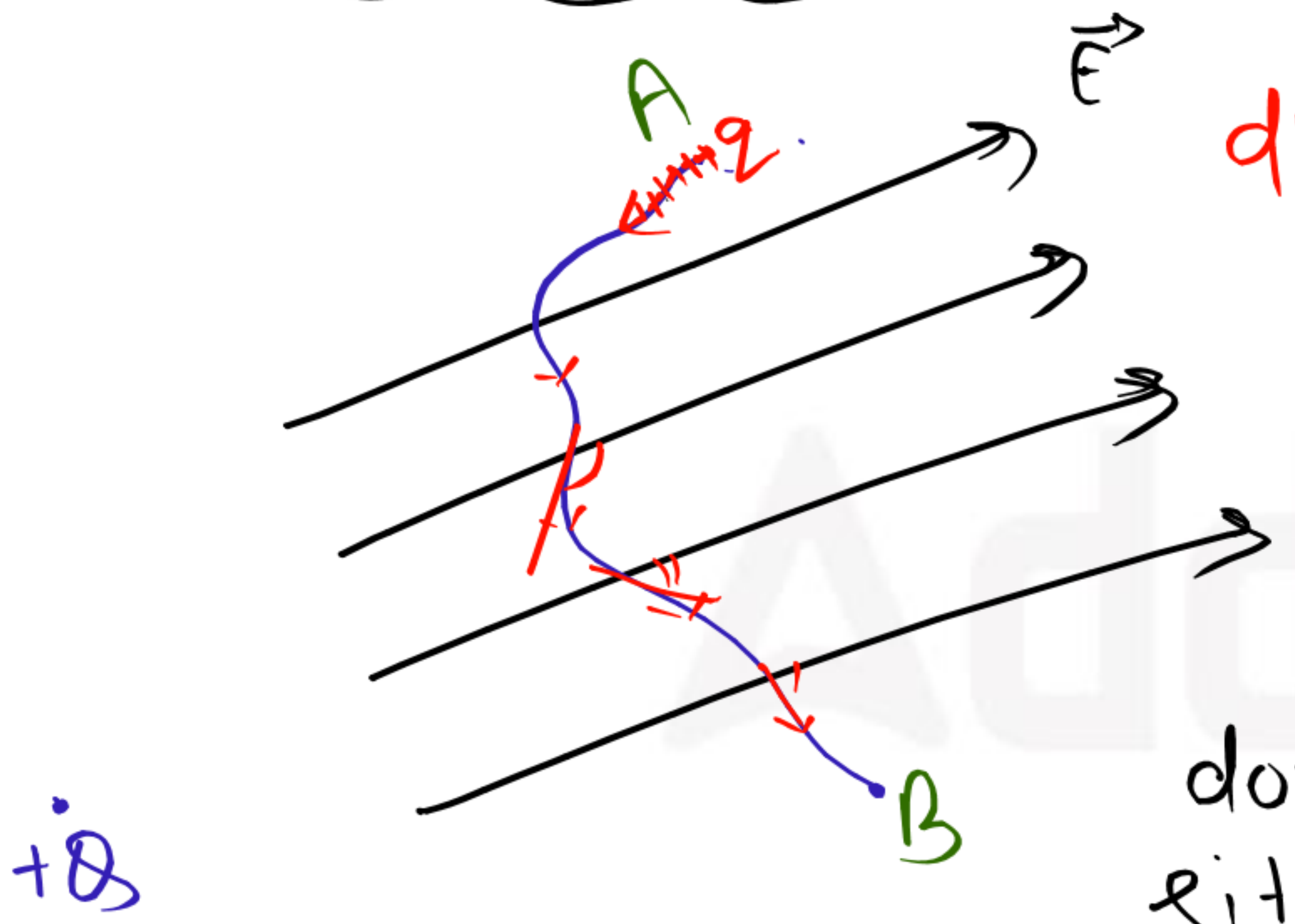
Electric Potential :-

$\vec{F} = q\vec{E}$

$\vec{F_e} \rightarrow$ Newton

$\vec{E} \rightarrow N/C$

$\vec{D} \rightarrow Cou/m^2$



$dw = -\vec{F} \cdot \vec{dl}$

Here -ve sign represents that work is done by external agent in opposition of field.

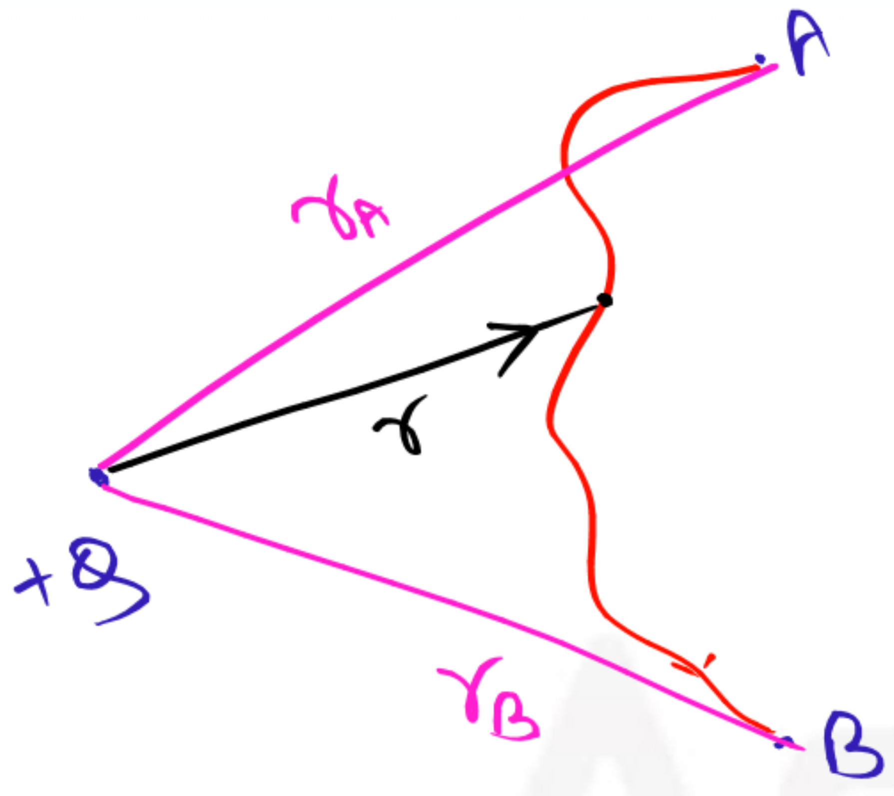
$$W_{AB} = \int_A^B dw = -q\int_A^B \vec{E} \cdot \vec{dl}$$

$$W_{AB} = -q\int_A^B \vec{E}\cdot\vec{dl}$$

Here W_{AB} is the work done to displace the charge 'q' from 'A' to 'B'.

Potential difference :→ It is work done to displace a test charge from point 'A' to 'B' in electric field space

$$= \frac{W_{AB}}{q} = -\int_A^B \vec{E}\cdot\vec{dl}$$



$$V_{BA} = -\int_A^B \vec{E}\cdot\vec{dl}$$

$$V_{BA} = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \cdot \left(dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi\right)$$

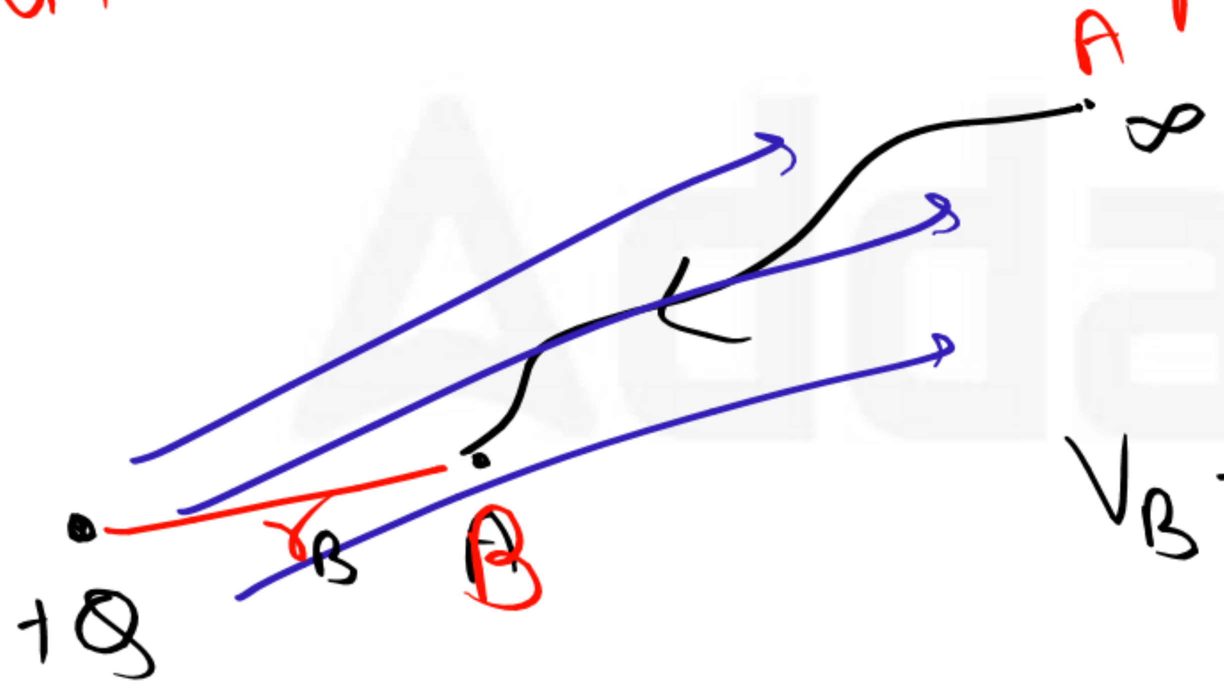
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

$$V_{BA} = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}dr = +\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_A^B$$

$$V_{BA} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = \frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}$$

$$= V_B - V_A$$

Potential at a point :→ potential difference with respect to a refference point at which potential is zero is potential at this point.



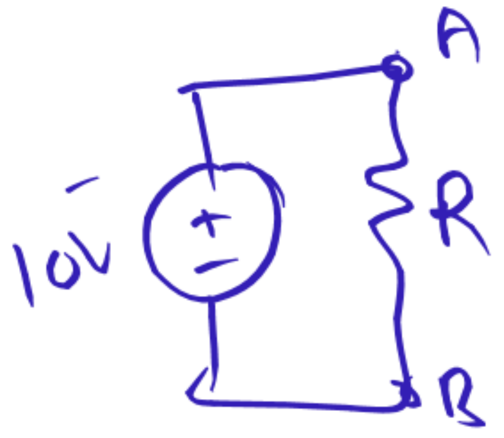
$$V_B = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_A^B$$

$$\frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}\Bigg|_{r_A \to \infty}$$

$$= \frac{Q}{4\pi\varepsilon_0 r_B}$$

q

$$V = \frac{q}{4\pi\varepsilon_0 r}$$



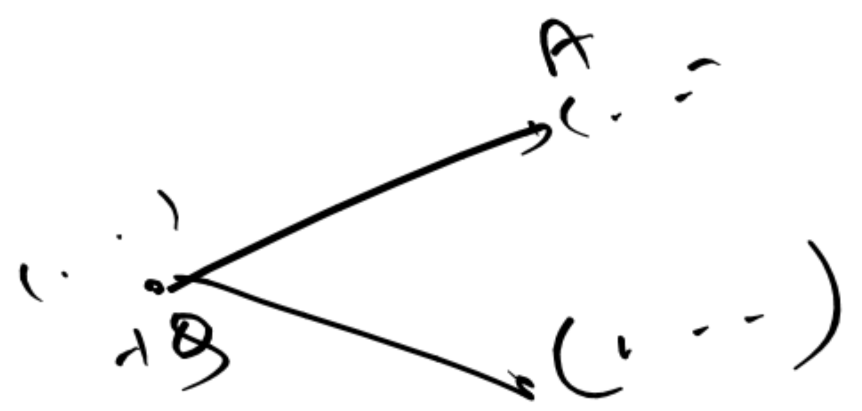
$V_{AB} = 10V$

$V_{BA} = -10V$

$$V_{BA} = -\int_A^B \vec{E}\cdot\vec{dl} = V_B - V_A$$

$V_A - V_B = 10V$

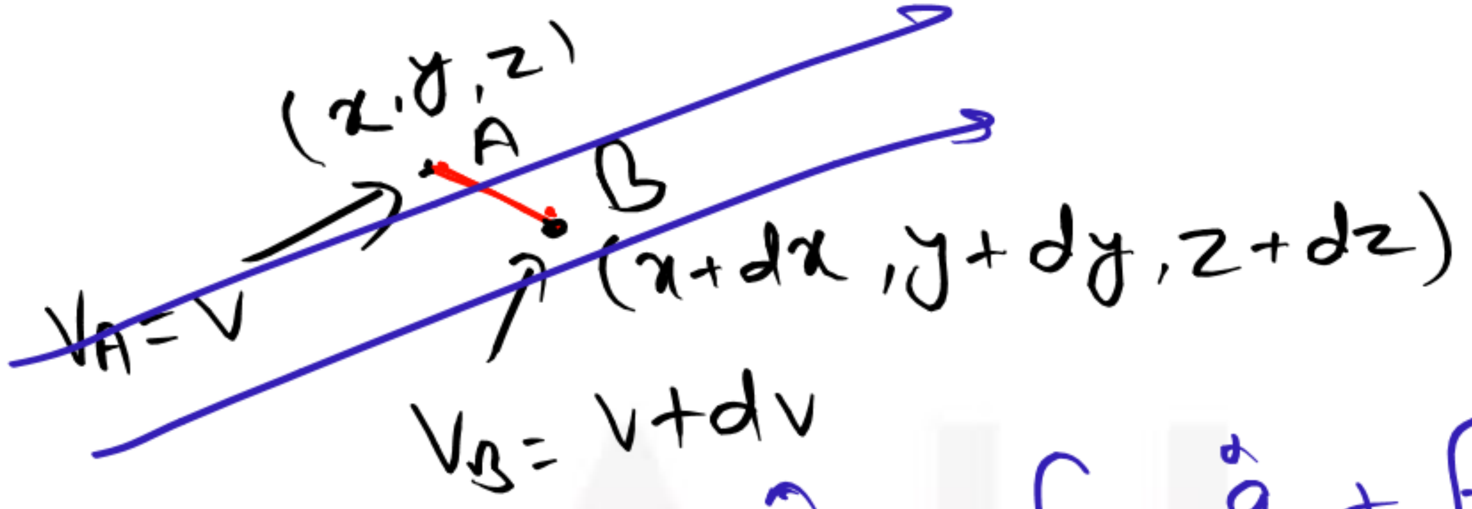
① $V_{BA} = -\int_A^B \vec{E} \cdot \vec{dl}$



② $V_{BA} = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r_B} - \dfrac{1}{r_A}\right)$

③ $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

Relation Between Electric Field Intensity and Electric Potential



$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$dW = -\vec{F} \cdot \vec{dl}$$

$$dV = \frac{dW}{q} = -\vec{E} \cdot \vec{dl} = -(E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z) \cdot (dx\, \hat{a}_x + dy\, \hat{a}_y + dz\, \hat{a}_z)$$

$$dV = -(E_x dx + E_y dy + E_z dz) \quad -(1)$$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \quad \text{—(2)}$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$V(x, y, z)$

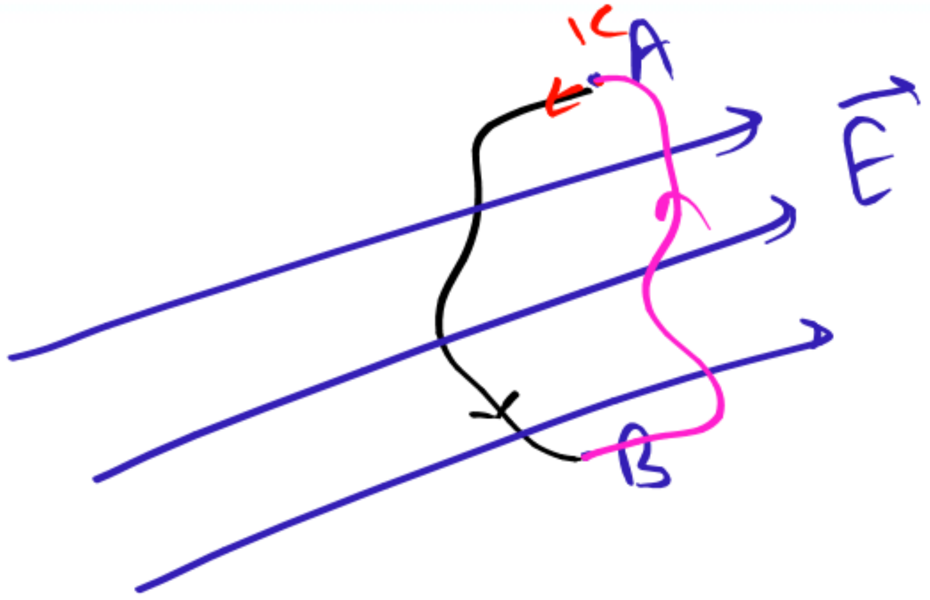
$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$= -\left(\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z\right)$$

$$\boxed{\vec{E} = -\nabla V}$$

* Electric field vector will be oriented in the direction in which electric potential is decreasing.

* Electric field is oriented from high potential to low potential.



$$V_{BA} = -\int_A^B \vec{E}\cdot\vec{dl}$$

$$V_{AB} = -\int_B^A \vec{E}\cdot\vec{dl} \Rightarrow V_{AB} = -V_{BA}$$

$$V_{AB} + V_{BA} = 0$$

$$\oint \vec{E}\cdot\vec{dl} = 0$$

Applying stoke's theorem

$$\int(\nabla\times\vec{E})\cdot\vec{ds} = \int 0\cdot\vec{ds}$$

integral form

$$\Rightarrow \nabla\times\vec{E} = 0$$

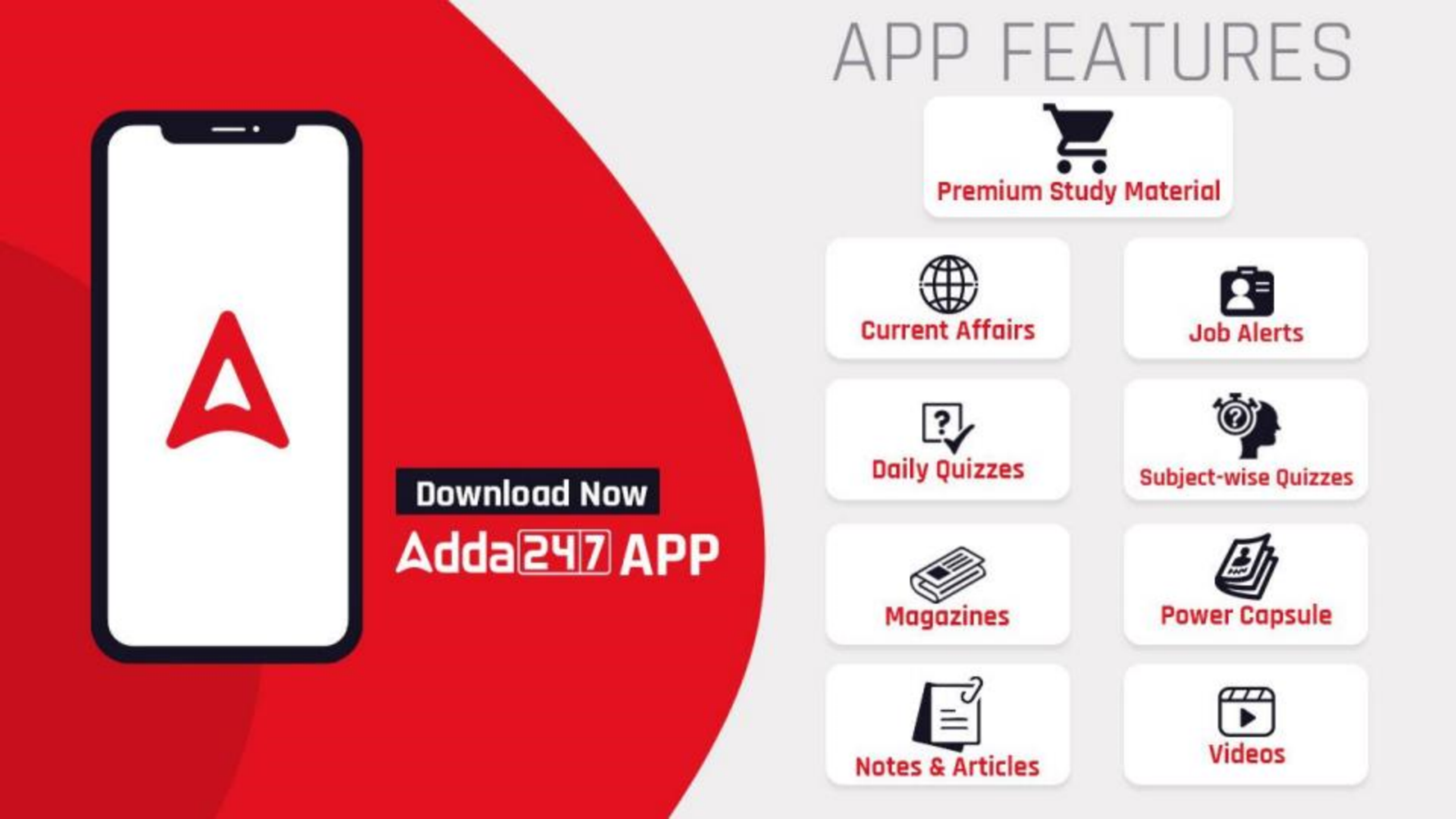
maxwell's 2nd eqn in differential form

$\nabla \cdot \vec{D} = \rho_v$ ← gauss law

$\nabla \times \vec{E} = 0$

⇒ Static electric field is having curl zero so it is irrotational vector i.e. it is straight vector.

* As electric field is conservative so its line integral is independent of path.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