Question Practice on Electrostatic Field in Free Space

Recap

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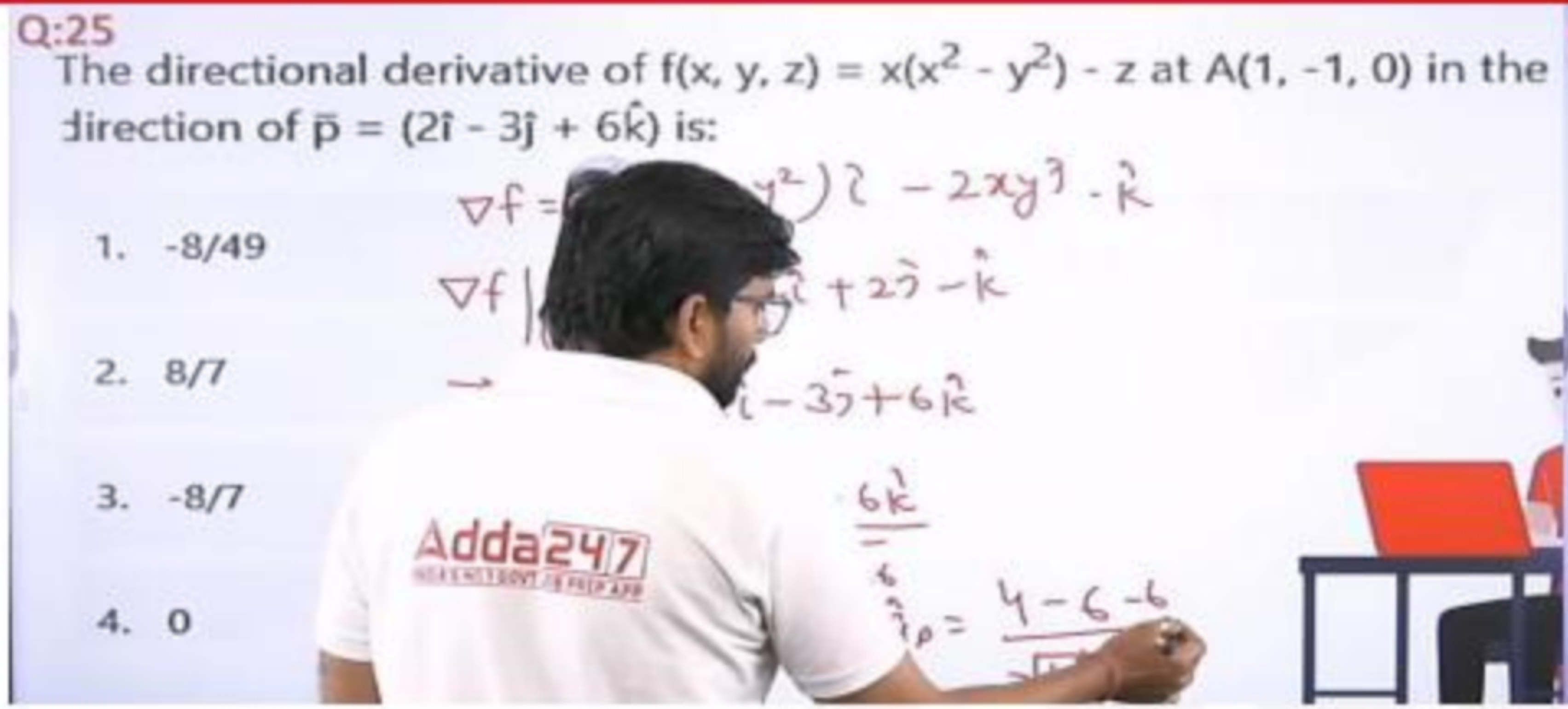
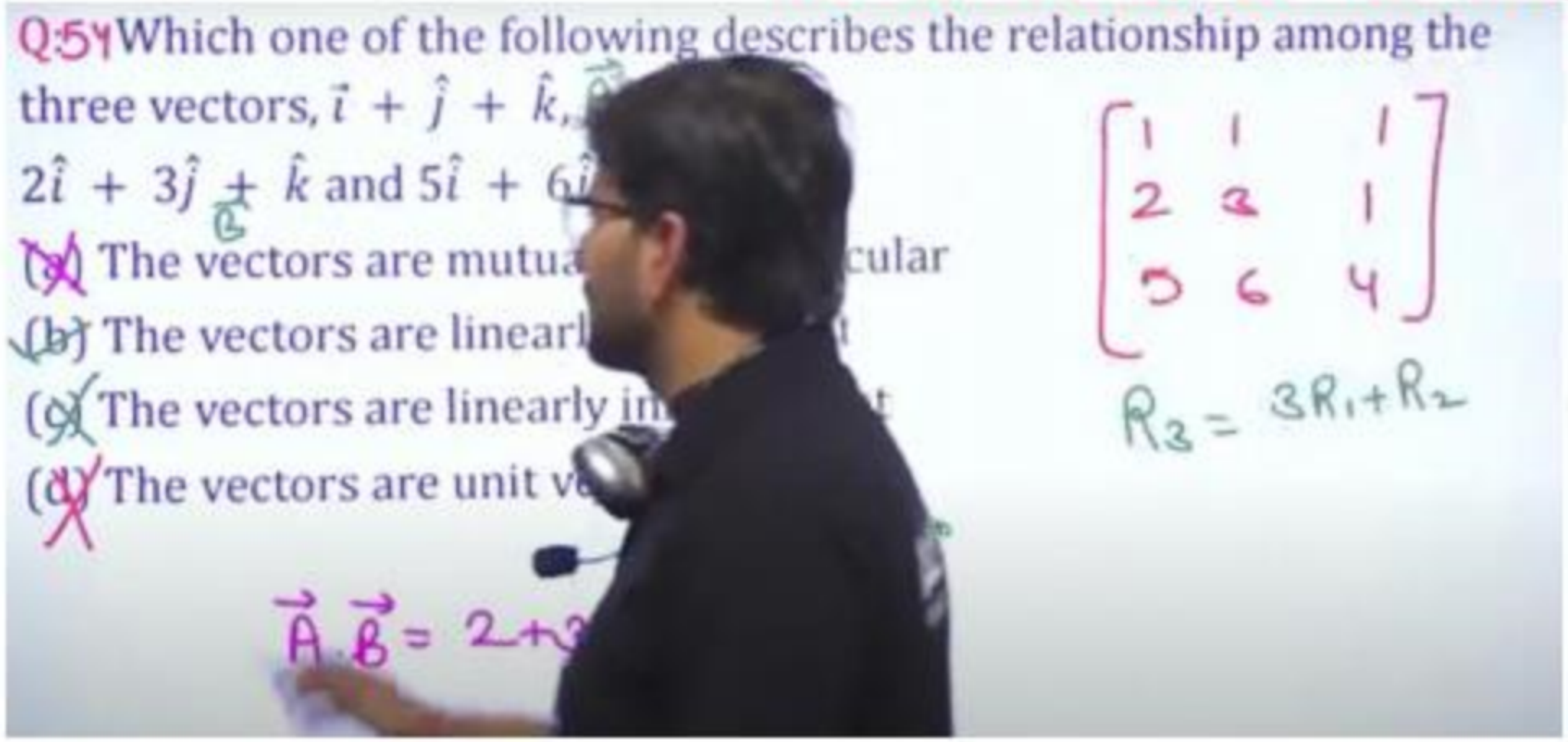
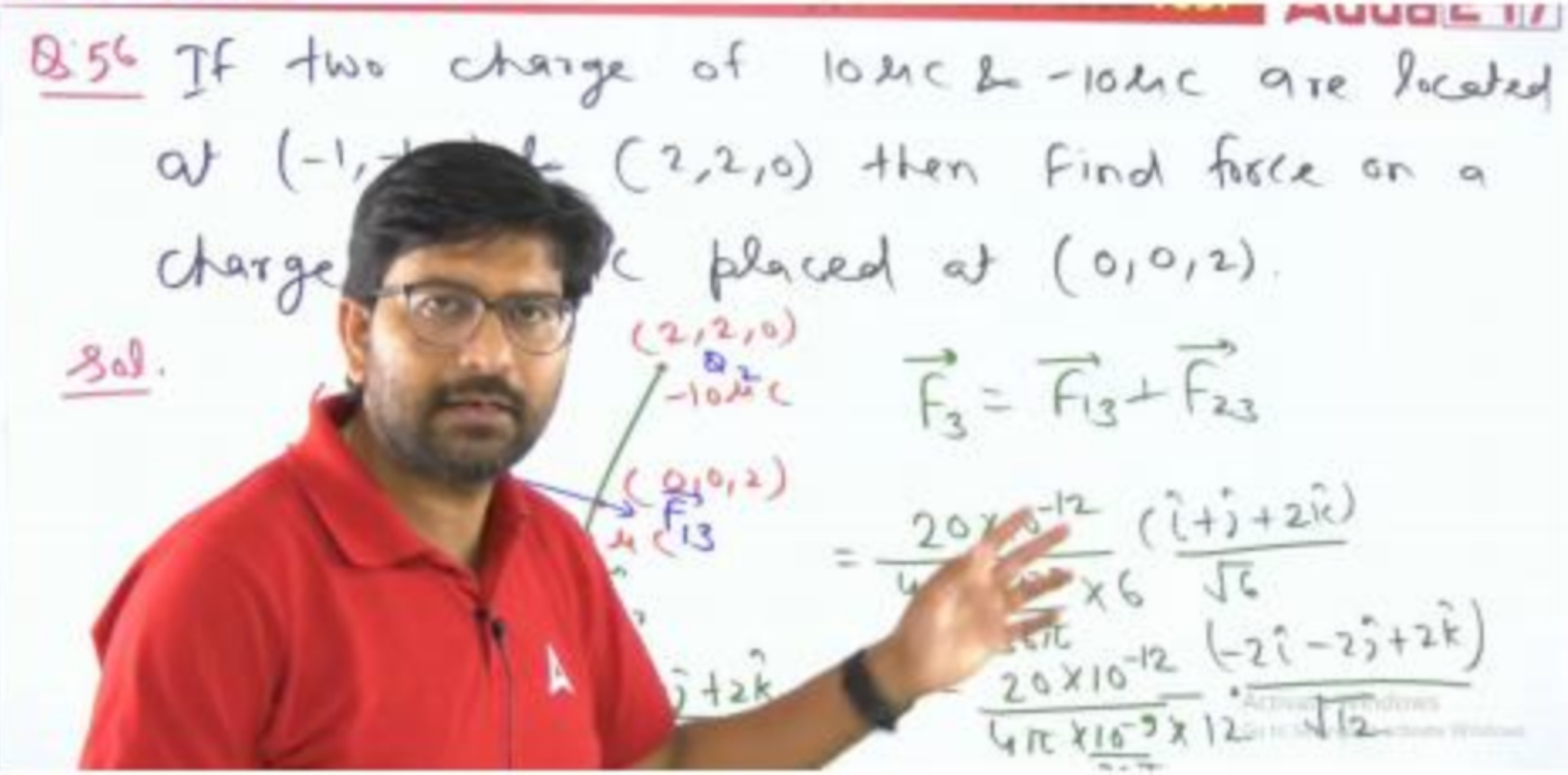


Number of Questions covered-56





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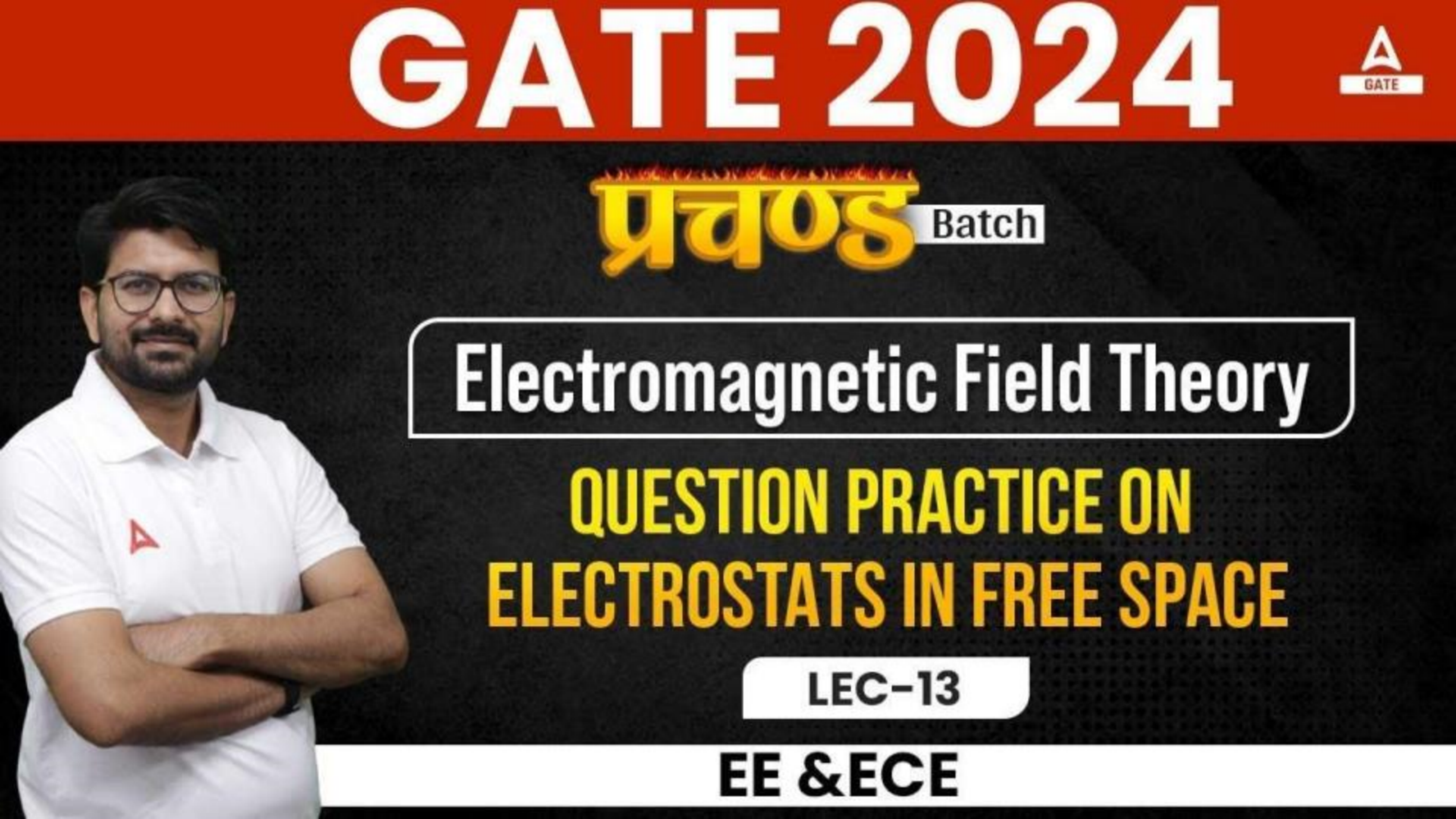
GATE 2024
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प्रचण्ड Batch
Electromagnetic Field Theory
QUESTION PRACTICE ON
ELECTROSTATS IN FREE SPACE
LEC-13
EE &ECE

1. Basic introduction of Fields
2. Basics of Vectors
3. Coordinate Systems
4. Vector Integrals
5. Vector differentials
6. Coulomb's law and Gauss law
7. Field due to line, Surface and Spherical Volume Charge
8. Electric Potential

Question Practice on Electrostatic Field in Free Space

Up to Now → $\underline{\underline{56}}$

Q:57 An electrostatic field is said to be conservative when:

(a) The divergence of the field is equal to zero

(b) The curl of the field is equal to zero

(c) The curl of the field is equal to $-\frac{\partial E}{\partial t^2}$

(d) The Laplacian of the field is equal to $\mu\varepsilon \frac{\partial^2 E}{\partial t^2}$

$\nabla \times \vec{A} = 0$

$\nabla \times \vec{E} = 0$

$V_{AB} + V_{BA} = 0$

$\oint \vec{E} \cdot \vec{dl} = 0$

$\int (\nabla \times \vec{E}) \vec{ds} = 0$

$\nabla \times \vec{E} = 0$

Q:58 For a uniformly charged sphere of radius R and charge density ρ, the ratio of magnitude of electric fields at distances R/2 and 2R from the center, i.e., $\frac{E(r=R/2)}{E(r=2R)}$ is 2.

$$E = \begin{cases} \frac{\rho_v}{3\varepsilon_0} r & r < R \\ \frac{\rho_v}{3\varepsilon_0} \frac{R^3}{r^2} & r > R \end{cases}$$

$$E\left(r=\frac{R}{2}\right) = \frac{\rho_v}{3\varepsilon_0} \frac{R}{2}$$

$$E[r=2R] = \frac{\rho_v}{3\varepsilon_0} \frac{R^3}{4R^2} = \frac{\rho_v}{3\varepsilon_0} \frac{R}{4}$$

$$\frac{E\left[r=\frac{R}{2}\right]}{E[r=2R]} = \frac{\frac{1}{2}}{\frac{1}{4}} = 2:1$$

Q:59 In the infinite plane, y = 6 m, there exists a uniform surface charge density of $(1/6000\pi)\mu C/m^2$. The associated electric field strength is

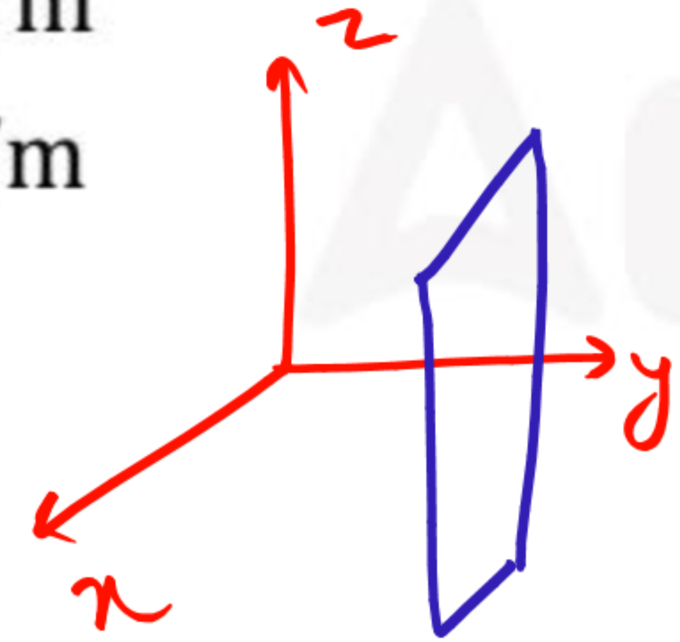
(a) $30\hat{i}$ V/m

(b) $3\hat{j}$ V/m

(c) $30\hat{k}$ V/m

(d) $60\hat{j}$ V/m

Sol: $\vec{E} = \dfrac{\rho_s}{2\varepsilon_0}\hat{a}_n$



$$\vec{E} = \frac{1}{6000\cancel{\pi}} \times 10^{-6} \times \frac{1}{\frac{2\times10^{-9}}{36\cancel{\pi}}}\hat{j}$$

$$= \frac{36}{12}\hat{j} = 3\hat{j}$$

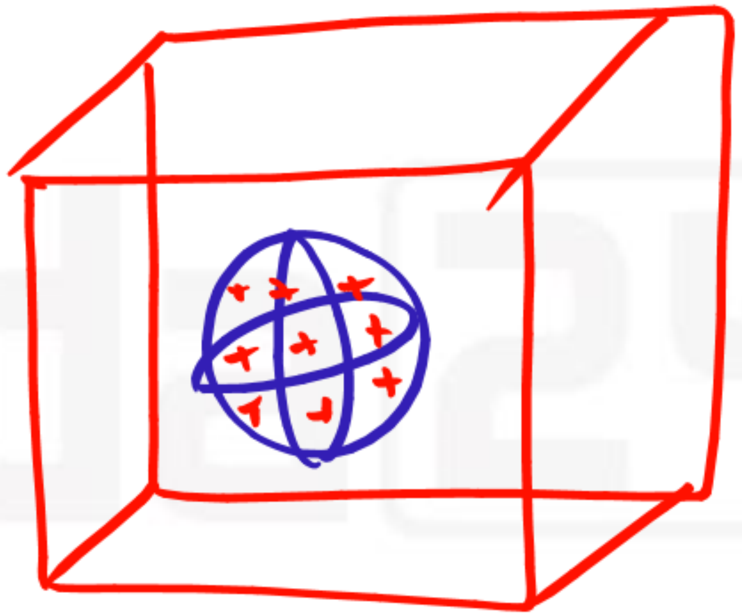
Q:60 A metal sphere with 1 m radius and a surface charge density of 10 Coulombs/m^2 is enclosed in a cube of 10 m side. The total outward electric displacement flux normal to the surface of the cube is

(a) 40π Coulombs

(b) 10π Coulombs

(c) 5π Coulombs

(d) None of the above



$\psi_{net} = ??$

from gauss law $\psi_{net} = Q_{enc} = 10 \times 4\pi (1m)^2$

$= 40\pi$ Coulomb

Q: 61. If the electric field intensity is given by

$\vec{E} = (x\hat{u}_x + y\hat{u}_y + z\hat{u}_z)$ Volt /m the potential difference between X(2, 0, 0) and Y(1, 2, 3) is

(a) + 1 volt

(b) - 1 volt

(c) + 5 volt

(d) + 6 volt

(E) -5 volt

$$V_{BA} = -\int_A^B \vec{E}\cdot\vec{dl} = V_B - V_A$$

potential diff. between 'B' and 'A'

$B \rightarrow X, \quad A \rightarrow Y$

$$= -\int_{(1,2,3)}^{(2,0,0)} (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

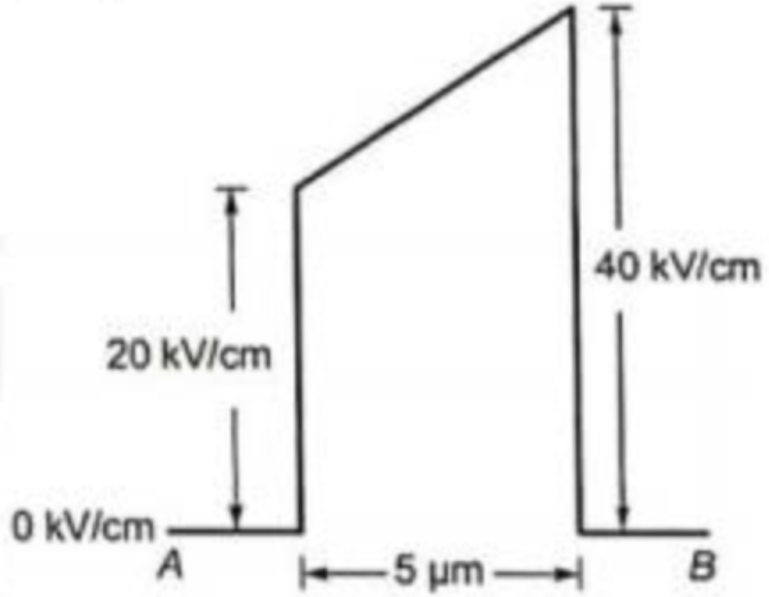
$$-\left[\int_1^2 x\,dx + \int_2^0 y\,dy + \int_3^0 z\,dz\right]$$

$$- \left[\left(\frac{x^2}{2}\right)_1^2 + \left(\frac{y^2}{2}\right)_2^0 + \left(\frac{z^2}{2}\right)_3^0 \right]$$

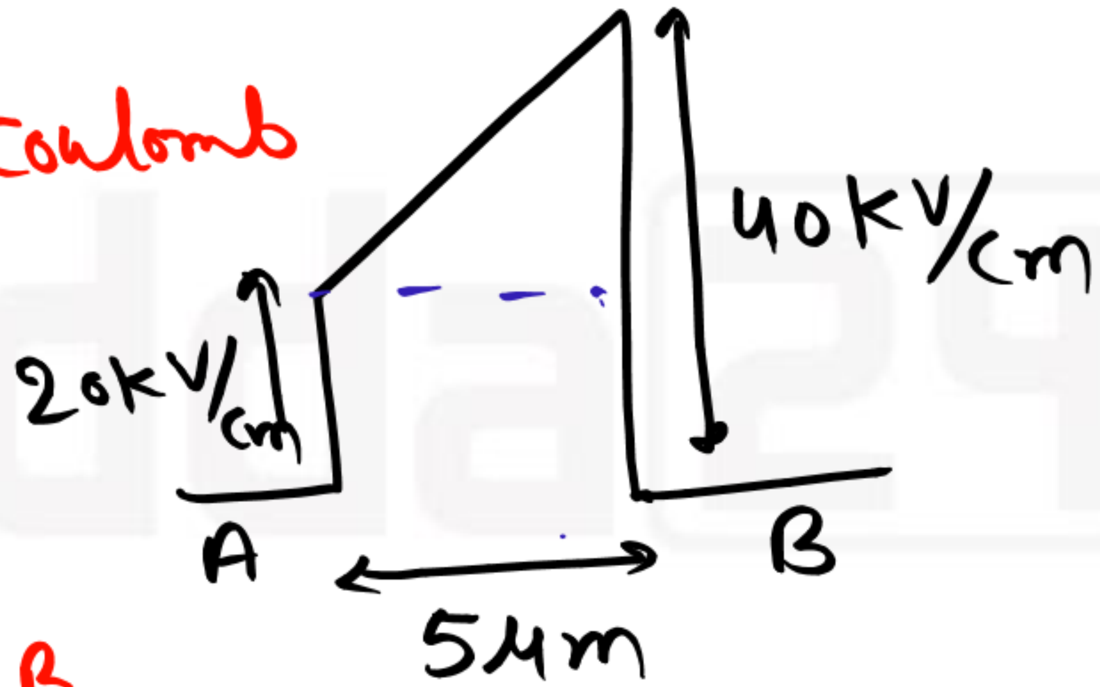
$$- \left(\frac{3}{2} - \frac{4}{2} - \frac{9}{2}\right)$$

$$= +5 \ \underline{\text{Volt}}$$

Q:62 The electric field (assumed to be one - dimensional) between two points A and B is shown. Let ψ_A and ψ_B be the electrostatic potentials at A and B, respectively. The value of $\psi_A - \Psi_B$ in Volts is ________.



Sol: $\vec{E} = \frac{\vec{F}}{q} \rightarrow$ Newton/Coulomb



$-\int \vec{E}\cdot d\vec{l} =$ Volt

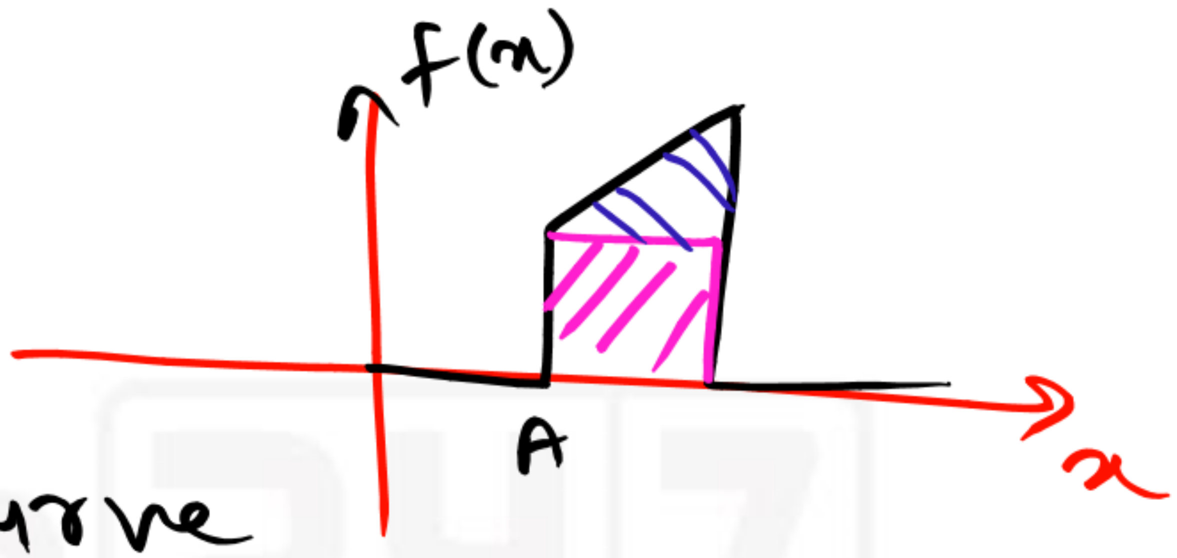
$\cdot\, E \rightarrow V/m$

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = \Psi_B - \Psi_A$$

$$\vec{E} = f(x)\,\hat{a}_x$$

$$\psi_A - \psi_B = \int_A^B \vec{E}\cdot\vec{dl}$$

$$= \int_A^B f(x)\,dx$$



= Area under curve

$$= 5\times10^{-6} \times \frac{20\times10^{3}}{10^{-2}} + \frac{1}{2}\times 5\times10^{-6}\times\frac{20\times10^{3}}{10^{-2}}$$

$$= 10 + 5$$

$$= 15\,V$$

Q:63 If $\vec{E} = -(2y^3 - 3yz^2)\hat{x} - (6xy^2 - 3xz^2)\hat{y} + (6xyz)z$ is the electric field in a source free region, a valid expression for the electrostatic potential is

(a) $xy^3 - yz^2$

(b) $2xy^3 - xyz^2$

(c) $y^3 + xyz^2$

(d) $2xy^3 - 3xyz^2$

$\vec{E} = -\nabla V$

$V_{BA} = -\int_A^B \vec{E} \cdot \vec{dl}$

Option: A $\quad -\nabla V = y^3 \hat{a}_x$

Option: B $\quad -(2y^3 - yz^2)\hat{a}_x$

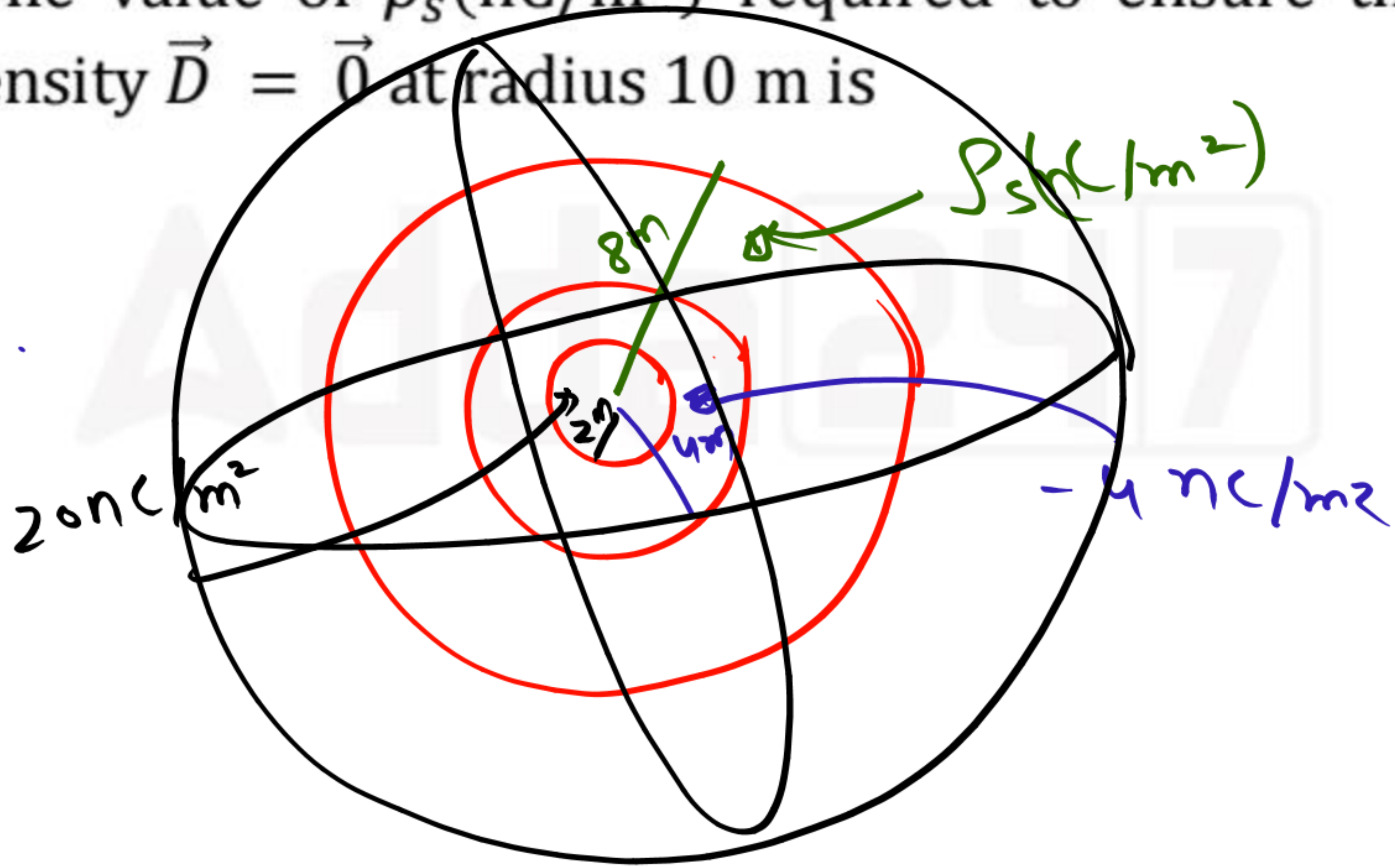
option: c

yz^2

option: D

$-(2y^3 - 3yz^2)\hat{a}_x - (6xy^2 - 3xz^2)\hat{a}_y + 6xyz\,\hat{a}_z$

Q:64 Concentric spherical shells of radii 2 m, 4 m, and 8 m carry uniform surface charge densities of $20 nC/m^2, -4 nC/m^2$ and ρ_s, respectively. The value of $\rho_s (nC/m^2)$ required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is



$$\Psi_{net} = Q_{enc}$$

$$\oint \vec{D} \cdot \vec{ds} = Q_{enc}$$

$$\rho_s = -0.25\ nC/m^2$$

required $\vec{D} = 0$

$$Q_{enc} = 0$$

$$Q_{enc} = Q_1 + Q_2 + Q_3 = 0$$

$$20 \times 4\pi(2)^2 - 4 \times 4\pi(4)^2 + \rho_s \times 4\pi(8)^2 = 0$$

$$80 - 64 + \rho_s \times 64 = 0$$

$$\rho_s = -\frac{16}{64} = -\frac{1}{4}$$

Q:65 Consider the vector field $\vec{F} = \hat{a}_x(4y - c_1 z) + \hat{a}_y(4x + 2z) + \hat{a}_z(2y + z)$ in a rectangular coordinate system (x, y, z) with unit vectors $\hat{a}_x, \hat{a}_y$, and $\hat{a}_z$. If the field $\vec{F}$ is irrotational (conservative), then the constant c_1 (in integer) is 0

Sol:

$$\nabla \times \vec{F} = 0$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4y - c_1 z & 4x + 2z & 2y + z \end{vmatrix} = 0\hat{a}_x + c_1\hat{a}_y + 0\hat{a}_z = 0$$

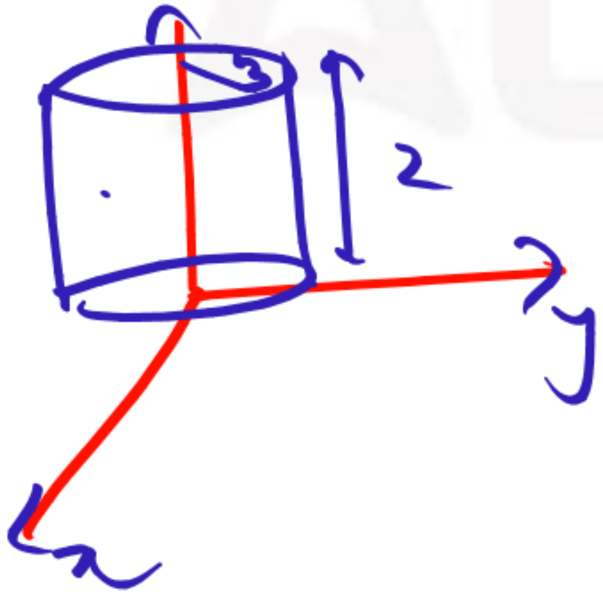
$$c_1 \hat{a}_y = 0$$

$$\Rightarrow c_1 = 0$$

Q:66 For a vector field $\vec{D} = \rho\cos^2\varphi\,\hat{a}_p + z^2\sin^2\varphi\,\hat{a}_\varphi$ in a cylindrical coordinate system (ρ, φ, z) with unit vectors $\hat{a}_\rho, \hat{a}_\varphi$ and $\hat{a}_z$, the net flux of $\vec{D}$ leaving the closed surface of the cylinder (ρ = 3, 0 ≤ z ≤ 2) (rounded off to two decimal places) is

Sol:

$$\vec{D} = \rho\cos^2\phi\,\hat{a}_\rho + z^2\sin^2\phi\,\hat{a}_\phi$$



$$\oint \vec{D}\cdot\vec{ds} = \int_{top} \vec{D}\cdot\vec{ds} + \int_{bottom} \vec{D}\cdot\vec{ds} + \int_{cyl} \vec{D}\cdot\vec{ds}$$

divergence theorem

$$\int (\nabla\cdot\vec{D})\,dv$$

$$\nabla \cdot \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot \rho \cos^2\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(z^2 \sin^2\phi) + \frac{\partial}{\partial z}(0)$$

$$= 2\cos^2\phi + \frac{z^2}{\rho} 2\sin\phi\cos\phi$$

$$\int \left(2\cos^2\phi + \frac{z^2}{\rho}\sin 2\phi\right)\rho\, d\rho\, d\phi\, dz$$

$$\int \left(\rho^2\cos^2\phi + \rho z^2 \sin 2\phi\right)^3 d\phi\, dz$$

$$\int \left(\rho z \cos^2\phi + z^3 \sin 2\phi\right)_0^2 d\phi$$

$$\int_0^{2\pi}\left(18\cos^2\phi+8\sin 2\phi\right)d\phi$$

$$\int_0^{2\pi}18\left(\frac{1+\cancel{\cos 2\phi}}{2}\right)d\phi+\int_0^{2\pi}\cancel{8\sin 2\phi\, d\phi}$$

$$0 \qquad\qquad 0$$

$$9\times 2\pi$$

$$=18\pi$$

Q:67 An electrostatic potential is given by $\phi = 2x\sqrt{y}$ volts in the rectangular co - ordinate system. The magnitude of the electric field at x = 1 m, y = 1 m is V/m.

Sol:

$\vec{E} = -\nabla V$

$V = \phi = 2x\sqrt{y}$

$\vec{E} = -\left[2\sqrt{y}\,\hat{i} + \frac{2x \cdot 1}{2\sqrt{y}}\hat{j}\right]$

$\vec{E}\Big|_{(x=1,y=1)} = -(2\hat{i}+\hat{j}) = -2\hat{i}-\hat{j}$

$|\vec{E}| = \sqrt{4+1} = \sqrt{5}$

Q: 68 In electrostatic field, $\nabla \times \vec{E} = 0$ (True/False).

Sol:

$\nabla \times \vec{E} = 0$

Q:69 If V, W, q stands for voltage, energy and charge, then V can be expressed as

(a) $V = \frac{dq}{dW}$

(b) $V = \frac{dW}{dq}$

(c) $dV = \frac{dW}{dq}$

(d) $dV = \frac{dq}{dW}$

$dw = -\vec{F} \cdot \vec{dl}$

$W_{AB} = -\int_A^B \vec{F} \cdot \vec{dl}$

$dv = \frac{dw}{dq}$

Q:70 Given the potential function in free space to be $V(x) = (50x^2 + 50y^2 + 50z^2)$ volts, the magnitude (in volts/metre) and the direction of the electric field at a point (1, - 1, 1), where the dimensions are in metres, are

(a) 100; $(\hat{i} + \hat{j} + \hat{k})$

(b) $\frac{100}{\sqrt{3}}$; $(\hat{i} - \hat{j} + \hat{k})$

(c) $100\sqrt{3}$; $[(\hat{i} + \hat{j} - \hat{k})/\sqrt{3}]$

(d) $100\sqrt{3}$; $[(-\hat{i} + \hat{j} - \hat{k})/\sqrt{3}]$

$\vec{E} = -\nabla V$

$\vec{E} = -[100x\hat{i} + 100y\hat{j} + 100z\hat{k})$

$\vec{E}\Big|_{(1,-1,1)} = -(100\hat{i} - 100\hat{j} + 100\hat{k})$

$|\vec{E}| = 100\sqrt{3}$

$\hat{a}_E = \frac{\vec{E}}{|\vec{E}|} = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$

Q:71 The electric field $\vec{E}$ (in volts/meter) at the point (1, 1, 0) due to a point charge of + 1μC located at (- 1, 1, 1) (co - ordinates in meters) is

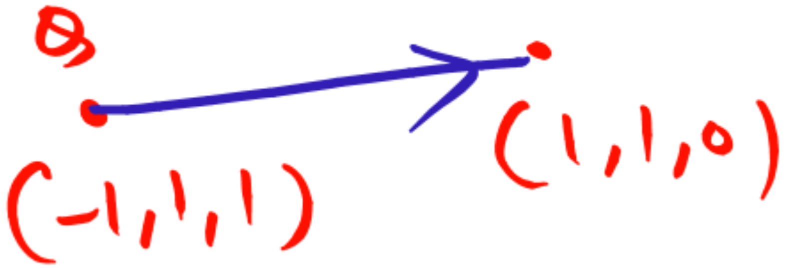
(a) $\frac{10^{-6}}{20\sqrt{5}\pi\varepsilon_0}(2i - k)$

(b) $\frac{10^{-6}}{20\pi\varepsilon_0}(2i - k)$

(c) $\frac{-10^{-6}}{20\sqrt{5}\pi\varepsilon_0}(2i - k)$

(d) $\frac{-10^{-6}}{20\pi\varepsilon_0}(2i - k)$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$



$$\vec{R} = 2\hat{i} - \hat{k}$$

$$r = \sqrt{5}, \quad \hat{a}_r = \frac{2\hat{i}-\hat{k}}{\sqrt{5}}$$

$$\vec{E} = \frac{10^{-6}}{4\pi \times \frac{10^{-9}}{36\pi} \times 5} \frac{2\hat{i}-k}{\sqrt{5}}$$

$$\vec{E} = \frac{(2\hat{i}-\hat{k}) \times 10^{-6}}{20\pi\varepsilon_0\sqrt{5}}$$

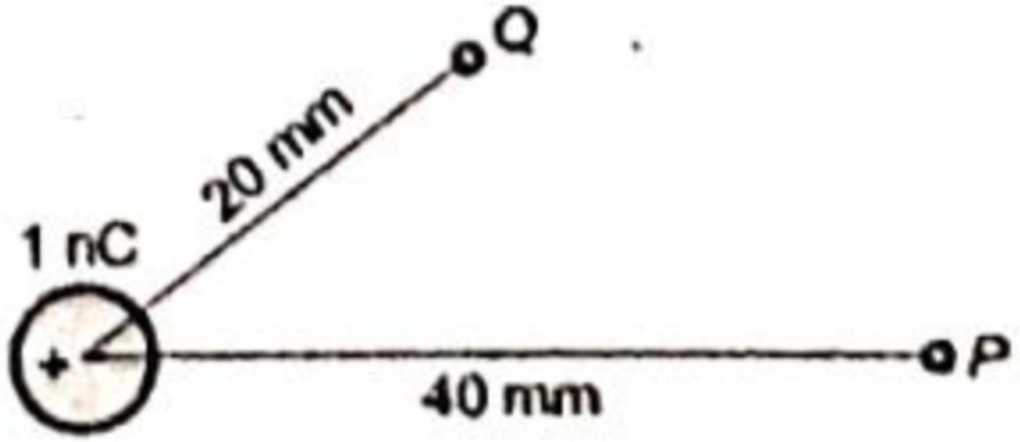
Q:72 A point charge of + 1nC is placed in a space with permittivity of 8.85×10^{-12} F/m as shown in figure. The potential difference V_{PQ} between two points P and Q at distances of 40 mm and 20 mm respectively from the point charge is

(a) 0.22kV

(b) - 225 V

(c) - 2.24kV

(d) 15 V



$$V_{BA} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$V_{PQ} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_Q}\right) = \frac{10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi}}\left(\frac{1}{40\times10^{-3}} - \frac{1}{20\times10^{-3}}\right)$$

$$9 \times \frac{-1}{40\times10^{-3}}$$

$$= - \frac{9000}{40} = -225 \text{ Volt}$$

Q:73A solid sphere made of insulating material has a radius R and has a total charge Q distributed uniformly in its volume. What is the magnitude of the electric field intensity, E, at a distance r(0<r<R) inside the sphere?

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$

(b) $\frac{3}{4\pi\varepsilon_0}\frac{Qr}{R^3}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{QR}{r^3}$

Q:74 Two electric charges Q and - 2Q are placed at (0, 0) and (6, 0) on the x - y plane. The equation of the zero equipotential curve in the x - y plane is

(a) x = - 2

(b) y = 2

(c) $x^2 + y^2 = 2$

(d) $(x + 2)^2 + y^2 = 16$

Q:75 A positive charge of 1nC is placed at (0, 0, 0.2) where all dimensions are in meters. Consider the x - y plane to be a conducting ground plane. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. The Z component of the E field at (0, 0, 0.1) is closed to

(a) 899.18 V/m

(b) - 899.18 V/m

(c) 999.09 V/m

(d) - 999.09 V/m

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