



today's
topics

Dipole

Fields in Material Space

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Q:25 The directional derivative of $f(x, y, z) = x(x^2 - y^2) - z$ at $A(1, -1, 0)$ in the direction of $\vec{p} = (2\hat{i} - 3\hat{j} + 6\hat{k})$ is:

- 8/49
- 8/7
- 8/7
- 0

Handwritten solution:

$$\nabla f = (2xy^2)\hat{i} - 2xy\hat{j} - \hat{k}$$

$$\nabla f|_A = (2(-1)^2)\hat{i} - 2(1)(-1)\hat{j} - \hat{k} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{p} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$|\vec{p}| = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

$$\vec{p} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

$$\text{Directional derivative} = \nabla f \cdot \vec{p} = (2\hat{i} + 2\hat{j} - \hat{k}) \cdot \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} = \frac{4 - 6 - 6}{7} = \frac{-8}{7}$$

Q:21 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$

(i) Circular path $x^2 + y^2 = 1$ described clockwise.

(ii) The square formed by the lines $x = \pm 1, y = \pm 1$, counter clockwise.

Handwritten solution for (i):

$$\int_C \frac{y dx - x dy}{x^2 + y^2}$$

$$= \int_0^{2\pi} \frac{-\sin^2 \theta - \cos^2 \theta}{1} d\theta = \int_0^{2\pi} -1 d\theta = -2\pi$$

Handwritten solution for (ii):

$$d\vec{r} = -\sin \theta d\theta \hat{i} + \cos \theta d\theta \hat{j}$$

$$\vec{F} = \frac{\sin \theta \hat{i} - \cos \theta \hat{j}}{1}$$

$$\vec{F} \cdot d\vec{r} = -\sin^2 \theta + \cos^2 \theta = \cos 2\theta$$

$$\int_0^{2\pi} \cos 2\theta d\theta = \left[\frac{\sin 2\theta}{2} \right]_0^{2\pi} = 0$$

Number of Questions covered-72

Q:54 Which one of the following describes the relationship among the three vectors, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 4\hat{k}$

(a) The vectors are mutually perpendicular

(b) The vectors are linearly independent

(c) The vectors are linearly independent

(d) The vectors are unit vectors

Handwritten solution:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{bmatrix}$$

$$R_3 = 3R_1 + R_2$$

$\vec{A} \cdot \vec{B} = 2 + 3 + 4 = 9 \neq 0$

Q:56 If two charges of $10\mu\text{C}$ & $-10\mu\text{C}$ are located at $(-1, 1, 0)$ & $(2, 2, 0)$ then find force on a charge $2\mu\text{C}$ placed at $(0, 0, 2)$.

Handwritten solution:

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$= \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 12} \frac{(\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{6}} + \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 12} \frac{(-2\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{12}}$$

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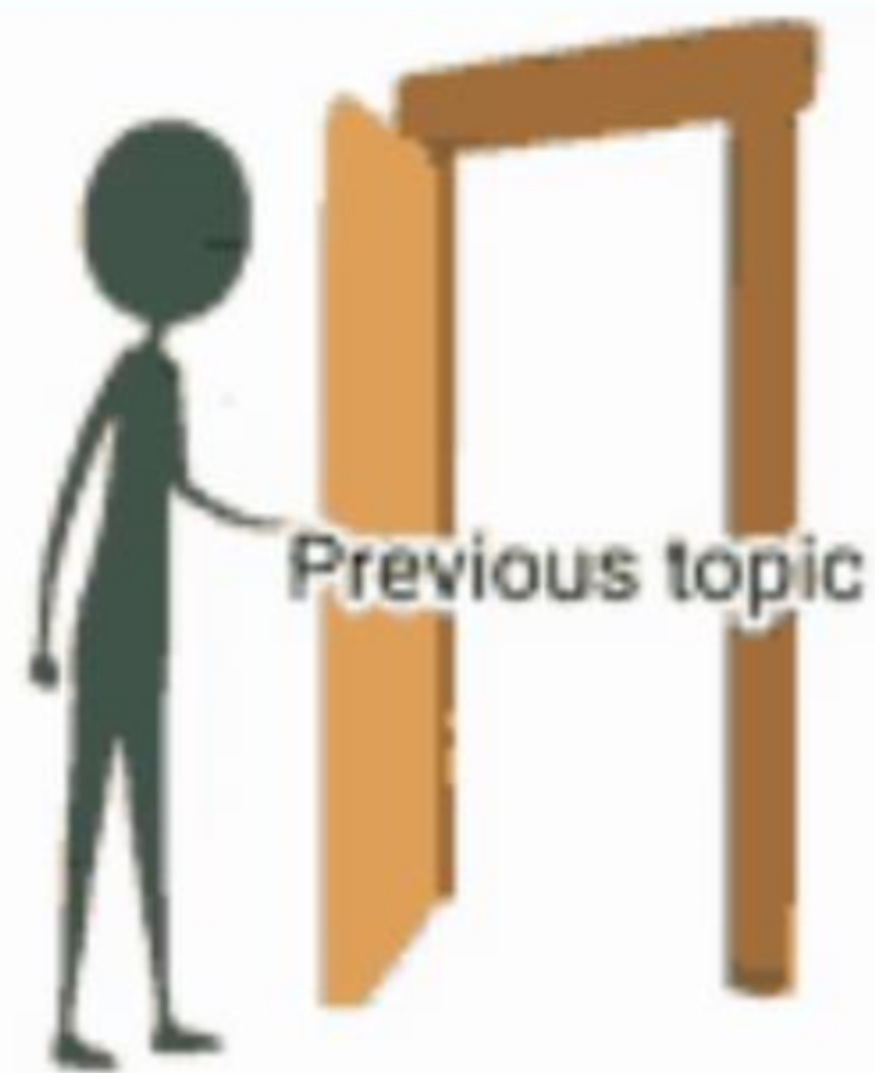
Electromagnetic Field Theory

FIELDS IN MATERIAL SPACE

LEC-14

EE & ECE





- 1. Basic introduction of Fields**
- 2. Basics of Vectors**
- 3. Coordinate Systems**
- 4. Vector Integrals**
- 5. Vector differentials**
- 6. Coulomb's law and Gauss law**
- 7. Field due to line, Surface and Spherical Volume Charge**
- 8. Electric Potential**



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Dipole

Fields in Material Space

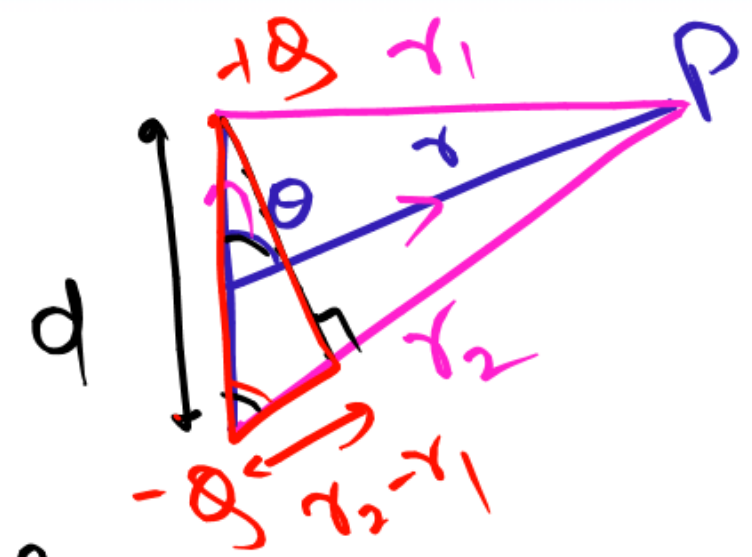
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Dipole and Fields due to dipole

Dipole: \rightarrow It is combination of same amount of positive and negative charge placed at very small distance with each other.



Electric field calculations due to dipole at a point placed at distance r and oriented at an angle ' θ ' from dipole axis.



$$V = V_1 + V_2$$

$$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$$

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{r_2 - r_1}{r_1 r_2} \right) \approx \frac{Q d \cos\theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{Q \vec{d} \cdot \vec{r}}{4\pi\epsilon_0 r^3}$$



$$V = \frac{Q}{4\pi\epsilon_0 r}$$

lim
 $d \rightarrow 0$

\vec{p} i.e. dipole moment is directed from -ve pole to +ve pole.

$Q\vec{d} = \vec{p}$
dipole moment of dipole

① monopole



$$Q \frac{1}{r}$$

$$E \frac{1}{r^2}$$

② Dipole



$$Q \frac{1}{r^2}$$

$$E \frac{1}{r^3}$$

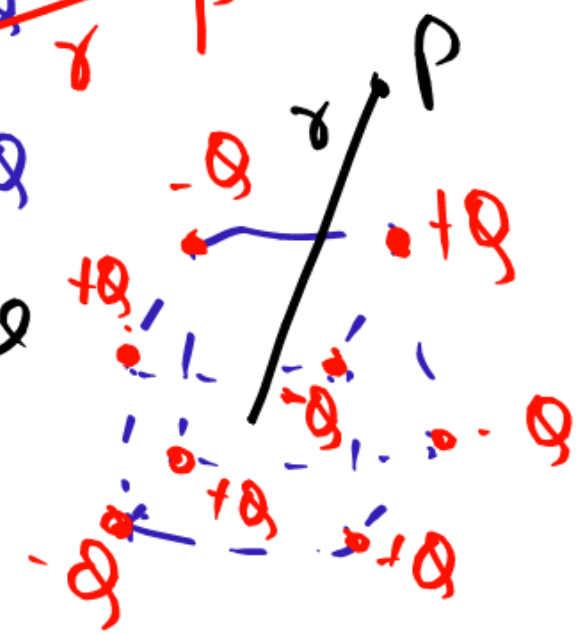
③ Quadrupole



$$Q \frac{1}{r^3}$$

$$E \frac{1}{r^4}$$

④ Octopole



$$Q \frac{1}{r^4}$$

$$E \frac{1}{r^5}$$

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Types of Materials

1. Conductors $\rightarrow \sigma \gg 1$

2. Insulators / dielectric $\rightarrow \sigma \ll 1$

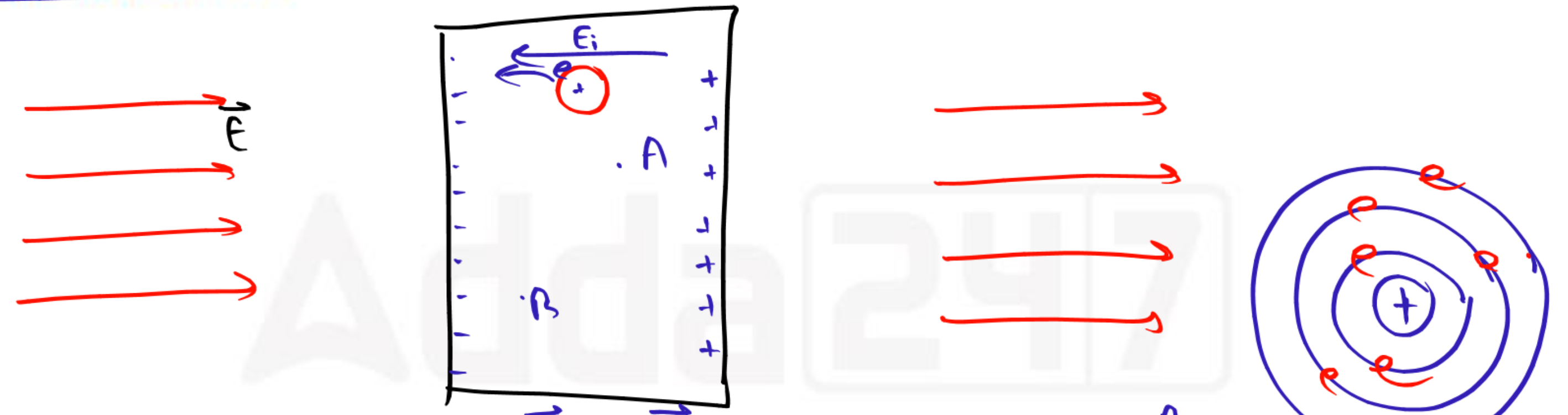
3. Semiconductors \rightarrow Semiconductor physics \rightarrow E.D.C.

Dielectric constant $\rightarrow \epsilon$

Electric Fields in Conductor Space

Isolated Conductors

$$\vec{F} = q\vec{E}$$



At equilibrium

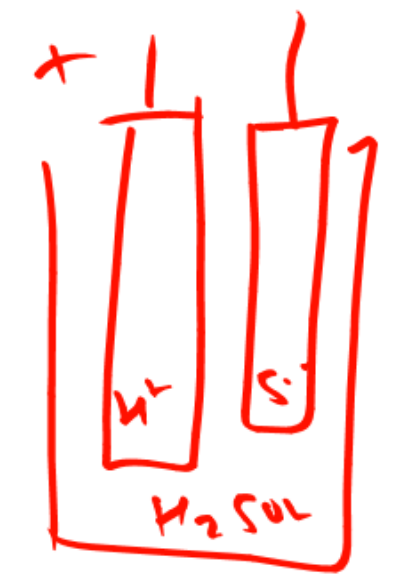
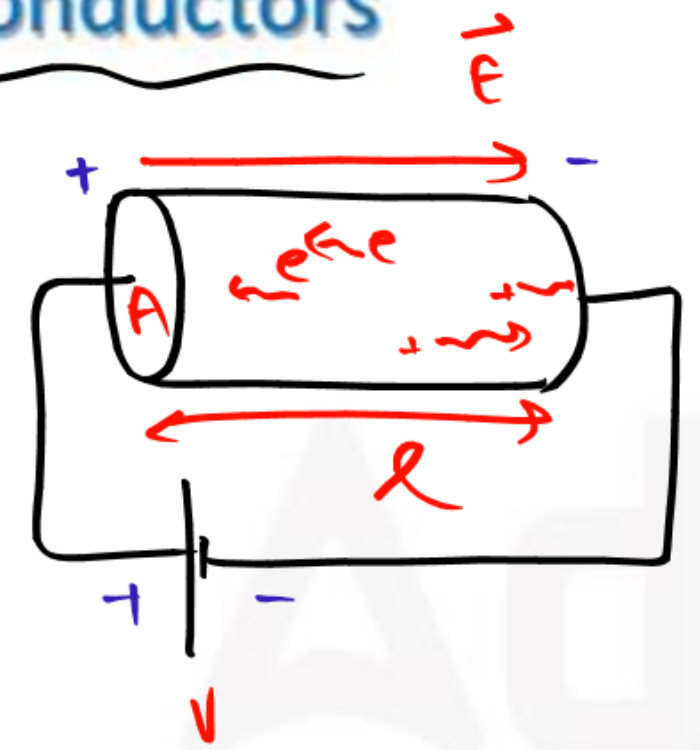
$$\vec{E}_i = -\vec{E} \text{ so } E_{\text{total}} = 0$$

$$V_{BA} = -\int_A^B \vec{E} \cdot d\vec{l} = 0$$

- * electric field inside conductor = 0
- * All inside points are equipotential.

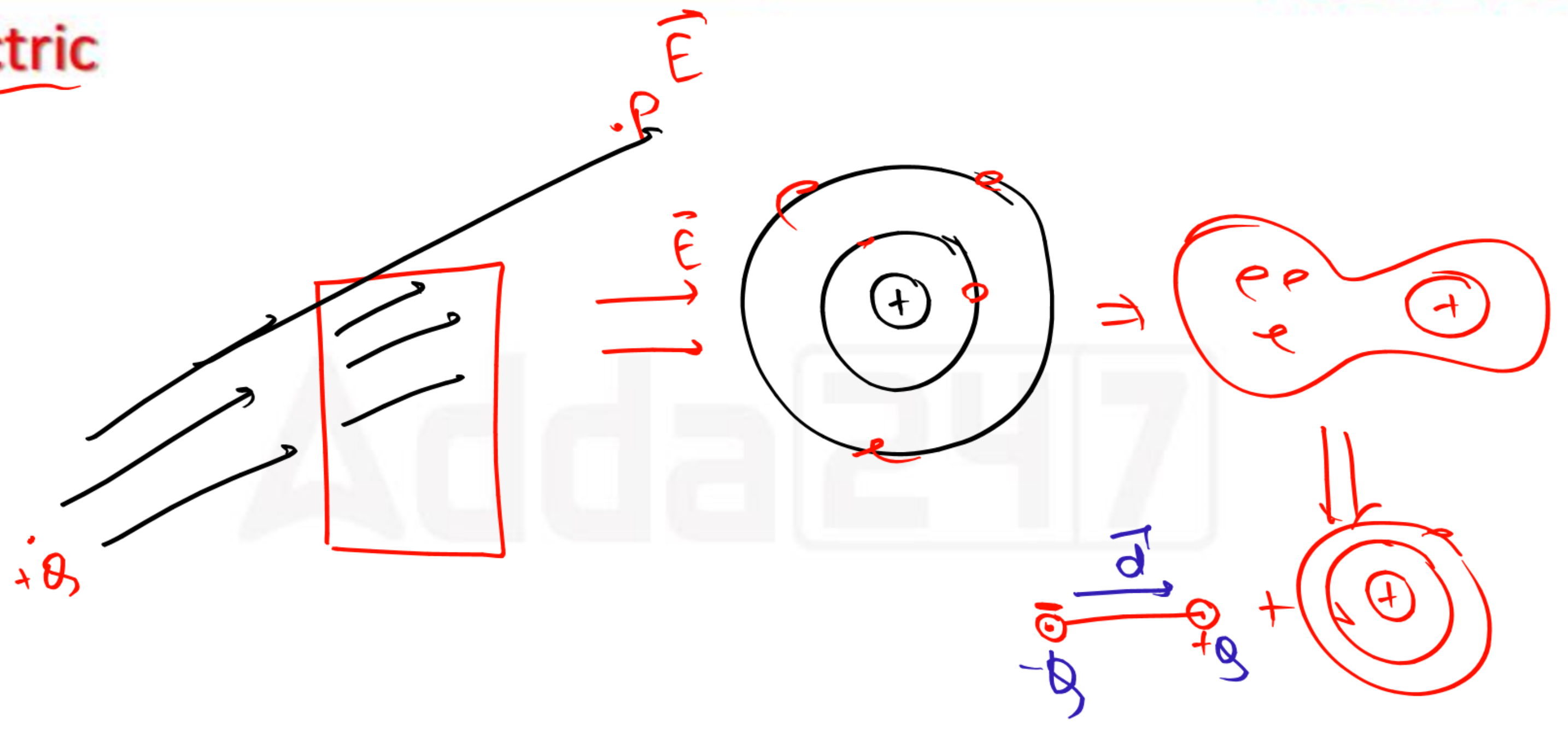
Electric Fields in Conductor Space

Connected Conductors



$$I = \frac{Q}{t}$$
$$I = \frac{V}{R}$$
$$R = \frac{\rho l}{A}$$

Dielectric



$$\vec{P} = \frac{Q\vec{d}_1 + Q\vec{d}_2 + Q\vec{d}_3 + \dots + Q\vec{d}_n}{\Delta V}$$

$$\vec{P} = \frac{\sum_{i=1}^n Q\vec{d}_i}{\Delta V}$$

← Polarisation Vector

$$\vec{P} \propto \vec{E}$$
$$\vec{P} = \chi_e \vec{E}$$

χ_e is called electric susceptibility.
It is the measure of how much a material gets polarized when placed in electric field.

$$\vec{E}_{\text{total}} = \vec{E} + \vec{P}$$

$$\vec{D} = \epsilon_0 \vec{E}_{\text{total}}$$

$$= \epsilon_0 (\vec{E} + \chi_e \vec{E})$$

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E}$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$\epsilon = \epsilon_0 \epsilon_r$
 $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$
 $\epsilon_r \rightarrow$ relative permittivity of material. (unit less)
 $\epsilon \rightarrow$ permittivity of material. (F/m)
 $\epsilon_r = 1 + \chi_e$ (unit less)

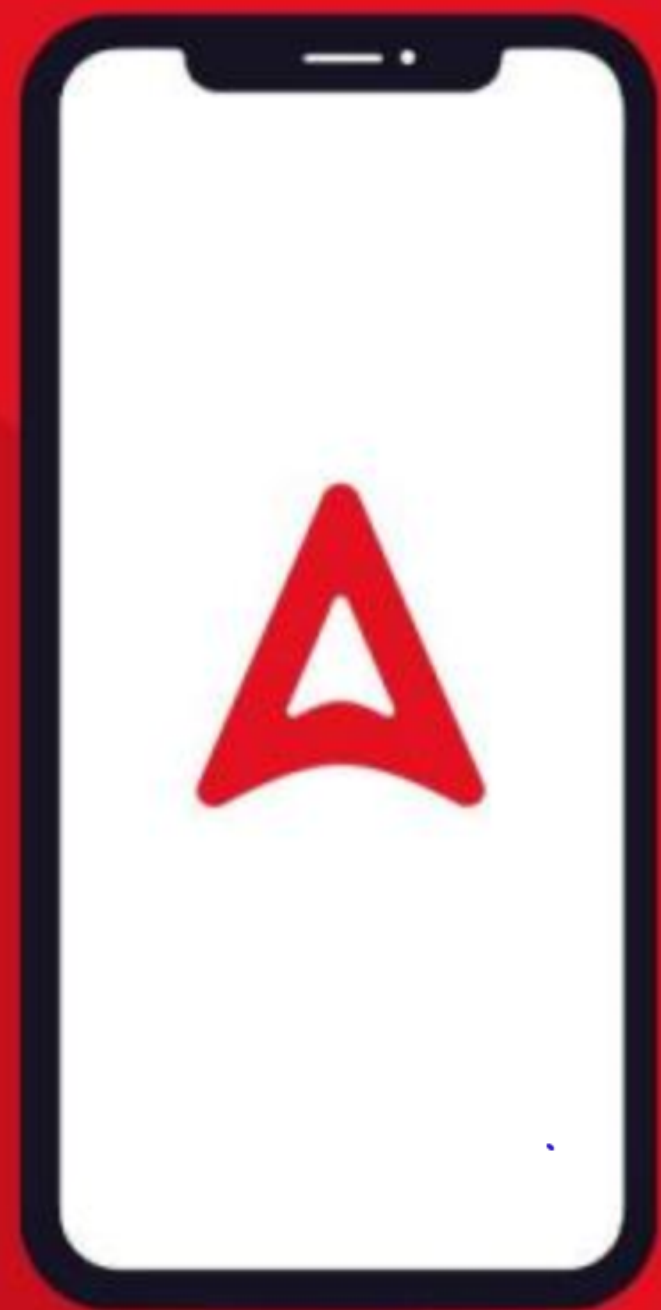
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$$\vec{D} = \epsilon \vec{E}$$

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