Questions on Eigen Values and Eigen vectors

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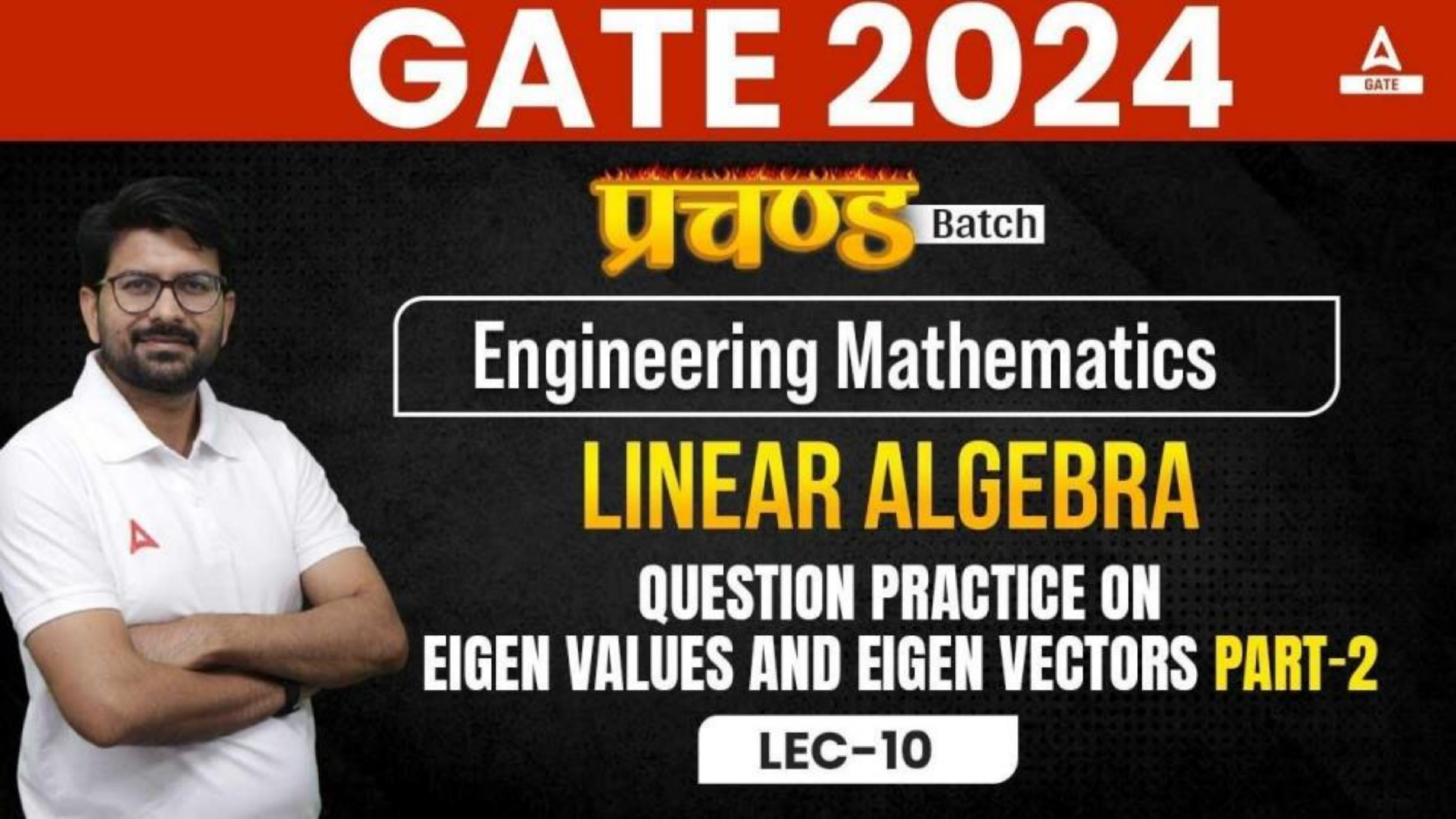
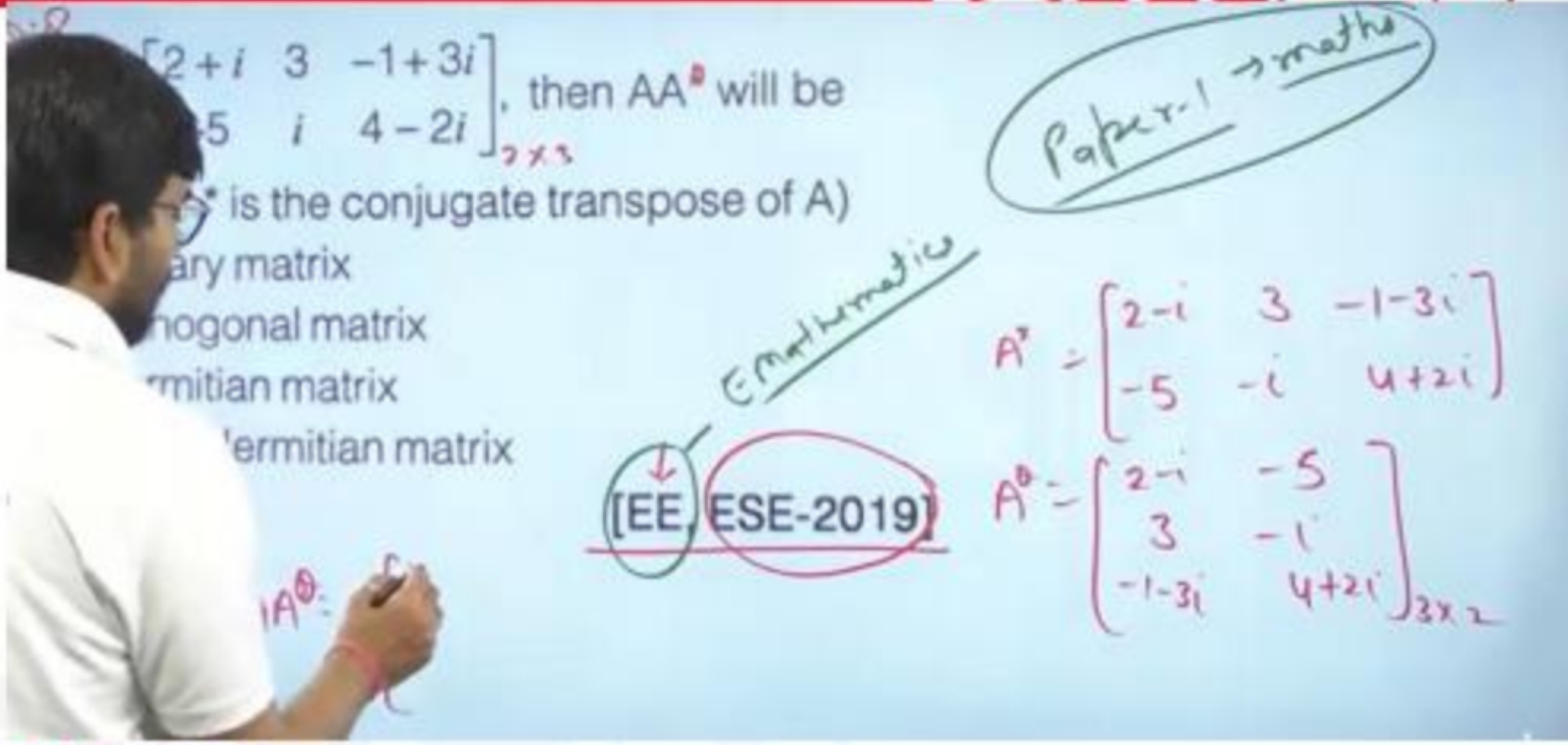
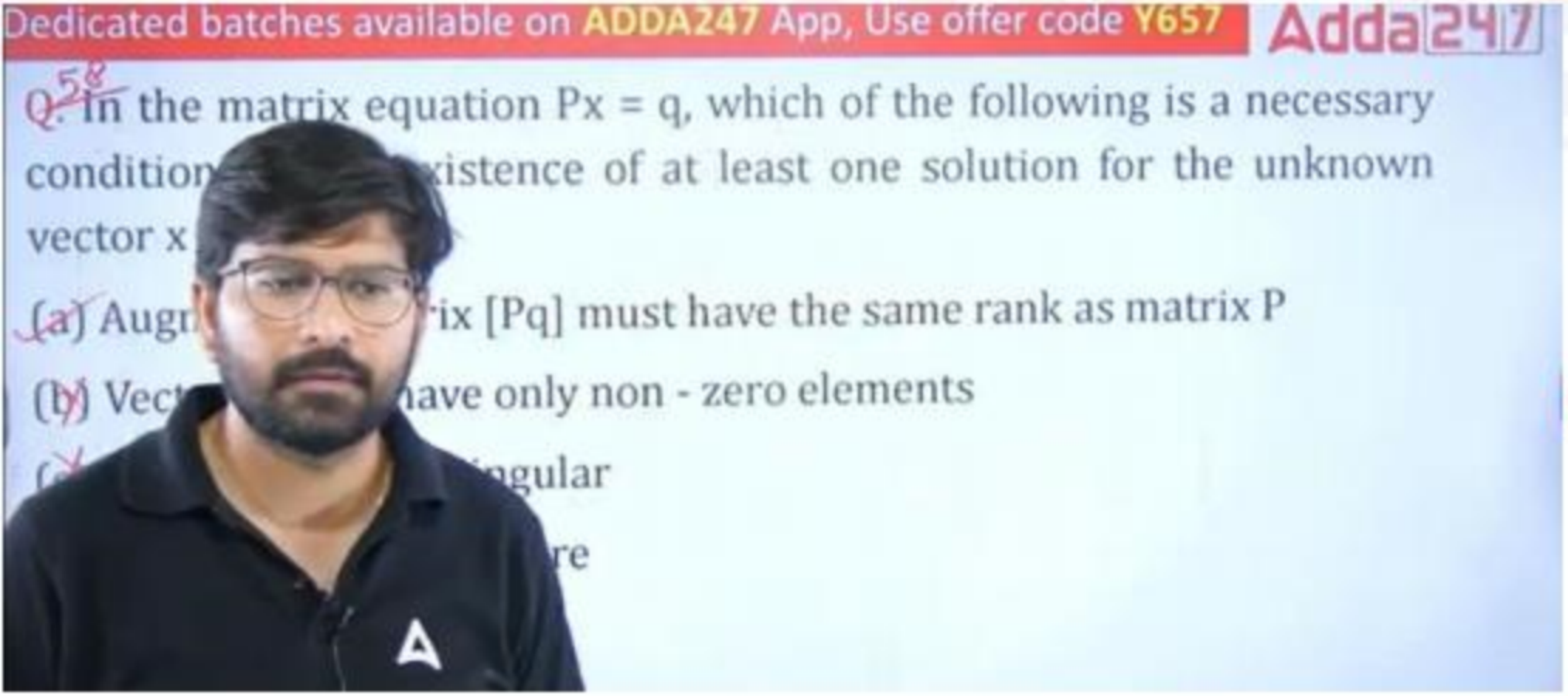

GATE 2024
GATE
प्रचण्ड Batch
Engineering Mathematics
LINEAR ALGEBRA
QUESTION PRACTICE ON
EIGEN VALUES AND EIGEN VECTORS PART-2
LEC-10

Recap



ESE Question Practice →



← GATE Question Practice

Number of questions covered-94

1. Introduction to Linear Algebra
2. Classification of Matrices
3. Transpose, Determinant, Inverse of a matrix
4. Question practice on Basics of Matrices
5. Rank and dimension of null space of Matrix
6. System of linear simultaneous equations
7. Eigen Values and Eigen Vectors

Questions on Eigen Values and Eigen vectors

Properties of Eigen Values :→

① $A_{n\times n} \longrightarrow \alpha_1, \alpha_2 - - - \alpha_n$

$B_{n\times n} \longrightarrow \mu_1, \mu_2, \mu_3 - - - \mu_n$

$C = (A_{n\times n} B_{n\times n})_{n\times n} \longrightarrow p_1, p_2, p_3 - - - p_n$

$p_1 = \alpha_1 \mu_1, \alpha_2 \mu_2 \cdots$

Properties of Eigen Values

② $A \longrightarrow \lambda_1, \lambda_2 - - - \lambda_n$

$A^2 \longrightarrow \lambda_1^2, \lambda_2^2 - - - - \lambda_n^2$

$\vdots$

$A^m \longrightarrow (\lambda_1)^m, (\lambda_2)^m - - - (\lambda_n)^m$

③ $A \rightarrow \lambda_1, \lambda_2 - - - - \lambda_n$

$kA \longrightarrow k\lambda_1, k\lambda_2 - - - k\lambda_n$

$A_{n\times n} \rightarrow |A| = \lambda_1 \times \lambda_2 \times - - \lambda_n$

$kA \rightarrow k^n|A| = k\lambda_1 \times k\lambda_2 \times - - - k\lambda_n$

$= k^n(\lambda_1 \cdot \lambda_2 \cdot \lambda_3 - - - \lambda_n)$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad trace(A) = a+d$

$3A = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$ $\quad trace(3A) = 3(a+d)$

Properties of Eigen Values

(4) Eigen values of a hermitian matrix are always pure real.

$$A^{\theta} = A$$

If a matrix is real symetric matrix

$$A^{T} = A \qquad A^{*} = A$$

$$(A^{*})^{T} = A^{T} = A$$

$$\Rightarrow A^{\theta} = A$$

* A real symetric matrix is hermitian as well. So its eigen values will also be pure real.

Properties of Eigen Values

(5) For skew hermitian matrix eigen values are pure imaginary or zero.

$A^{\theta} = -A$

$(A^{*})^{T} = -A$

A real skew symetric matrix is skew hermitian always so its eigen values will also be pure imaginory or zero.

(6) $A \rightarrow \lambda_1, \lambda_2 - - \lambda_n$

$A^{T} \rightarrow \lambda_1, \lambda_2 - - - \lambda_n$

Properties of Eigen Values

(7) For othogonal matrix

$$AA^T = I$$

eigen values of $AA^T \longrightarrow 1, 1, \ldots$

$$\lambda_1^2 = 1 \Rightarrow \lambda_1 = \pm 1, \lambda_2 = \pm 1$$

⇒ eigen values of orthogonal matrix will be either +1 or -1.

(8) For unitory matrix eigen values having magnitude equal to '1'.

$$i, 1, -1, -i, \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}$$

Properties of Eigen Values

(9)

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

⇓

linear transformed

$$B = \begin{bmatrix} a & b & c \\ 0 & e' & f' \\ 0 & 0 & i' \end{bmatrix}$$

$|A| = |B|$

trace(A) = $a + e + i$

trace(B) = $a + e' + i'$

* With linear transformations in a matrix eigen values are changed.

Q: 95 Eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigenvalues of the matrix $S^2 = SS$?

(a) 1 and 25

(b) 6 and 4

(c) 5 and 1

(d) 2 and 10

$S \rightarrow 5, 1$

$S^2 \rightarrow (5)^2, (1)^2$

$25, 1$

Q:96. If a square matrix A is real and symmetric, then the eigenvalues -

(a) are always real

(b) are always real and positive

(c) are always real and non - negative

(d) occur in complex conjugate pairs

Q:97 At least one eigenvalue of a singular matrix is

(a) positive

(b) zero

(c) negative

(d) imaginary

Q: 98 The trace and determinant of a 2 × 2 matrix are known to be - 2 and - 35 respectively. It eigenvalues are

(a) - 30 and - 5

(b) - 37 and - 1

(c) - 7 and 5

(d) 17.5 and - 2

→ -35

→ -15.5 X

Q:99 All the four entries of the 2 × 2 matrix P = $\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ are nonzero, and one of its eigenvalues is zero. Which of the following statements is true?

(a) $p_{11}\ p_{22} - p_{12}\ p_{21} = 1$

(b) $p_{11}\ p_{22} - p_{12}\ p_{21} = -1$

(c) $p_{11}\ p_{22} - p_{12}\ p_{21} = 0$

(d) $p_{11}\ p_{22} + p_{12}\ p_{21} = 0$

$\lambda_1 \times \lambda_2 = 0$

$|P| = 0$

$p_{11} p_{22} - p_{12} p_{21} = 0$

Q:100 The eigen values of the following matrix are $\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ 3×3

(a) 3, 3 + 5j, 6 - j

(b) - 6 + 5j, 3 + j, 3 - j

(c) 3 + j, 3 - j, 5 + j

(d) 3, - 1 + 3j, - 1 - 3j

5+2i, 5-7i → 5-2i, 5+7i

3+(-1+3i)+(-1-3i)

3+(-1)+(-1) = 1

3×(-1+3i)(-1-3i)

|A| = +3(1+9) = 30

3(1+9) = 30

Q:101 A real (4 × 4) matrix A satisfies the equation $A^2 = I$, where I is the (4 × 4) identity matrix. The positive eigen value of A is ___+1___ .

Sol:

$A^2 = I$ $\quad A \rightarrow \alpha_1, \alpha_2, \alpha_3, \alpha_4$

$(\alpha_1)^2, (\alpha_2)^2, (\alpha_3)^2, (\alpha_4)^2 = 1, 1, 1, 1$

$(\alpha_1)^2 = 1$

$\alpha_1 = \pm 1$

Q:102 The value of x for which all the eigen - values of the matrix given below are real is -

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) 5 + j

(b) 5 - j

(c) 1 - 5j

(d) 1 + 5j

$-5, +2, 1$

$\frac{5}{2}$

Q:103 A system matrix is given as follows.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}_{3\times3}$$

The absolute value of the ratio of the maximum eigenvalue to the minimum eigenvalue is ___3___.

$\lambda_1 + \lambda_2 + \lambda_3 = trace(A) = -6$

$-3, -2, -1$

$\lambda_1 \times \lambda_2 \times \lambda_3 = |A| = -6$

$\frac{3}{1}$

Q:104 Consider the following matrix.

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(a) x = 4, y = 10

(b) x = 5, y = 8

(c) X = - 3, y = 9

(d) x = - 4, y = 10 ✓

$2 + y = 12$

$y = 10$

$20 - 3x = 32$

$3x = -12$

$x = -4$

Q:105 Consider the matrix as given below:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the CORRECT values of the eigenvalues of the matrix ?

(a) 1, 4, 3

(b) 3, 7, 3

(c) 7, 3, 2

(d) 1, 2, 3

Q: 106. Which one of the following statements is TRUE about every n × n matrix with only real eigenvalues?

(a) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

$d_1 + d_2 + d_3 + \dots d_n = +ve$

$d_1 \times d_2 \times d_3 \times \dots d_n = -ve$

(b) If the trace of the matrix is positive, all its eigenvalues are positive.

(c) If the determinant of the matrix is positive, all its eigenvalues are positive.

(d) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

$trace(A) \times |A| = +ve$

$trace(A) = +ve, \ |A| = +ve$

$trace(A) = -ve, \ |A| = -ve$

Q: 107 Consider the following 2 × 2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are - 1 and 7. What are the values of a and b?

$$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$$

(a) a = 6, b = 4

(b) a = 4, b = 6

(c) a = 3, b = 5

(d) a = 5, b = 3

$\lambda_1 + \lambda_2 = trace(A)$

$6 = a + 1$

$a = 5$

$\lambda_1 \times \lambda_2 = |A|$

$-7 = 5 - 4b$

$+4b = 12$

$b = 3$

Q:108 The value of x for which the matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$$

3×3

has zero as an eigen value is ________ .

$\lambda_1 \times \lambda_2 \times \lambda_3 = |A|$

$|A| = 0$

$3(-63+7x+52) - 2(-81+9x+78) + 4(-36+42) = 0$

$-33 + 21x + 6 - 18x + 24 = 0$

$3x = 3$

$x = 1$

13×6
98

GATE 2024

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Th, Fr, Sat
9 p.m.

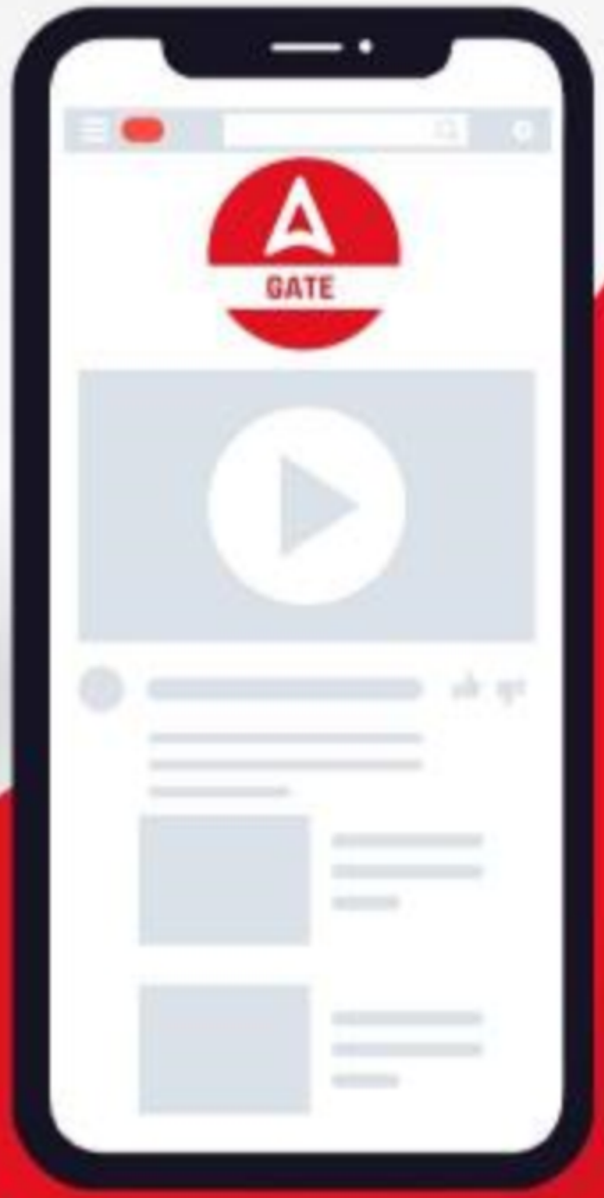
प्रचण्ड Batch

Electromagnetic Field Theory

QUESTION PRACTICE ON VECTOR CALCULUS

LEC-09

EE &ECE



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