A

19231 **120 MINUTES**

1. $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^m$, where m is a finite number is equal to
   A) e^x B) e^{-x} C) 0 D) 1

2. Which of the following statement is **not** true?
   A) A function which is uniformly continuous on a set is also continuous on that set.
   B) If a function is uniformly continuous on a bounded set, then the function is bounded on the set.
   C) The product of any two uniformly continuous functions is uniformly continuous.
   D) The function $f(x) = x^2, x \in R$ is not uniformly continuous on R.

3. $\int_0^\infty sinx\, dx =$
   A) 0 B) 1 C) $\frac{\pi}{2}$ D) does not exist

4. Which of the following statements are true?
   1. The union of any collection of open sets is an open set.
   2. The intersection of any collection of open sets is an open set.
   3. The union of any collection of closed sets is a closed set.
   4. The intersection of any collection of closed set is a closed set.

   A) 1 and 3 only B) 1 and 4 only C) 2 and 3 only D) 2 and 4 only

5. Let S_1 and S_2 be two subspaces of $\Re^m$. Then which of the following statement(s) is/are true.
   1. $S_1 \cup S_2$ is always a subspace of $\Re^m$.
   2. $S_1 \cap S_2$ is always a subspace of $\Re^m$.
   3. The sum $S_1 + S_2$ is the smallest subspace of $\Re^m$ that contains $S_1 \cup S_2$.

   A) 1 only B) 2 only C) 1 and 3 only D) 2 and 3 only

6. Let V be a vector space of dimension n, and U and W be two subspaces of V with dimensions n_1 and n_2 respectively such that $n_1 < n_2$. Then the maximum dimension of $U \cap W$ is
   A) n_1 B) n_2 C) n D) $n_1 + n_2 - n$

7. Assume that the sum of two idempotent matrices is again idempotent. Then product of the matrices is
   A) a non-zero idempotent matrix B) a zero matrix
   C) an identity matrix D) none of these

8. If A and B are square matrices of order n, then
   A) $Rank(AB) \geq Rank(A) + Rank(B) - n$
   B) $Rank(AB) \leq Rank(A) + Rank(B) - n$
   C) $Rank(AB) = Rank(A) + Rank(B) - n$
   D) None of the above

9. Let A^+ be the Moore-Penrose g-inverse of a matrix A. Which of the following statements are true:

1. A^+ is unique 2. $(A^+)^+ = A$ 3. $(A^+)^T = (A^T)^+$

A) 1 and 2 only B) 1 and 3 only
C) 2 and 3 only D) 1, 2 and 3

10. Which of the following statement is **not** true?
A) Similar matrices have identical characteristic polynomials
B) If A and B are $n \times n$ matrices then characteristic polynomial of AB and BA are same
C) Characteristic polynomial of a matrix and its transpose are same.
D) Product of characteristic roots of a matrix is equal to its trace.

11. The matrix A is a positive definite matrix. Then
A) $|A| > 0$
B) A^{-1} is positive definite
C) All the eigen values of A are positive
D) All the above

12. Let $\{A_n\}$ be a nondecreasing sequence of sets, then $\lim A_n$ is:

A) $\bigcup_{n=1}^{\infty} A_n$ B) $\bigcap_{n=1}^{\infty} A_n$ C) $\emptyset$ D) Does not exist

13. Let E_1 and E_2 be Lebesgue measurable sets, then which of the following statement(s) is/are true?
1. $E_1 \cup E_2$ is Lebesgue measurable
2. $E_1 \cap E_2$ is Lebesgue meaurable

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

14. Let A, B and C be three events such that $P(A) = P(B) = P(C) = \frac{1}{4}$, $P(A \cap B) = P(B \cap C) = 0$ and $P(A \cap C) = \frac{1}{8}$. Then probability that at least one of the events A, B and C occurs is

A) $\frac{1}{8}$ B) $\frac{3}{8}$ C) $\frac{5}{8}$ D) $\frac{7}{8}$

15. Let A, B and C be three mutually exclusive and exhaustive events such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$. Then $P(A) =$

A) $\frac{2}{3}$ B) $\frac{8}{9}$ C) $\frac{4}{13}$ D) $\frac{4}{9}$

16. The odds against an event A is $4:3$ and odds in favour of another event B is $7:4$, A and B are independent. Then the probability that neither A nor B occurs is

A) $\frac{61}{77}$ B) $\frac{16}{77}$ C) $\frac{21}{77}$ D) $\frac{25}{77}$

17. The probability that a 3-card hand drawn at random and without replacement from an ordinary deck consists entirely black cards is

A) $\frac{1}{17}$ B) $\frac{2}{17}$ C) $\frac{3}{26}$ D) $\frac{4}{17}$

18. Among three urns, the first urn contains 7 white and 10 black balls, the second contains 5 white and 12 black balls, and the third contains 17 white balls and no black ball. An urn is chosen and a ball is drawn from the selected urn. It is found that the ball is white. Then probability that the ball came from the second urn is

A) $\frac{5}{29}$ B) $\frac{7}{29}$ C) $\frac{17}{29}$ D) $\frac{25}{29}$

19. Which of the following is **not** a probability density function?

A) $f(x) = \left\{\frac{1}{2}e^{-|x|}, -\infty < x < \infty\right.$

B) $f(x) = \begin{cases} \frac{1}{\sigma}e^{-(x-\theta)/\sigma}, & x > \theta,\ \sigma > 0 \\ 0, & otherwise \end{cases}$

C) $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$

D) $f(x) = \begin{cases} x(2-x), & 0 < x < 2 \\ 0, & otherwise \end{cases}$

20. Consider the function $F(x) = \begin{cases} 0, & x < 0 \\ (x+1)/2, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$. Then $F(x)$ is

A) Distribution function of a discrete random variable
B) Distribution function of a continuous random variable
C) Distribution function of a mixed type random variable
D) Not a distribution function

21. Which measure is the most unreliable indicator of central tendency if the distribution is skewed?

A) Mean B) Median C) Mode D) Range

22. Let X be a random variable with distribution function $F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \frac{1}{e^{2x}}, & x > 0 \end{cases}$.

Median of the distribution is

A) 0 B) $\frac{\log 2}{2}$ C) $2\log\frac{1}{2}$ D) $\frac{e-1}{e}$

23. Which of the following statements are true for the variance σ^2 of a random variable X?
1. $\sigma^2 > 0$ for all non-degenerate random variables
2. If the distribution of X is concentrated near $E(X)$ then σ^2 will be small.
3. A small value of σ^2 means the probability is small that X will deviate much from its mean.
4. Large value of σ^2 means the probability is large that X will be far from the mean.

A) 1and 2 only B) 1, 2 and 3 only
C) 1, 2 and 4 only D) 1, 2, 3 and 4

24. Let X be integer valued random variable with probability generating function $P(s)$, $|s| \leq 1$. Then the second factorial moment of X is given by

A) $P'(1)$ B) $P''(1)$
C) $P''(1) - [P'(1)]^2$ D) $P''(1) - [P'(1)]^2 + P'(1)$

25. Which of the following statement is false?
A) Two random variables X and Y have the same moment generating function, then X and Y must have the same distribution.
B) If moment generating function of a random variable exist, then moments of all orders exist.
C) If moments of all orders exist, then moment generating function exist in some open neighborhood of zero.
D) All the above statements are true.

26. Let $\{X_n\}$ be a sequence of random variables for which $E(X_1^2) < \infty$ and let $S_n = \sum_{k=1}^{n} X_k, n \geq 1$. A necessary and sufficient condition for the sequence $\{X_n\}$ to satisfy weak law of large numbers is

A) $E\left\{\frac{S_n^2}{1+S_n^2}\right\} \rightarrow 0, as\ n \rightarrow \infty$ B) $E\left\{\frac{S_n^2}{1+S_n^2}\right\} \rightarrow 1, as\ n \rightarrow \infty$

C) $E\left\{\frac{S_n^2}{n^2+S_n^2}\right\} \rightarrow 0, as\ n \rightarrow \infty$ D) $E\left\{\frac{S_n^2}{n^2+S_n^2}\right\} \rightarrow 1, as\ n \rightarrow \infty$

27. Let $X_1, X_2, \ldots$ be iid random variables with mean 0 and variance 1. Let $\Phi(x)$ denote the cumulative distribution function of a standard normal random variable. Then for any $x > 0$, $\lim_{n\to\infty} P(|\sum_{i=1}^{n} X_i| < nx)$ equals

A) $\Phi(x)$ B) $1 - 2\Phi(x)$ C) $2\Phi(x) - 1$ D) 1

28. Binomial distribution is positively skewed if the probability of success is

A) less than $\frac{1}{2}$ B) greater than $\frac{1}{2}$ C) equal to $\frac{1}{2}$ D) equal to 1

29. The mean of zero-truncated Poisson(λ) variable is

A) λ B) $\frac{\lambda}{e^{-\lambda}}$ C) $\frac{\lambda}{1-e^{-\lambda}}$ D) $\frac{1}{e^{-\lambda}}$

30. Which of the following distribution possesses lack of memory property?

A) Poisson B) Binomial

C) Hypergeometric D) Geometric

31. The mean and variance of negative binomial distribution are related as:

A) mean is greater than variance B) mean is less than variance

C) mean is equal to variance D) None of the above

32. Suppose that the amount of time a customer spends at a fast food restaurant has an exponential distribution with mean of six minutes. Then the probability that the customer will spend more than 12 minutes in the restaurant given that she has been there for more than 6 minutes is

A) $\frac{1}{e}$ B) $\frac{1}{e^2}$ C) e D) e^2

33. Which of the following statements are true?

1. If X and Y are iid $\text{Cauchy}(1, 0)$ random variables,. Then $\text{X} + \text{Y}$ follows $\text{Cauchy}(2, 0)$ distribution
2. If X and $\frac{1}{\text{X}}$ have the same distribution, then X is $Cauchy(1,\ 0)$ random variable.
3. Let X be a uniform random variable over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then $Y = \tan X$ is a Cauchy random variable.

A) 1 and 2 only B) 1 and 3 only

C) 2 and 3 only D) 1, 2 and 3

34. Let X has Pareto distribution with pdf $f(x) = \begin{cases} \frac{\beta\alpha^\beta}{(x+\alpha)^{\beta+1}}, & x > 0, \alpha, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$. Then $E(X)$ exists only when

A) $\alpha > 1$ B) $\alpha < 1$ C) $\beta > 1$ D) $\beta < 1$

35. Let the random variable X has lognormal distribution with parameters μ and σ^2. Then moment generating function of X is

A) $\exp\left(\mu t + \frac{1}{2}t^2\sigma^2\right)$ B) $\left(\mu t + \frac{1}{2}t^2\sigma^2\right)$

C) $\log\left(\mu t + \frac{1}{2}t^2\sigma^2\right)$ D) does not exist

36. Let X and Y be independent random variables with pdfs f_1 and f_2, respectively. Then the pdf of $Z = XY$ is

A) $f_Z(z) = \int_{-\infty}^{\infty} f_1(x) f_2(xz) |x|\, dx$ B) $f_Z(z) = \int_{-\infty}^{\infty} f_1(x) f_2(xz) \frac{1}{|x|}\, dx$

C) $f_Z(z) = \int_{-\infty}^{\infty} f_1(x) f_2\left(\frac{z}{x}\right) |x|\, dx$ D) $f_Z(z) = \int_{-\infty}^{\infty} f_1(x) f_2\left(\frac{z}{x}\right) \frac{1}{|x|}\, dx$

37. Let $X_1, X_2, \dots, X_n$ be a random sample taken from discrete uniform distribution with pmf $P_N(x) = \frac{1}{N},\ x = 1, 2, \dots, N,\ N \geq 1$. Then the pmf of the n^{th} order statistic is

A) $P_N^{(n)}(x) = \frac{nx^{n-1}}{N^n},\quad x = 1, 2, \dots, N$

B) $P_N^{(n)}(x) = \frac{x^n}{N^n},\quad x = 1, 2, \dots, N$

C) $P_N^{(n)}(x) = \frac{x^n - (x-1)^n}{N^n},\quad x = 1, 2, \dots, N$

D) $P_N^{(n)}(x) = \frac{x^n - (x-1)^{n-1}}{N^n},\quad x = 1, 2, \dots, N$

38. Consider a system of n identical batteries operating independently in a system, and the batteries operate in series. Suppose the length of life of the batteries have the common distribution function $F(x)$ and pdf $f(x)$. Then the pdf of length of life of the system has the form

A) $n[1 - F(x)]^{n-1} f(x)$ B) $n[F(x)]^{n-1} f(x)$

C) $n\{1 - [1 - F(x)]^{n-1}\} f(x)$ D) None of the above

39. Let $X_{1:3},\ X_{2:3},\ X_{3:3}$ be the order statistics corresponding to the random sample $X_1,\ X_2,\ X_3$ taken from a population with pdf $f(x) = \lambda e^{-\lambda x},\ x > 0,\ \lambda > 0$. Define $Y_1 = 3X_{1:3}$, $Y_2 = 2(X_{2:3} - X_{1:3})$ and $Y_3 = (X_{3:3} - X_{2:3})$. Then which of the following statements are true?

1. $(X_1,\ X_2,\ X_3)$ and $(Y_1,\ Y_2,\ Y_3)$ are identically distributed.
2. Y_1, Y_2 and Y_3 are independent.
3. $Y_i,\ i = 1, 2, 3$ has the pdf $f(x)$.

A) 1 and 2 only B) 1 and 3 only C) 2 and 3 only D) 1, 2 and 3

40. The mean of a non-central chi-square random variable with degrees of freedom n and noncentrality parameter δ is

A) n B) $n + \delta$ C) $n + 2\delta$ D) $2n + \delta$

41. Which of the following statement is false for *t* distribution with n degrees of freedom?
A) Mean of the distribution is zero for all $n \geq 1$.
B) Pdf of the distribution is symmetric about zero.
C) Pdf of the distribution can be approximated by a standard normal density for large n.
D) For small n, t-distribution assigns more probability to its tails compared with standard normal distribution.

42. If $X \sim Cauchy(1, 0)$ then $X^2 \sim$

A) $Cauchy\ (1, 0)$ B) $t(1)$ C) $t(2)$ D) $F(1, 1)$

43. Let (X,Y) be bivariate random vector. Then the estimate $\phi(X)$ of Y based on X, which minimizes the MSE $E(Y-\phi(X))^2$ is

A) $E(X)$ B) $E(Y)$ C) $E(X|Y)$ D) $E(Y|X)$

44. Let $(X,Y)\sim bivariate\ normal(\mu_1,\mu_2,\sigma_1,\sigma_2,\rho)$. Then $V(X|Y)=$

A) σ_1^2 B) $\sigma_1^2(1-\rho)$ C) $\sigma_1^2(1-\rho^2)$ D) $\sigma_1^2(\rho^2-1)$

45. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous population with distribution function $F(.)$. Define

$$\hat{F}_n(x)=\frac{\text{Number of } X_i\text{'s} \le x}{n}, \qquad x\in R$$

Then

A) $\hat{F}_n(x)$ is unbiased but not consistent for $F(x)$
B) $\hat{F}_n(x)$ is consistent but not unbiased for $F(x)$
C) $\hat{F}_n(x)$ is unbiased and consistent for $F(x)$
D) $\hat{F}_n(x)$ is neither unbiased nor consistent for $F(x)$

46. Let T_1 be an unbiased estimator of the parameter θ of a family of distribution $\{F_\theta, \theta\in\Theta\}$ such that $E_\theta(T_1^2)<\infty$, and T be a sufficient statistic for $\{F_\theta,\theta\in\Theta\}$. Also let $T_2=E_\theta(T_1|T)$. Then

A) T_2 is unbiased estimator with $V(T_1)\ge V(T_2)$
B) T_2 is not unbiased estimator but $V(T_1)\ge V(T_2)$
C) T_2 is unbiased estimator with $V(T_1)\le V(T_2)$
D) T_2 is the UMVUE

47. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Poisson distribution with mean θ. Then the Cramer-Rao lower bound of unbiased estimator of $e^{-\theta}$ is

A) $e^{-\theta}$ B) $\frac{e^{-2\theta}}{n}$ C) $\frac{\theta e^{-2\theta}}{n}$ D) $\frac{e^{-\theta}(1-e^{-\theta})}{n}$

48. The MLE of θ based on a random sample $X_1, X_2, \ldots, X_n$ taken from the PDF

$$f(x)=\theta x^{-2} I_{(\theta,\infty)}(x),\ \theta>0\ ,$$

where $I_A(.)$ is the indicator function defined on the set A is

A) $\frac{\sum_{i=1}^n X_i}{n}$ B) $\left(\frac{\prod_{i=1}^n X_i^2}{n}\right)^{1/n}$ C) $\min_i X_i$ D) $\max_i X_i$

49. State whether the following statements are true (T) or false (F)
1. Maximum likelihood estimate is always unique.
2. Maximum likelihood estimate is unbiased if it is unique.
3. Maximum likelihood estimate itself is a sufficient statistic.
4. Asymptotic distribution of maximum likelihood estimate is always normal.

A) 2 & 3 true, 1 & 4 false B) 1 & 2 True, 3 & 4 false
C) All the statements are true D) All the statements are false

50. Let $X_1, X_2, \dots., X_n$ be a random sample taken from population with pdf

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1, \theta > 0 \\ 0, & otherwise \end{cases}.$$

If $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ then the moment estimator of θ is

A) $\bar{X}$ B) $\frac{\bar{X}-1}{\bar{X}}$ C) $\frac{\bar{X}}{1-\bar{X}}$ D) $\frac{\bar{X}}{\bar{X}-1}$

51. An urn contains 10 balls, of which M are red and $10 - M$ are black. To test $H_0: M = 5$ against the alternative $H_1: M = 6$, one draws 3 balls from the urn without replacement. The null hypothesis is rejected if the sample contains 2 or 3 red balls; otherwise it is accepted. Then the power of the test is

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{5}{8}$

52. Which of the following distribution does not have monotone likelihood ratio property?

A) Uniform over $[0, \theta]$ B) Poisson
C) Binomial D) Cauchy $(1, \theta)$

53. The critical region of the UMP test for testing $H_0: \theta \geq \theta_0$ against $H_0: \theta < \theta_0$, based on random sample $X_1, X_2, \dots., X_n$ taken from the pdf $f(x) = \frac{1}{(\theta-1)!} x^{\theta-1} e^{-x}, x > 0$, $\theta > 0$ has the form

A) $\sum_{i=1}^{n} X_i > k$ B) $\sum_{i=1}^{n} X_i < k$
C) $\prod_{i=1}^{n} X_i > k$ D) $\prod_{i=1}^{n} X_i < k$

54. Which test is the nonparametric analogue of the analysis of variance F test for the two way classification?

A) Kruskal-Wallis test B) Friedman test
C) Shapiro-Wilk test D) Freund-Ansari test

55. The decision in a sequential probability ratio test depends on

A) P(type I error)
B) P(type II error)
C) P(type I error) and P(type II error)
D) None of the above probabilities

56. Let A denote the region of acceptance of an α-level UMP test of $H_0: \theta = \theta_0$ against $H_0: \theta \neq \theta_0$. For each observation, $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$, let $S(\boldsymbol{x})$ is the set $S(\boldsymbol{x}) = \{\theta: \boldsymbol{x} \in A\}$. Then

A) $S(\boldsymbol{x})$ is UMA confidence interval with confidence level $1 - \alpha$.
B) $S(\boldsymbol{x})$ is confidence interval with confidence level $1 - \alpha$ but not UMA
C) $S(\boldsymbol{x})$ is confidence interval with a different confidence level
D) $S(\boldsymbol{x})$ is not a confidence interval

57. Choose the conjugate priors for the distributions in List I from List II and select the correct answer using the codes given below.

List I		List II	
a.	Poisson distribution with unknown mean	1.	Normal distribution
b.	Binomial distribution with unknown probability of success	2.	Gamma distribution
c.	Uniform over [0,θ], θ>0 is unknown	3.	Beta distribution
d.	Normal distribution with unknown mean and known variance	4.	Pareto distribution

A) a-2, b-3, c-4, d-1 B) a-4, b-2, c-3, d-1
C) a-4, b-1, c-3, d-2 D) a-2, b-4, c-3, d-1

58. In a hypothesis testing about a population mean, the *p*-value is found to be 0.04. Assume that the population mean given the null hypothesis is μ_0. Which of the following is/are true about the population mean?
A) The 95% confidence interval includes μ_0
B) The 99% confidence interval includes μ_0
C) The 90% confidence interval includes μ_0
D) All the above are true

59. Suppose a finite population contains 7 items and 3 items are selected at random without replacement, then the number of all possible samples will be
A) 21 B) 35 C) 14 D) 7

60. Consider a population of N units, in which the proportion of units possessing a given characteristic is P. A random sample of size n is taken from the population using simple random sampling without replacement. Then the variance of the unbiased estimate of P is equal to

A) $\frac{(N-n)P(1-P)}{nN}$ B) $\frac{(N-n)P(1-P)}{n(N-1)}$

C) $\frac{N^2(N-n)P(1-P)}{n(N-1)}$ D) $\frac{P(1-P)}{N}$

61. The manager of the customer service division of a major consumer electronics company is interested in determining whether the customers who have purchased a LED television made by the company over the past 12 months are satisfied with their products. If there are 5 different types of LED televisions made by the company, the best strategy would be to use a

A) simple random sample
B) stratified random sample
C) cluster sample
D) systematic sample

62. Which of the following statement(s) is/are true for Horvitz-Thomson estimator?

1. Horvitz-Thomson estimator is the best estimator in the class of linear unbiased estimators of the population mean.
2. Horvitz- Thomson estimator is the UMV in the class of all unbiased estimators

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

63. The ratio estimator of population mean is unbiased if

A) The variable under study and the auxiliary variable have a positive correlation
B) The variable under study and the auxiliary variable have a negative correlation
C) The variable under study and the auxiliary variable are linearly related
D) None of the above

64. Which of the following statement is/are true for a connected design?

1. All treatment contrasts of a connected design are estimable.
2. The rank of the C-matrix of a connected incomplete block design with v treatments is $v-1$.
3. Latin square designs are connected.

A) 1 and 2 only B) 1 and 3 only
C) 2 and 3 only D) 1, 2 and 3

65. Assuming no bias, the total variation in a response variable is due to error (unexplained variation) plus differences due to treatments (known variation). If known variation is large compared to unexplained variation, which of the following conclusions is the best

A) There is no evidence for a difference in response due to treatments.
B) There is evidence for a difference in response due to treatments.
C) There is significant evidence for a difference in response due to treatments
D) The treatments are not comparable.

66. In $p \times p$ Latin square design, the degrees of freedom of the error sum of square is equal to

A) $p(p-1)$ B) $(p-1)^2$ C) $(p-2)(p-1)$ D) $p(p-2)$

67. Which of the following statements is/are true for symmetric BIBD?

1. Number of treatment is equal to the number of blocks.
2. Number of replications of each treatment is equal to the block size.
3. The number of treatments common in any two blocks is equal to the number of blocks in which each pair of treatments occurs together.

A) 1 only B) 1 and 2 only
C) 1 and 3 only D) 1, 2 and 3

68. Suppose a 2^6 design confounded in 8 blocks by selecting 3 effects to generate the blocks. Then the number of effects that are confounded in addition to the selected effects is equal to

A) 3 B) 4 C) 5 D) 6

69. The Gauss-Markov theorem will not hold if

A) the error term has the same variance given any values of the independent variables
B) the error term has an expected value of zero given any values of the independent variables
C) the independent variables have exact linear relationships among them
D) the regression model relies on the method of random sampling for collection of data

70. If $\boldsymbol{Y}$ is distributed as $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $(\boldsymbol{Y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}-\boldsymbol{\mu})$ has the distribution

A) χ^2 distribution with $(p-1)$ degrees of freedom
B) χ^2 distribution with p degrees of freedom
C) Wishart distribution with p degrees of freedom
D) $N_p(\boldsymbol{0}, \boldsymbol{\Sigma})$

71. Let $\boldsymbol{X} = (X_1, X_2, X_3)'$ be distributed as $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Which of the following statement is/are true?

1. X_1 and X_2 are independent.
2. (X_1, X_2) and X_3 are independent.

A) 1 only
B) 2 only
C) Both 1 and 2
D) Neither 1 nor 2

72. Let $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ be random sample from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\overline{\boldsymbol{X}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{X}_i$, and $\boldsymbol{\Sigma}$ is known. Then the test statistic to test $H_0\colon \boldsymbol{\mu} = \boldsymbol{\mu_0}$ against $H_1\colon \boldsymbol{\mu} \neq \boldsymbol{\mu_0}$ is

A) $(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)'\,\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)$
B) $(n-1)(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)'\,\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)$
C) $n(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)'\,\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)$
D) $\frac{1}{n}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)'\,\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0)$

73. The multiple correlation coefficients

A) can vary within the range from -1 to $+1$
B) can vary within the range from 0 to $+1$
C) can be any nonnegative value
D) cannot be zero

74. The state space of a stochastic process is

A) always discrete
B) always continuous
C) may be continuous or discrete
D) neither discrete nor continuous

75. Consider a Markov chain with transition probability matrix

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}.$$

Then

A) The Markov chain is irreducible
B) All states are periodic with period 2
C) All the states are persistent
D) All the above

76. If $\{X(t)\}$ is a Poisson process then $P\{X(2) = 1 | X(4) = 4\}$ is

A) $\frac{3}{4}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 0

77. In an M/M/1 queue with arrival rate λ, departure rate μ and infinite queue capacity, the steady state probabilities

A) exist when $\lambda < \mu$
B) exist when $\lambda > \mu$
C) always exist
D) none of the above

78. Cyclical variation in time series has a period of oscillation of

A) Less than one year
B) More than one year
C) Both A and B
D) None of the above

79. Method of simple averages for a time series data is used to measure

A) Seasonal variation
B) Trend
C) Cyclical variation
D) Irregular variation

80. Which of the following index number is generally expected to have an upward bias?

A) Paasche's index number
B) Fisher's index numbers
C) Laspeyre's index number
D) All the above