1.   Consider the two real valued functions defined on R: $f(x) = \begin{cases} 0, if\ x\ is\ irrational \\ 1, if\ x\ is\ rational \end{cases}$ and

$g(x)$, which is a continuous function. Then in the context of Riemann integrability:

A)   Both are integrable.
B) $f(x)$ is not integrable, but $g(x)$ is integrable
C) $g(x)$ is not integrable, but $f(x)$ is integrable
D)   Both are not integrable

2.   In R, a sequence is given to be convergent. Then:
A)   It must be Cauchy
B)   Not necessarily Cauchy
C)   Cauchy only when the limit is not zero
D)   Cauchy only when the limit is zero

3.   Consider $f(x) = [x], x \in R$, where $[x]$ denotes the integral part of x . Then the limit of $f(x)$ as $x \rightarrow 2$ is:
A)   1   B)   1.5   C)   2   D)   none of these

4.   If we use Latin Square Design to study the effects of k fertilizers on the yield of a certain variety of wheat, then the number of replications of each treatment and the number of plots required respectively are:
A) k, k B) k, k^2 C) k^2, k D) k^2, k^2

5.   For any two events A and B in a sample space if $P(A \cap B) \geq P(A) + P(B) + k,$ then k is:
A)   1   B)   -1   C)   2   D)   0

6.   Consider the following linear forms of treatment effects t_1, t_2 and t_3 :
1. $t_1 - 2t_2 + 2t_3$
2. $t_1 - 2t_2 + t_3$
3. $3t_1 + 4t_2 - 7t_3$
Which of these form(s) is/are linear contrasts?

A)   Only 1   B)   2 and 3 only   C)   1 and 2 only   D)   Only 3

7.   Let X be a random variable with mean 10 and variance 9. Then the lower bound to $P(4 < X < 16)$ given by Chebychev's inequality is:

A)   0.25   B)   0.90   C)   0.75   D)   0.50

8.   The characteristic function of a random variable X is real means that X is:
A)   Positive valued   B)   Continuous
C)   Positively skewed   D)   Symmetric in distribution

9. Which of the following is a wrong statement concerning the subsets of the set of real numbers?
   A) A set is open if and only if it is the union of countably many disjoint open intervals
   B) The intersection of any number of closed set is closed
   C) The intersection of any number of open set is open
   D) A set is closed if and only if it contains all its limit points

10. In SRSWOR if n units are selected from a population of size N, the number of samples possible is:
    A) N - n   B) NC_n C) Nn   D) N/n

11. A closed and bounded subset in real line is also called:
    A) Complete   B) Cauchy   C) Compact   D) All the above

12. In a discrete metric space M, every subset S is always:
    A) Compact   B) finite   C) Open   D) Closed

13. Laspeyre's price index number is computed with weights as:
    A) Current year price   B) Current year quantity
    C) Base year price   D) Base year quantity

14. Given the following set of vectors in $\mathbb{R}^3$
    (a) [100, 010, 001]   (b) [111, 010, 101]
    (c) [100, 010, 111]   (d) [110, 001, 101]

    A) Each of (a) and (c) are basis set of vectors of $\mathbb{R}^3$
    B) (a) alone is a basis set of vectors of $\mathbb{R}^3$
    C) Each of (b) and (c) are basis set of vectors of $\mathbb{R}^3$
    D) All sets are basis sets of vectors of $\mathbb{R}^3$

15. Family budget method is adopted for computation of
    A) Chain base index number   B) Cost of living index number
    C) Both A and B   D) None of these

16. Match List I with List II
    List I   List II
    a. Diagonal matrix   1. Eigen values are diagonal elements
    b. Idempotent matrix   2. Inverse exists
    c. Nonsingular matrix   3. Absolute value of its determinant is 1
    d. Orthogonal matrix   4. Rank is the sum of its diagonal elements

    A) a-2, b-1, c-3, d-4   B) a-1, b-4, c-2, d-3
    C) a-4, b-1, c-3, d-2   D) a-1, b-2, c-4, d-3

17. Two unbiased coins are tossed. If one of them shows head, the probability that the other also shows head is:
    A) 1/4   B) 1/2   C) 1/8   D) none of these

18. A man has five coins, one of which has two heads. He randomly takes out a coin and tosses it 3 times. What is the probability that it will fall head upward all the time?
A) 0.30 B) 0.13 C) 0.25 D) none of these

19. In order to test whether a coin is unbiased, it is tossed 5 times. The null hypothesis of unbiasedness is rejected if and only if more than four heads are obtained. Then the probability of type I error (rounded off to two decimal places) is:
A) 0.05 B) 0.01 C) 0.03 D) none of these

20. In the test for independence of attributes with a 3×4 contingency table, the test statistic follows:
A) Chi-square with 7 df. B) Chi-square with 6 df
C) Students t with 7 df. D) Students t with 12 df

21. The number of telephone calls during a time period $(0,t]$ at an enquiry counter is an example for a stochastic process having:
A) continuous parameter space and continuous state space
B) continuous parameter space and discrete state space
C) discrete parameter space and continuous state space
D) discrete parameter space and discrete state space

22. In stratified sampling scheme, providing an estimate of the population mean with the desired degree of precision for a minimum cost or to provide an estimate with maximum precision for a given cost falls under:
A) Optimum allocation B) Neyman allocation
C) Proportional allocation D) None of these

23. The stationary distribution of the Markov chain whose transition probability matrix $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ is:

A) $\pi = (1,0)$ B) $\pi = (1,1)$ C) $\pi = (0,0)$ D) None of these

24. The interval between two successive occurrences within a Poisson process follows:
A) Gamma distribution B) Poisson distribution
C) Normal distribution D) Negative exponential distribution

25. Suppose P_1 and P_2 are two probability measures on $(\Omega, \mathcal{F})$ then which of the following is a correct statement?
A) $P_1 - P_2$ is a probability measure on $(\Omega, \mathcal{F})$

B) $P_1 + P_2$ is a probability measure on $(\Omega, \mathcal{F})$

C) $\propto P_1 + (1-\propto) P_2$, for $0 \leq \propto \leq 1$, is a probability measure on $(\Omega, \mathcal{F})$

D) $P_1 \pm P_2$ probability measure on $(\Omega, \mathcal{F})$

26. Consider the following statements:

    1. Characteristic functions of a random variable always provides all the moments of that random variable.
    2. Probability generating function exists for any random variable
    3. Moment generating function exists for any random variable
    Which of the following is correct?

    A) Statement 1 is correct where as 2 and 3 are wrong
    B) Statement 1 and 3 are correct where as 2 is wrong
    C) All statements are correct
    D) All statements are wrong

27. Let T be a statistic for estimating the parameter θ. Then expected value of the quantity $T - \theta$ is called:
    A) Bias   B) Risk function C) Efficacy   D) MSE

28. The set of all limit points of the interval (a,b) in R is:
    A) $\{a,b\}$ B) null set   C) (a,b) itself   D) $[a,b]$

29. Consider the two statements:
    (1) A set is closed if and only if it contains all its cluster points.
    (2) A set is closed if and only if it contains all its adherent points.
    Then:

    A) Only (1) is true   B) Only (2) is true
    C) Both are false   D) Both are true

30. Let F be an open covering of a closed and bounded set S in R. Then:

    A) No finite sub collection of F can cover S
    B) There must exist a finite sub collection of F that can cover S
    C) S must be null set
    D) F must contain finite number of sets

31. For a distribution first and second moments about the number 27 are 0 and 256 respectively. Then its mean and variance are respectively:

    A) 0, 256   B) 27, 256
    C) 0, 229   D) cannot determine

32. If $\{X_n\}$ is a sequence of independent and identically distributed random variables with $Var(X_n) = \sigma^2 < \infty$ and $S_n = \sum_{i=1}^{n} X_i$. Consider the following lists and select the correct match

	List I		List II
a.	Kolmogorov's SLLN	1.	$\frac{S_n}{n} \xrightarrow{P} E(x)$
b.	Khintchin's WLLN	2.	$P(|\bar{X}_n - E(\bar{X}_n)| > \epsilon) < \frac{V(\bar{X}_n)}{\epsilon^2}$
c.	Chebychev's inequality	3.	$\frac{S_n}{\sqrt{n}} \xrightarrow{d} Z \sim N(0, 1)$
d.	Lindberg –Levy CLT	4.	$\frac{S_n}{n} \xrightarrow{a.s} E(x)$

A) a-4, b-1, c-2, d-3   B) a-3, b-2, c-1, d-4
C) a-1, b-2, c-3, d-4   D) a-1, b-3, c-2, d-4

33. ------- are short term variations with period less than one year.
A) Trends   B) Irregular variations
C) Seasonal variations   D) None of these

34. ------- is the overall tendency of the time series data to increase or decrease over a long period of time.
A) Moving average   B) Random variation
C) Cyclical movements   D) Trend

35. If A is an orthogonal matrix then:
A) $|A| = 0$ B) $|A|$ is always positive
C) $|A|$ is always negative   D) None of these

36. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$ is:

A) 1   B) 2   C) 3   D) none of these

37. The solution to the system of linear equations: $x + y + z = 3; x + 2y + 3z = 4$ and $x + 4y + 9z = 6$ is:
A) $x = 2; y = 1; z = 0$
B) $x = 1; y = -1; z = 2$
C) Solution doesn't exists since the system is inconsistent
D) Infinitely many solutions

38. Given that A is a square matrix. Then:
   A) A is always of full rank.
   B) A satisfies its characteristic equation only when it is orthogonal
   C) A always satisfies its characteristic equation
   D) None of the above is true

39. Let W be a collection of k number of vectors in R^n. Consider the statements:
   (1) If any of the vectors is a null vector then the vectors in W are linearly dependent
   (2) If $k < n$, then the vectors in W are always linearly dependent.
   Then:

   A) Only (1) is true
   B) Only (2) is true
   C) Both (1) and (2) are true
   D) Both (1) and (2) are false

40. Determine the value of k such that the function $f(x, y) = kxy$, $0 < x < 1$, $0 < y < x$ is a probability density function.
   A) 1   B) 7   C) 8   D) 6

41. A two digit number is formed from among the digits 1 to 9 without replacement. What is the probability that the number is less than 60 and even?
   A) $\dfrac{1}{4}$ B) $\dfrac{1}{12}$ C) $\dfrac{1}{20}$ D) $\dfrac{1}{5}$

42. Which of the following statements on non-parametric test are true?
   1. The Kolmogorov – Smisnov test assumes continuity of the data
   2. The chi-square test can be applied when the data are discrete or continuous
   3. Wilcoxon signed rank test can be applied only for the problem of location

   A) 1 only   B) 1 and 2 only   C) 1 and 3 only   D) All are correct

43. The distribution function (cdf) of a random variable (rv) X is given by
   $F(x) = \begin{cases} 0, & if\ x < c \\ 1, & if\ x \ge c \end{cases}$, $\forall x \in R$. Then which of the following is always true?

   A) F is mixture type.
   B) Pdf of X is not defined
   C) X has point binomial distribution
   D) X is degenerate rv

44. For a continuous type random variable X with distribution function F(x) and density function f(x), which of the following is always true?
   A) f(x) is continuous but F(x) may or may not be continuous
   B) F(x) is continuous but f(x) may or may not be continuous
   C) Both F(x) and f(x) are not necessarily continuous, but should have finite number of discontinuities.
   D) Both F(x) and f(x) are continuous

45. Which of the following defines the geometric mean G of a random variable X with $P(X > 0) = 1$?

A) $G = E(\ln X)$

B) $G = \exp\{E(X)\}$

C) $G = \exp\{-\ln E(X)\}$

D) $G = \exp\{E(\ln X)\}$

46. The mean deviation of a random variable X about a point c will be the least if c is the:

A)   Median of X   B)   Mode of X   C)   Mean of X   D)   zero

47. Suppose $X_{(1)}, X_{(2)}, \ldots\ldots X_{(n)},$ are order statistics of i.i.d random variables distributed $U(0, \theta), \theta > 0$. Then a consistent estimator of θ is given by:

A) $X_{(n)}$ B) $X_{(1)}$ C) $\bar{X}$ D) $X_{(n)} - X_{(1)}$

48. In the case of constant estimators, which of the following is always true?

A)   Mean square consistency implies weak consistency

B)   A consistent estimator, if it exists, is always unique

C)   All consistent estimators are unbiased also

D)   Weak consistency implies mean square consistency

49. Which among the following is a statistic whose distribution doesn't depend upon the population parameter θ under investigation?

A)   Minimal sufficient   B)   UMVUE

C)   Complete sufficient   D)   Ancillary

50. A distribution with second, third and fourth central moments respectively as 1.4, -0.5 and 8.5 is:

A)   symmetric   B)   leptokurtic   C)   platykurtic   D)   pyrokurtic

51. Which of the following is a method of local control?

A)   blocking   B)   confounding   C)   replication   D)   none of these

52. RBD is preferred over CRD when:

A)   Sample size is small   B)   Sample size is large

C)   Data is homogeneous   D)   None of these

53. Which of the following is a generalization of chi-square distribution?

A)   Wishart distribution   B)   Mahalanobis D^2

C)   Hotteling's T^2 D)   none of these

54. Let X have a multivariate normal distribution. Consider the two statements:

(1)   Linear combinations of components of X are normally distributed.

(2)   The conditional distribution of the components are multivariate normal.

Then:

A)   Both statements are true   B)   Only (1) is true

C)   Only (2) is true   D)   Both statements are false

55. If A, B and C are three events in the same sigma field such that $A \subset B$, then:
   A) $P(A|C) \le P(B|C)$ B) $P(A) < P(B)$ always
   C) $P(A) \ge P(B)$ D) $P(A \cap B) \ge P(B)$

56. A binomial distribution has mean 4 and variance 3. Then it has:
   A) Only one mode as 4   B) Two modes as 3 and 4
   C) Only one mode as 4.25   D) Mode doesn't exist

57. If X and Y are independent uniform variates over (0,1), then X+Y follows:
   A) uniform over (0,2)
   B) triangular over (0,2)
   C) beta distribution of first kind
   D) none of these

58. Which of the following cannot be a moment generating function (in the real variable t)?

   A) $\dfrac{1}{1-t^2}$ B) $\dfrac{t}{1-t}$ C) $e^{4(e^t - 1)}$ D) $\dfrac{1}{8}(1+e^t)^3$

59. The mean age of a combined group of men and women is 25. If the mean age of men is 26 and that of the group of women is 21, then the percentage of women in the group is:
   A) 80   B) 20   C) 55   D) none of these

60. A distribution with first, second and third quartiles as 25.8, 49 and 64.2 respectively is:
   A) Cannot determine   B) Positively skewed
   C) Negatively skewed   D) Symmetric

61. In the case of normal distribution $N(\mu, \sigma^2)$, where μ is known, the quantity $\sum (x_i - \mu)^2$ is:
   A) Sufficient but not complete for σ^2
   B) Complete but not sufficient for σ^2
   C) Complete and sufficient for σ^2
   D) Neither complete nor sufficient for σ^2

62. Let $Y = ZB + U$, where Y is an $n \times 1$ vector, Z is an $n \times p$ matrix of known real numbers $(n \ge p)$, B is a $p \times 1$ vector of parameters and U is an $n \times 1$ random vector. Also U follows $N(\mathbf{0}, \sigma^2 I)$, Then

   1. Least square estimate of B is $\hat{B} = (Z'Z)^{-1}Z'Y$ only if $Z'Z$ has rank p
   2. The matrix $(I - Z(Z'Z)^{-1}Z')$ is symmetric and indempotent
   3. $\hat{B}$ follows $N_p(B, \sigma^2 (Z'Z)^{-1})$

   Choose the correct answer from the following:

   A) Only the statement 1 and 2 are correct
   B) Only the statement 1 and 3 are correct
   C) Only the statement 2 and 3 are correct
   D) All the three statements are correct

63. If $X \sim U(-\theta, \theta)$, then a sufficient statistic for θ is:

A) $X_{(n)}$ B) $X_{(1)}$ C) $(X_{(1)}, X_{(n)})$ D)   none of these

64. Consider two statements about UMVUE of a parameter θ:
1.   If UMVUE exists, it is unique
2.   If observations are iid, and if UMVUE exists, it must be a symmetric function of observations.
Then:

A)   Both (1) and (2) are true   B)   Only (1) is true
C)   Only (2) is true   D)   Both are false

65. A sample of size n is drawn from the population with pdf
$f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1; \theta > 0$. Then the moment estimator of θ is:

A) $\dfrac{\overline{X}}{1-\overline{X}}$ B) $\overline{X}$ C) $\dfrac{\overline{X}}{n^2}$ D) $1 - \overline{X}$

66. The value of $1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \dfrac{1}{2^4} + \dots$ is:

A)   2   B)   1.5   C)   2.4   D) $\sqrt{2}$

67. Let $X \sim N(\mu, \sigma^2)$, where $\sigma^2 > 0$ and μ is known. Then MLE of σ^2 is:

A) $\dfrac{1}{n}\sum(x_i - \overline{x})^2$ B) $\dfrac{1}{n-1}\sum(x_i - \mu)^2$

C) $\dfrac{1}{n-1}\sum(x_i - \overline{x})^2$ D) $\dfrac{1}{n}\sum(x_i - \mu)^2$

68. In a simple linear regression model having nonzero intercept term, with usual notations the unbiased estimate of the model error variance σ^2 is given by:

A) $\dfrac{1}{n-1}\sum e_i^2$ B) $\dfrac{1}{n-2}\sum e_i^2$

C) $\dfrac{1}{n}\sum e_i^2$ D) None of these

69. Convenience sampling is an example of:
A)   Probabilistic sampling   B)   Stratified sampling
C)   Nonprobabilistic sampling   D)   Cluster sampling

70. The square of a Student's t statistic with n df follows:
A)   F with $(1, n)$ df   B)   chi-square with n df
C) t with n^2 df   D) t with n df itself.

71. If the power of the test is not less than its size, it is called:
A)   Biased   B)   Unbiased
C)   Most powerful   D)   None of these

72. Consider a sample of size n from $N(\mu, \Sigma)$. Let $\bar{X}$ be the sample mean vector and

$S = \frac{1}{n-1}\sum_{i=0}^{n}(X_i - \bar{X})^1(X_i - \bar{X})'$ and μ_0 is a given vector of order px1.

Then $n(\bar{X} - \mu_0)^1 S^{-1}(\bar{X} - \mu_0)$
is distributed as;

A) $\frac{(n-1)}{(n-p)}F_{p,\ n-p}$

B) $\frac{np}{(n-p)}F_{p,\ n-p}$

C) $\frac{(n-1)p}{(n-p)}F_{n-p,\ p}$

D) $\frac{(n-1)p}{(n-p)}F_{p-1,\ n-p}$

73. Which among the following density functions does not possess the monotone likelihood ratio property through a statistic?
A)   Binomial   B)   Uniform   C)   Cauchy   D)   Normal

74. The empirical distribution function as a nonparametric estimator of the population distribution function is:
A)   unbiased and consistent   B)   unbiased but not consistent
C)   biased but consistent   D)   biased and not consistent

75. For a symmetric distribution, the third central moment is:
A)   Always negative   B)   Always positive
C)   Always zero   D)   Same as the mean of the distribution

76. If $u = ax + b$ and $v = -cy + d$, where c and a are positive real numbers and if the coefficient of correlation between u and v is 0.4, then the correlation between x and y is:
A)   -0.4   B)   0.4   C)   -1   D)   none of these

77. If the coefficient of rank correlation between marks in Mathematics and Physics obtained by a group of students is 0.8 and if the sum of the squares of difference in ranks is given to be 33, then the total number of students in that group is:
A)   10   B)   100   C)   50   D)   none of these

78. If the correlation coefficient between two variables X and Y is 0.4 and the regression coefficient of X on Y is 0.2, then the regression coefficient of Y on X is:
A)   0.5   B)   0   C)   0.75   D)   0.8

79. In a 3^3 experiment after confounding two interactions the following principal block:
{(0, 0, 0), (1, 2, 2), (2, 1, 1)} was obtained. Then state which among the following pairs of interacting is not confounded in the arrangement

A) AB, BC^2 B) AB^2, BC C) AC, BC^2 D) AB, AB^2C^2

80. If n people are seated around a round table, the probability that two particular individuals are seated next to each other is:
A) $\frac{2}{n}$ B) $\frac{2}{n-1}$ C) $\frac{1}{n-1}$ D)   none of these