A

20221 **120 MINUTES**

1. The roots of the equation $(a-b)x^2 - 2ax + (a+b) = 0$ are:
   A) *Equal* B) *Rational*
   C) *Irrational* D) *Imaginary*

2. The range of the function $f(x) = \frac{|x-5|}{x-5}$, $x \in \mathbb{R}$ and $x \neq 5$ is:

   A) $(-1, 1)$ B) $[-1, 1]$ C) $\{-1, 1\}$ D) $\mathbb{R} \setminus \{5\}$

3. If a straight line makes equal intercepts l on the axes and passes through the point $(3, 2)$, then the value of l is:

   A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) 6 D) 5

4. The straight line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ lies in the plane

   A) $x - 2y - 4z + 7 = 0$ B) $-x + 2y + 4z + 7 = 0$
   C) $x - 2y - 4z + 21 = 0$ D) $-x + 2y + 4z + 21 = 0$

5. The value of $\int_0^{\pi/2} \frac{(\sin x)^4}{(\sin x)^4 + (\cos x)^4} dx$ is:

   A) $\pi/4$ B) $\pi/2$ C) π D) 2π

6. The area bounded by the curves $y = \tan x$, $y = \cot x$, where $0 \leq x \leq \pi/2$ and the x −axis is:
   A) $\log 2$sq: units B) $\log \sqrt{2}$ sq: units.
   C) 2 sq: units. D) $\sqrt{2}$ sq: units.

7. Two dice are thrown once. The probability of getting an even number on the first die and a total of 8 is:

   A) $1/2$ B) $5/9$ C) $5/36$ D) $1/12$

8. The lim sup and lim inf for the sequence $\{(-1)^n\}$ are respectively:
   A) 1 and 1 B) 1 and 0 C) 1 and -1 D) -1 and -1

9. The series $\sum_{n=1}^{\infty} \frac{n!}{(n+1)^n} x^n$ where $x > 0$ is convergent if:

   A) $x > e$ B) $x \geq e$ C) $x \leq e$ D) $x < e$

10. Let $f(x) = 1$ for $x = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and $f(x) = 0$ elsewhere on $[0, 1]$. Then

A) f is Riemann integrable on $[0, 1]$ and $\int_0^1 f = 4$.

B) f is Riemann integrable on $[0, 1]$ and $\int_0^1 f = 0$.

C) f is Riemann integrable on $[0, 1]$ and $\int_0^1 f = \frac{9}{5}$.

D) f is not Riemann integrable on $[0, 1]$.

11. 1. Every countable set has measure zero
2. There exists uncountable sets of measure zero

Which of the above statements holds true?

A) Both 1and 2 B) 1 only

C) 2 only D) Neither 1 nor 2

12. Let the function f be defined on $[0, 1]$ by $f(0) = 0$ and $f(x) = x \sin\frac{\pi}{x}$ when $x \neq 0$. Then

A) f is continuous and is of bounded variation $[0, 1]$

B) f is not continuous but f is of bounded variation $[0, 1]$

C) f is continuous but f is not of bounded variation $[0, 1]$

D) f is not continuous and f is not of bounded variation $[0, 1]$

13. The fixed points of the Mobius transformation $w = \frac{8z-15}{z}$ are:

A) $4, \ 2$ B) $\pm 4i$ C) $5, 3$ D) $5 \pm 3i$

14. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{2^{n^2}}$ is:

A) 2 B) ∞ C) 0 D) $\frac{1}{2}$

15. Which of the following is true?

A) The function $f(z) = e^z$ has an isolated essential singularity at $z = 0$.

B) The function $f(z) = e^{1/z}$ has an isolated essential singularity at $z = 0$.

C) The function $f(z) = \frac{1}{z(z^2+4)}$ has an isolated essential singularity at $z = 0$.

D) The function $f(z) = \frac{\sin z}{z}$ has an isolated essential singularity at $z = 0$.

16. Let γ be the circle of radius 1 centered at $z = -i$. The value of the integral

$\int_\gamma \frac{e^z}{z^2+1} dz$ is

A) πe^i B) πe^{-i} C) $-\pi e^i$ D) $-\pi e^{-i}$

17. Which of the following is an element of the subgroup generated by $(1\ 2\ 3)$ and $(2\ 3\ 4)$ in the symmetric group S_4

A) $(1\ 2)$ B) $(1\ 3)$

C) $(1\ 2\ 3\ 4)$ D) $(1\ 3)(2\ 4)$

18. Let G be the multiplicative group of non-zero rational numbers. Which of the following is true of G

A) $G\ is\ cyclic$

B) G has a subgroup of order 2

C) G has a subgroup of order 3

D) G has a subgroup of order 5

19. Which of the following groups is isomorphic to $\mathbb{Z}_{20} \times \mathbb{Z}_{25}$

A) $\mathbb{Z}_{500}$ B) $\mathbb{Z}_{10} \times \mathbb{Z}_{50}$ C) $\mathbb{Z}_5 \times \mathbb{Z}_{100}$ D) $\mathbb{Z}_4 \times \mathbb{Z}_{125}$

20. The number of solutions for x in the equation $x^2 + x = 0$ in the ring $\mathbb{Z}_{12}$ is

A) 2 B) 3 C) 4 D) 5

21. Which of the following is a generator of a maximal ideal in the ring $\mathbb{Z}_{100}$

A) 10 B) 15 C) 20 D) 25

22. Which of the following is an irreducible polynomial in $\mathbb{Z}_3[x]$?

A) $x^3 + 2x^2 + 1$ B) $2x^3 + x^2 + 1$

C) $2x^3 + 2x^2 + x + 1$ D) $2x^3 + x^2 + 2x + 1$

23. Which of the following pairs of fields are isomorphic?

A) $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$

B) $\mathbb{Q}(\sqrt{2} + \sqrt{3})$ and $\mathbb{Q}(\sqrt{6})$

C) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ and $\mathbb{Q}(\sqrt{2}, \sqrt{5})$

D) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ and $\mathbb{Q}(\sqrt{2} + \sqrt{3})$

24. Which of the following is not the order of a finite field?

A) 7 B) 9 C) 11 D) 12

25. The system of equations $x - 3z = 1, y + z = 4$ is

A) inconsistent

B) consistent and has a unique solution

C) consistent and has an infinite number of solutions

D) consistent and the solution set is a subset of $\mathbb{R}^2$

26. Let A be a $n \times n$ square matrix which is row equivalent to the $n \times n$ identity matrix. Then which one of the following is not true.

A) The equation $AX = 0$ has only the trivial solution.

B) The equation $AX = b$ has at least one solution for each $b \in \mathbb{R}^n$.

C) A is an invertible matrix.

D) The columns of A form a linearly dependent set.

27. Which one of the following is the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

A) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

28. Let W be the subspace of the solutions of the system of equations $x - 2y + z = 0,\ 2x - 3y + z = 0$ in $\mathbb{R}^3$. Then $\dim W$ is:
A) 0 B) 1 C) 2 D) 3

29. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear with $T(1,0) = (1,3)$ and $T(1,1) = (2,5)$. What is $T(2,3)$?
A) $(5,\ 12)$ B) $(3,\ 8)$ C) $(1,\ 2)$ D) $(1,\ 8)$

30. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1,\ x_2,\ x_3) = (\,x_1 - x_3\,,\ -x_2,\ 0\,)$ for $(x_1,\ x_2,\ x_3) \in \mathbb{R}^3$. Then the null space of T is

A) $\{\ (0,\ 0,\ a\,) : a \in \mathbb{R}\ \}$ B) $\{\ (\,a,\ 0,\ 0\,) : a \in \mathbb{R}\ \}$

C) $\{\ (\,a, a,\ 0\,) : a \in \mathbb{R}\ \}$ D) $\{\ (a,\ 0,\ a\,) : a \in \mathbb{R}\ \}$

31. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $(\,2a_1 + 3a_2 - a_3,\ a_1 + a_3\ )$ for $(a_1,\ a_2,\ a_3) \in \mathbb{R}^3$. Find the matrix which represents T where $\mathbb{R}^3$ and $\mathbb{R}^2$ are assigned with standard ordered basis.

A) $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -2 & -3 & 1 \\ -1 & 0 & -1 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 1 \end{bmatrix}$ D) $\begin{bmatrix} -2 & -1 \\ -3 & 0 \\ 1 & -1 \end{bmatrix}$

32. Let $B = \{(2,\ 1),\ (3,\ 1)\}$ be an ordered basis for $\mathbb{R}^2$. If $B^* = \{\,f_1,\ f_2\,\}$ is the dual basis for B, then f_2 is given by
A) $f_2(x,\ y) = x - 2y$ B) $f_2(x,\ y) = 2x - 4y$
C) $f_2(x,\ y) = 3y - x$ D) $f_2(x,\ y) = 4y - 2x$

33. Let T be the operator on $\mathbb{R}^2$ defined by $T(x,\ y) = (\,x, -y)$ for $x,\ y \in \mathbb{R}^2$.
If $(0,\ 1)$ is an eigen vector corresponding to an eigen value λ of T, then the value of λ is
A) 2 B) -2 C) 1 D) -1

34. Which one of the following is not true in relation with (,), the $g.c.d$. of two numbers.
A) If $(a,\ b) = (a,\ c) = 1,$ then $(a,\ bc) = 1.$
B) $(2,\ n) = 1$ for any odd integer n
C) $(a,\ 1) = (1,\ a) = 1$
D) $(a,\ 0) = (0,\ a) = 0$

35. If $\varphi(n)$ is the Euler 's totient function, then, for any odd prime p, the value of $\varphi(p^2 + p)$ is
A) $p\varphi(p+1)$ B) $(p-1)\varphi(p+1)$
C) $p\varphi(p)$ D) $(p-1)\varphi(p)$

36. Which one of the following is not true?
A) $12^{16} \equiv 1 (mod\ 17)$ B) $16^{12} \equiv 1 (mod\ 13)$
C) $13\,.\,(12)^{16} \equiv 13 (mod\ 221)$ D) $13.\,(12)^{16} \equiv 1 (mod\ 221)$

37. Consider the system of linear congruences :$2x \equiv 1 (mod\ 3)$, $3x \equiv 1 (mod\ 4)$, $4x \equiv 1 (mod\ 5)$. Then the number of solutions congruent $mod(60)$ is:
A) Infinite B) 1 C) 2 D) 3

38. A particular solution of the equation $y'' + y = \sec x$ is:
A) $y = x \sin x + \cos x \ \log \cos x$
B) $y = x \cos x + \sin x \log \cos x$
C) $y = x \sin x + \cos x \ \log \sin x$
D) $y = x \cos x + \sin x \log \sin x$

39. The equation $[(x-y)^2 + 1]u_{xx} + 2u_{xy} + [1-(x-y)^2]u_{yy} = 0$ is:
A) Parabolic B) Elliptic C) Hyperbolic D) None of these

40. The characteristic equation of the one-dimensional wave equation
$u_{xx} - 4\,u_{tt} = 0\,,\ -\infty < x < \infty,\ t > 0;$ are

A) $2x - t = \eta$ and $2x + t = \xi$
B) $x - 2t = \eta$ and $x + 2t = \xi$
C) $4x - t = \eta$ and $4x + t = \xi$
D) $x - 4t = \eta$ and $x + 4t = \xi$

41. Consider the norms $||\ ||_1,\ ||\ ||_2,\ ||\ ||_\infty$ on $\mathbb{R}^2$. Then for all $x \in \mathbb{R}^2$, which one of the following is not true.

A) $||\mathrm{x}||_1 \leq \sqrt{2}\,||x||_2$

B) $||x||_2 \leq \sqrt{2}||x||_\infty$

C) $||\mathrm{x}||_\infty \leq ||x||_2 \leq ||x||_1$

D) $||\mathrm{x}||_1 \leq ||x||_2 \leq ||\mathrm{x}||_\infty$

42. Which of the following statements is true?

A) The set of all real numbers $\mathbb{R}$ with usual metric is not complete.

B) The open interval $(0, 1)$ with usual metric is not complete .

C) The set of all real numbers $\mathbb{R}$ with discrete metric is not complete.

D) The Euclidean $m -$ space $\mathbb{R}^m$ with usual metric is not complete.

43. Let the set of all real numbers $\mathbb{R}$ be equipped with the discrete metric d defined by

$$d(x,\ y) = \begin{cases} 1 & if \quad x \neq y \\ 0 & if \quad x = y \end{cases}$$

The open ball $B(3,\ 2)$centered at 3 and radius 2 is:

A) $\{3\}$ B) $\mathbb{R}$ C) $(1, 5)$ D) $(3, 5)$

44. Let X be an infinite set with the co-finite topology (In which the closed sets are the finite sets and X). Then

A) X is a $T_2 -$ space and is not compact.

B) X is a $T_2 -$ space and is compact.

C) X is a $T_1 -$ space and is not compact.

D) X is a $T_1 -$ space and is compact.

45. Let $X = \left(\mathbb{R}^3,\ ||\ ||_1\right)$ and $Y = \left(\mathbb{R}^2,\ ||\ ||_1\right)$ be the normed spaces. Let $F\ :\ X\ \rightarrow Y$ be defined by $F(x_1,\ x_2,\ x_3) = (x_2, -x_3)$.Then which one of the following is true.

A) F is surjective, closed but not open.

B) F is open and continuous, but not closed.

C) F is linear, continuous but not closed.

D) F is linear, continuous and open.

46. Let X be an inner product space and for $x, y\ \in X,$ let $\| x + y \| = 25,$ $\| x - y \| = 5,$ $\| x \| = 15.$ Then $\| y \|$ is:

A) 10 B) 15 C) 5 D) 9

47. Let H be the complex Hilbert space $l^2$for $n = 1, 2,\ \ldots,$ let $u_n = (\ 0,\ \ldots\ , 0, 1, 0,\ \ldots,\ ),$ where 1 occurs only in the n^{th}entry. Then which one of the following is not true.

A) For $x\ \in H,\ x = \sum_{n=1}^{\infty} \langle x,\ u_n \rangle u_n$

B) For $x\ \in H,\ ||\, x||^2 = \sum_{n=1}^{\infty} |\langle x,\ u_n \rangle|^2$

C) If $x\ \in H$ and $\langle x,\ u_n \rangle = 0$ for all n, then $x = 0.$

D) $\overline{span\,\{u_n : n = 1,\ 2, \ldots\}}\ \neq\ H$

48. Let H be the real Hilbert space $L^2([-1,\ 1])$ and $f: H\ \rightarrow \mathbb{R}$ be defined by $f(x) = \int_{-1}^{1} x(t).t\ dt$. Then $\| f \|$ is

A) 1 B) ${}^{1}\!/\!_{\sqrt{2}}$ C) $\frac{\sqrt{2}}{\sqrt{3}}$ D) 2

49. Let H be the real Hilbert space l^2 and $A : H \to H$ be defined by
$A(x(1), x(2), x(3), \ . \ . \ . \ ) = \ (x(1), 0, x(3), 0 ,. \ . \ . \ )$.
Then which one of the following is not true.

A) $A^2 = A$
B) The null space of A is closed
C) The null space of A is finite dimensional
D) $\| A \| = 1$

50. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is
A) (1,0) B) (−1,0) C) (−2,0) D) (0,2)

51. The value of $\sqrt{2i}$ is equal to:
A) 2+i B) 2−i C) 1+i D) 1−i

52. The shortest distance of the line $y - x - 1 = 0$ from the parabola $y^2 = x$ is:
A) $2\sqrt{3}$ B) $\frac{2\sqrt{3}}{4}$ C) $\frac{3\sqrt{2}}{8}$ D) $\frac{13}{4}$

53. The volume generated by rotating the triangle with vertices at (0,0), (3,0) and (3, 3) about x-axis is:
A) 18π B) 2π C) 36π D) 9π

54. If A is orthogonal, then:
A) A^T and A^{-1} are both orthogonal
B) A^T is orthogonal, but A^{-1} is not
C) A^{-1} is orthogonal, but A^T is not
D) None of these

55. Let rank (AB) = k, then:
A) rank (A) ≤ k and rank (B) ≥ k
B) rank (A) ≤ k and rank (B) ≤ k
C) rank (A) ≥ k and rank (B) ≤ k
D) None of these

56. Z be the set of integers and define $a \oplus b = a + b + 1$ and $a \odot b = a + b + ab$, then the ring $(Z, \oplus, \odot)$ is:
A) Commutative ring B) Integral domain
C) Field D) None of these

57. If $\gamma : [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z-a}$ is

A) Integer B) Rational number
C) Real number D) Complex numbers

58. Let $J_p(x)$ denotes the Bessel function, then $\frac{d}{dx}(x^p J_p(x))$ is:

A) $x^p J_{p-1}(x)$ B) $x^{p-1} J_{p-1}(x)$

C) $x^{p-1} J_p(x)$ D) $x^{p-1} J_{p-2}(x)$

59. How many elements of order 7 are there in a group of order 168?

A) 48 B) 7 C) 24 D) 168

60. The automorphism group $Aut(Z_2 \times Z_2)$ is isomorphic to:

A) Z_2 B) S_3 C) Q D) Z

61. The complete solution of $z = p^2 + q^2$ is given by:

A) $z(1 + a^2) = x + ay + b$ B) $4z(1 + a^2) = (x + ay + b)^2$

C) $z(1 + a) = (x + b)y$ D) None of the above

62. The equation of the envelope of the family of curves represented by general solution of a differential equation is called

A) particular solution B) singular solution

C) complementary solution D) None of the above

63. If V and W are vector spaces of dimension m and n respectively over F, then the dimension of $Hom(V, W)$ is:

A) m B) m + n C) m / n D) mn

64. If K is an extension field of F and $a \in K$ is algebraic of degree n over F, then

A) [F(a): F] = n B) [F(a): K] = n

C) [K : F(a)] = n D) [F,F(a)] = n

65. Let T be a linear map on a finite vector space V, then

A) $Rank(T) < dim(V)$ B) $Rank(T) = dim(V)$

C) $Rank(T) > dim(V)$ D) $Rank(T) = nullity(V)$

66. The ideal $< (x^2+4) >$ in Q(x) is

A) a maximal ideal B) a prime ideal

C) not a prime ideal D) none of these

67. If $f(x) = x^3 + 3x^2 - x - 1 \in Q(x)$, then the Galois group is

A) S_3 B) A_3 C) Z_3 D) None of these

68. Which of the following is a subspace of R^3

A) $W = \{(a, b, c) : a \leq 0\}$ B) $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$

C) $W = \{(a, b, c) : a, b, c \in Q\}$ D) None of these

69. The sum of the series $\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \cdots$ is

A) e^3 B) $\frac{1}{2}(e^3 - e^{-3})$ C) $3e$ D) $2e$

70. An integrating factor for the differential equation $(1+\ x^2)\frac{dy}{dx}+y=e^{tan^{-1}x}$ is

A) sec^2x B) $e^{2tan^{-1}x}$ C) $tan^{-1}x$ D) $e^{tan^{-1}x}$

71. The value of λ for which the diff. equation $(xy^2+\lambda x^2y)\,dx+(x+y)\,x^2dy=0$ is exact is

A) 1 B) -1 C) 2 D) 3

72. Every finite Hausdorf space is

A) connected B) totally disconnected
C) disconnected D) normal

73. A topological space is said to be locally compact if

A) each of its open set is compact
B) each of its closed set is compact
C) each of its subspace is compact
D) each of its points has a neighbourhood with compact closure

74. A metric space is compact if and only if

A) it is complete
B) it is totally bounded
C) it is complete and totally bounded
D) None of these

75. A bag contains a number of marbles of which 80 are red, 24 are white and the rest are blue. If the probability of randomly selecting a blue marble from this bag is 1/5, how many blue marbles are there in the bag?

A) 25 B) 26 C) 27 D) 28

76. At which point the tangent of the curve $y=2x^2-x+1$ will be parallel to $y=3x+9$

A) (2, 9) B) (3, 9) C) (1, 2) D) (2, 1)

77. The series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{8}$ C) π D) 1

78. The minimum value of $f(x,y)=\ x^2\ +\ y^2\ +6x\ +12$

A) 3 B) 6 C) 8 D) 9

79. Find the points of inflection of the function $f(x)=\ x^4-12x^3\ +6x-9$ on the interval $-2\ \leq x\ \leq 10$

A) $x\ =0,6$ B) $x=0,-6$ C) $x=\pm\sqrt{6}$ D) $x=\pm\sqrt{12}$

80. A ball is dropped from a height of 12 m and it rebounds 1/2 of the distance it falls. If it continues to fall and rebound in this way, how far will it travel before coming to rest?

A) 36 m B) 30 m C) 48 m D) 60 m