

MATHEMATICS SYLLABUS 2021

Unit-1 : Algebra, Number Theory and Linear Algebra

Group Theory : Groups, Subgroups, Normal Subgroups and Quotient Groups, Homomorphisms and applications, Permutation groups, Conjugacy and Class equation, Simple group, Sylow's Theorem, Ring Theory : Rings, Special Classes of rings, Homomorphisms, Ideals and Quotient rings, Maximal and Prime ideals, Polynomial rings Principal Ideal Domain, Unique Factorization theorem Field : Field of Quotients of an Integral Domain, Polynomials over the rational field, Algebraic Extension of Fields : Irreducible polynomials, Symmetric function of roots, Solution of Cubic and Biquadratic Equations. Number Theory : Integers, g.c.d., Fundamental Theorem of Arithmetic, Euclidean Algorithm, Arithmetical functions (Euler-function, Mobius function-), Dirichlet multiplication, Linear Congruences, Euler-Fermat Theorem, Linear Diophantine Equations, Fermat's Theorem, Fermat's Little Theorem, Polynomial Congruence, Lagrange's Theorem, Chinese Remainder Theorem, Wilson's Theorem and Applications.

Vector space, Subspace, Linear Dependence, Independence, Dimension and Basis, Linear Transformation, Range and Kernel, Rank and Nullity, Inverse of Linear Transformation, Linear Map associated with matrix. Elementary Row Operations, Rank and Nullity of Matrix, Inverse of a Matrix, Determinants and product of Determinates, Eigen values, Eigen vectors, Characteristic roots. Canonical forms, Triangular form, Nilpotent Transformations, Similarity of Matrices, Quadratic form. Traces and Transpose, Hermitian, Unitary and Normal Transformation.

Unit –II: Real Analysis

Basic Topology : Finite, Countable and Uncountable sets, Metric Spaces, Topological Spaces, Basis, Closed sets, Open Sets, Limit Points, Properties of Connected Spaces and Compact Spaces, Heine Borel Theorem. Sequence and Series : Convergent Sequences, Subsequences, Convergence of Monotone Sequences, Cauchy Sequences, Upper and Lower limits of Sequences, Bolzano Weirstrass Theorem, Series of non-negative terms, Convergence tests, Power Series, Cauchy Convergence Criterion, Absolute Convergence, Alternating Series. Continuity and Differentiability : Properties of Continuous Function, Continuity and Compactness, Continuity and Connectedness, Discontinuity, Monotonic Functions, Mean Value Theorem, Taylor Series, Function of Several Variables : Continuity, Differentiability, Extreme Values, Maxima and Minima, Line Integral, Surface Integral, Volume Integral, Applications of Green's Theorem, Stokes Theorem and Gauss Theorem.

Riemann stieltjes integral: Existence of the integral, Properties of the integral, Fundamental theorem of calculus, change of variables in an integral, Differentiation of integral. Sequence and series of functions uniform convergence of sequence of functions, Cauchy criterion for uniform convergence, weierstrass test for uniform convergence, uniform convergence and continuity, uniform convergence and differentiation, construction of continuous function on the real line which is nowhere differentiable. Measure Theory Lebesgue outer measure. Properties of outer measure, Measurable sets, Cantor set, Borel set and sets, Non measurable sets, Measurable functions, Properties of measurable functions. Lebesgue integration and L^p spaces comparison of Lebesgue and Riemann integral, Lebesgue integral of bounded, measurable functions over sets of finite measure, Bounded convergence theorem, Lebesgue integral for nonnegative measurable function. Fatou's Lemma, Monotone convergence theorem, L^p spaces, essential supremum of a function, Minkowski and Holder inequalities, Absolute summable and summable series in a normal linear space completeness in L^p .

Unit-III: Numerical Analysis and Differential Equations

Root Finding for Non-Linear Equations : Newton's Method, Secant Method, One-point Iteration Method, Multiple Roots, Newton Methods of Non-Linear Systems.

Interpolation Theory : Finite Differences, Newton's Forward and Backward differences, Newton's Divided differences, Lagrange's Interpolation, Errors in data and Forward differences, Hermite Interpolation, Piece-wise linear Interpolation.

Numerical Integration : Newton-cotes integration formula, trapezoidal rule, Simpson's rule. Gaussian quadrature, Asymptotic error formulas and their applications.

Numerical Methods for Ordinary Differential Equations : Euler's Method, Multistep Methods, Midpoint Method, Trapezoidal Method, Single Step Method and Runge-Kutta Method.

Linear differential Equations with constant coefficients and variable coefficients, system of Linear Differential Equations. Laplace Transformation : Linearity of the Laplace transformation, Laplace transforms of derivatives and integrals, shifting theorems, Differentiation and integration of transforms , Convolution theorem, Solution of integral equations and systems of differential equations using Laplace Transformation, Series Solution of differential equations: Power series method, Bessel, Legendre and Hypergeometric equations, Bessel, Legendre functions and their properties Sturm- Liouville problem, Orthogonality of eigen functions. Orthogonality of Bessel functions and Legendre polynomials. Partial Differential Equations of the 1st order. Lagrange's solution. Partial Differential equations of second order. Homogeneous and non-homogeneous equations with constant coefficients, Monge's method. Fourier Series and Fourier Transforms to Boundary value problems. Solution of Laplace equation, wave equation and heat conduction equations.

Unit-IV: Operation Research and Discrete Mathematics

Linear Programming : Simplex Method, Computation Procedure, Use of Artificial Variables. Duality in Linear Programming : General Primal-dual pair, Duality Theorems, Complementary Slackness Theorem, Duality and Simplex Method, Dual Simplex Method.

Games and Strategies : Two-person-Zero Sum Games, Minimax-Maximin Principle, Games with Saddle Points, Mixed Strategies, Graphical Solutions, Dominance Property, Arithmetic Method of $n \times n$ Games, General Solution of $n \times n$ rectangular Games, Transportation and Assignment : General Transportation Problem, Finding Initial Basic Feasible Solution, Test of Optimality, Transportation Algorithm, Transportation Problems, Mathematical Formulation of Assignment Problem, Method of Solution of Assignment Problem, Travelling Salesman Problem.

Fundamental of logic, Normal forms, Logical Inferences, Methods of proof, Mathematical Induction, Rules of Inferences for quantified propositions. Lattice and Boolean Algebra- Binary Relation, Equivalence relations, Poset, Lattice, Hasse Diagram, Algebraic properties of Lattice, Paths and closures, Directed graphs and adjacency matrix, Boolean Algebra, Boolean functions, Minimization of Boolean functions, Recurrence relation- Generating functions of sequences, Calculating coefficients of generating functions, Recurrence relation, solving recurrence relations by substitution and generating functions. Solution by the method of characteristic roots. Graph Theory – Tree and their properties, spanning trees, Binary trees, Euler's formula, Euler's circuits, Hamiltonian Graphs

Unit – V : Complex Analysis and Functional Analysis

Analytical Functions : Continuity, Differentiability, Cauchy-Reimann Equations, Analytic Function, Harmonic Functions. Bilinear Transformation: Elementary. Transformations, Bilinear Transformation, Mapping by Elementary Functions, Complex Integration: Cauchy's Theorem, Cauchy's Integral Formula, Maximum Modulus Theorem, Liouville's Theorem, Morera's Theorem, Related Problems. Singularities and Calculus of residues : Series Expansion, Taylor's Series, Laurent's Series, Zeros of Analytic Function, Singularities, Residues, Cauchy's Residue Theorem, Evaluation of Definite, Integrals.

Normed Linear space Linear spaces, Subspaces, Quotient spaces, properties of norm, Riesz Lemma, Continuity of linear maps, Bounded linear operations, Equivalent norms, Hahn Banach theorem and its consequences. Banach spaces Uniform boundedness principle, closed graph theorem and its consequences, open mapping theorem and its consequences. Spaces of Bounded linear functional Duals and transposes, Duals of l_p , $L^p [a,b]$, $C[a,b]$, Weak convergence, weak* convergence, Reflexivity. Hilbert space, Inner product spaces, Orthonormal sets, Gram Schmidt Orthonormalisation, Bessel's Inequality, Riesz. Fischer theorem, Projection theorem, Riesz representation theorem.