

**Question Booklet and Answer Key**

**For Recruitment Test**

**Held on 21.02.2015 (Evening)**

**Post: TGT Paper-II**

**(Mathematics)**

**'A' Series**



1. The vector spaces formed by the solution space of the set of equations  $x_1 + x_2 + x_3 = 0$ ,  $3x_1 + 2x_2 = 0$  and  $x_2 - x_3 = 0$  has dimension :  
 1) 0                      2) 1                      3) 2                      4) 3
2. The inequality  $|z - 2| < |z - 4|$  represents the half plane :  
 1)  $\operatorname{Re}(z) \geq 3$             2)  $\operatorname{Re}(z) = 3$             3)  $\operatorname{Re}(z) \leq 3$             4) none of these
3. If span of  $S = \{(1, 1, 0), (2, 1, 3)\}$  over  $\mathbb{R}$  is contained in  $\mathbb{R}^3$ , then which one of the following vectors is not in the span of  $S$  ?  
 1)  $(0, 0, 0)$             2)  $(3, 2, 3)$             3)  $(1, 2, 3)$             4)  $(4/3, 1, 1)$
4. The smallest +ve integer  $n$  for which  $\left(\frac{1+i}{1-i}\right)^n = 1$  is :  
 1) 8                      2) 16                      3) 12                      4) none of these
5. If two zeros of a polynomial,  $2x^4 - 3x^3 - 3x^2 + 6x - 2$ , are  $\sqrt{2}$  and  $\frac{1}{2}$  then other two zeros are :  
 1)  $-\sqrt{2}$  and  $-1$             2)  $-\sqrt{2}$  and  $-\frac{1}{2}$             3)  $-\sqrt{2}$  and  $1$             4) none of these
6. Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by  $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ , then rank of  $T$  is :  
 1) 0                      2) 2                      3) 1                      4) 3
7. A survey shows that 63% of the Americans like cheese and 76% like apples. If  $x\%$  of the Americans like both cheese and apples, then :  
 1)  $x = 39$             2)  $39 \leq x \leq 63$             3)  $x = 63$             4) none of these
8. If  $V$  and  $U$  are vector spaces of dimensions 4 and 6 respectively, then  $\dim \operatorname{hom}(V, U)$  is :  
 1) 4                      2) 6                      3) 10                      4) 24



9. Equations of the straight lines touching both  $x^2 + y^2 = 2a^2$  and  $y^2 = 8ax$  are :  
 1)  $x \pm y \pm 2a = 0$       2)  $x - y \pm 2a = 0$       3)  $x \pm y + 2a = 0$       4) None of these
10. Let  $G$  be a group of order 15. Then the number of sylow subgroups of  $G$  of order 3 is :  
 1) 0      2) 1      3) 3      4) 5
11. If  $x, y, z$  are in AP with common difference  $d$  and rank of the matrix  $\begin{bmatrix} 4 & 5 & x \\ 5 & 6 & y \\ 6 & k & z \end{bmatrix}$  is 2, then values of  $d$  and  $k$  are :  
 1)  $d = x/4; k = 7$       2)  $d$  is arbitrary;  $k = 7$       3)  $d = 5; k = 5$       4) none of these
12. The system of equations  $k \cdot x + y + z = 1, x + k \cdot y + z = k$  and  $x + y + k \cdot z = k^3$  does not have a common solution if  $k$  is equal to :  
 1) 0      2) 1      3) -1      4) -2
13. For which value of  $\lambda$  does the line  $y = x + \lambda$  touch the ellipse  $9x^2 + 16y = 144$  ?  
 1)  $\lambda = \pm 5$       2)  $\lambda = 5$       3)  $\lambda = -5$       4)  $\lambda = 1$
14. The minimal polynomial of the  $3 \times 3$  real matrix  $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$  is :  
 1)  $(x - a)(x - b)$       2)  $(x - a)^2(x - b)$       3)  $(x - a)(x - b)^2$       4) none of these
15. Equation of a parabola with vertex at origin, symmetric about  $y$ -axis and passes through a point  $(2, -3)$  is :  
 1)  $y^2 = \frac{9}{2}x$       2)  $x^2 = -\frac{4}{3}y$       3)  $x^2 + y^2 = 13$       4) none of these
16. If  $f(x) = \begin{cases} 1, & \text{if } x \text{ is a rational number} \\ 0, & \text{if } x \text{ is an irrational number} \end{cases}$ ; then what is the value of  $f \circ f(\sqrt{3})$ ?  
 1) 0      2) 1      3) both 0 and 1      4) none of these

17. How many elements of order 2 are there in a group of order 4?  
1) 1                      2) 2                      3) 3                      4) can't say
18. Which of the following matrix is not diagonalizable?  
1)  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$                       2)  $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$                       3)  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$                       4)  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
19. The least upper bound of any subset of the set of rational numbers is always :  
1) a rational number    2) a real number    3) a positive number    4) an irrational number
20. Value of  $\lim_{n \rightarrow \infty} \frac{1}{n} [1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \dots + n^{\frac{1}{n}}]$  is equal to :  
1) 0                      2)  $e$                       3) 1                      4)  $1/e$
21. How many proper normal subgroups are possible for a group of order 112?  
1) 0                      2) 5                      3) 10                      4) 16
22. Let  $G = \mathbb{Z}_4 \times \mathbb{Z}_6$  be a group and H be a subgroup of G generated by (0, 1). Then order of  $G/H$  is :  
1) 1                      2) 2                      3) 3                      4) 4
23. Every group G is isomorphic to a permutation group; this statement is :  
1) Cayley's theorem    2) Lagrange's theorem    3) Liouville's theorem    4) none of these
24. Every group of prime order is :  
1) a field                      2) abelian                      3) cyclic                      4) none of these
25. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  be defined as  $T(x, y, z, t) = (x + y + 5t, x + 2y + t, -z + 2t, 5x + y + 2z)$ .  
Then dimension of the eigen space of T is :  
1) 1                      2) 2                      3) 3                      4) 4



26. The order of 2 in the field  $\mathbb{Z}_{29}$  is :  
 1) 2                      2) 14                      3) 28                      4) 29
27. Let R be a commutative ring with unity of characteristic 3. For  $a, b \in R$ ,  $(a + b)^6$  is equal to :  
 1)  $a^6 + b^6$                       2)  $a^6 - a^3b^3 + b^6$                       3)  $a^6 + a^3b^3 + b^6$                       4) none of these
28. Let  $f(x) = x^2 + 1$ ,  $g(x) = x^3 + x^2 + 1$  and  $h(x) = x^4 + x^2 + 1$  then :  
 1)  $f(x)$  and  $g(x)$  are irreducible over  $\mathbb{Z}_2$                       2)  $g(x)$  and  $h(x)$  are irreducible over  $\mathbb{Z}_2$   
 3)  $f(x)$  and  $h(x)$  are irreducible over  $\mathbb{Z}_2$                       4)  $f(x)$ ,  $g(x)$  and  $h(x)$  are irreducible over  $\mathbb{Z}_2$
29. Let F be a field of order  $2^n$ . Then :  
 1) Char F = 0                      2) Char F = a prime number                      3) Char F = 2                      4) none of these
30. If  $\mathbb{Z}[i]$  is the ring of Gaussian integers, then the quotient  $\mathbb{Z}[i]/\langle 3 - i \rangle$  is isomorphic to :  
 1)  $\mathbb{Z}$                       2)  $\mathbb{Z}/3\mathbb{Z}$                       3)  $\mathbb{Z}/4\mathbb{Z}$                       4)  $\mathbb{Z}/10\mathbb{Z}$
31. The ring  $\mathbb{Z}[\sqrt{-11}]$  is a :  
 1) Euclidean Domain                      2) PID but not Euclidean Domain  
 3) UFD but not PID                      4) not a UFD
32. The order of normalizer of  $\sigma = (12)(34)$  in  $S_6$  is :  
 1) 8                      2) 16                      3) 24                      4) 4
33. Let A be a square matrix of order  $n$ , then nullity of A is :  
 1)  $n - \text{rank } A$                       2)  $\text{rank } A - n$                       3)  $n + \text{rank } A$                       4) none of these
34. If  $n(A) = 115$ ,  $n(B) = 326$ ,  $n(A - B) = 47$ , then the value of  $n(A \cup B)$  equal to :  
 1) 373                      2) 165                      3) 370                      4) 394

35. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation given by  $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_3)$ , then dimension of null space of  $T$  is :
- 1) 0                      2) 1                      3) 2                      4) 3
36. If  $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$  and  $Y = \{9(n - 1) : n \in \mathbb{N}\}$  then  $X \cup Y$  is equal to :
- 1)  $X$                       2)  $Y$                       3)  $\mathbb{N}$                       4) none of these
37. Which one of the following functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  is injective ?
- 1)  $f(x) = |x|$               2)  $f(x) = x^2$               3)  $f(x) = 16$               4)  $f(x) = -x$
38. Points  $(a + 5, a - 4)$ ,  $(a - 2, a + 3)$  and  $(a, a)$  are not collinear only when  $a$  is :
- 1) 3                      2) 4                      3) any real number      4) 5
39. Matrix  $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$  is :
- 1) involutory              2) nilpotent              3) idempotent              4) none of these
40. The point where the line  $y = x + 1$  is a tangent to the curve  $y^2 = 4x$  is :
- 1) (2, 1)                      2) (2, 2)                      3) (1, 1)                      4) (1, 2)
41. If  $f(x) = e^x$  then  $f^{-1}(e)$  is equal to :
- 1) 1                      2)  $e$                       3) 0                      4)  $1/e$
42. The unit normal vector to the level surface  $x^2 + y - z = 4$  at the point  $(-3, 1, 6)$  is :
- 1)  $(-6i + j - k)$       2)  $\frac{1}{\sqrt{38}}(-6i + j - k)$       3)  $\frac{1}{\sqrt{38}}(-9i + j - k)$       4) none of these
43. The function  $y = f(x)$  has a (relative) minimum where :
- 1)  $f(x) = 0$  and  $f'(x) < 0$                       2)  $f'(x) = 0$  and  $f''(x) < 0$   
 3)  $f(x) = 0$  and  $f'(x) > 0$                       4)  $f'(x) = 0$  and  $f''(x) > 0$



44. Derivative of  $y^x$  with respect to  $x$  is :
- 1)  $y^x$                       2)  $\log y$                       3)  $y^x \log y$                       4)  $x y^{x-1}$
45.  $\vec{V} \cdot (\vec{V} \times \vec{V})$  will be equal to :
- 1) 0                      2)  $\vec{V} \cdot (\vec{V} \cdot \vec{V})$                       3)  $\nabla^2 \vec{V}$                       4) none of these
46. How many epimorphisms are possible from  $\mathbb{Z}_{12}$  to  $\mathbb{Z}_6$ ?
- 1) 0                      2) 6                      3) 4                      4) 2
47. For any real number  $x$  the value of  $\lim_{n \rightarrow \infty} \frac{x^n}{n!}$  is equal to :
- 1) 1                      2) 0                      3)  $e$                       4) none of these
48. Value of the  $\lim_{n \rightarrow \infty} (n)^{\frac{1}{n}}$  is :
- 1) 1                      2)  $e$                       3)  $1/e$                       4) 0
49. The sequence  $\langle x_n \rangle = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$  is :
- 1) convergent                      2) monotonically decreasing  
3) not convergent                      4) none of these
50. The series  $\sum \frac{\sqrt{n}}{n^2+1}$  is :
- 1) convergent                      2) divergent                      3) not convergent                      4) none of these
51. The series  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$  is convergent for :
- 1) all real  $x$                       2)  $|x| < 1$                       3)  $|x| \leq 1$                       4)  $-1 < x \leq 1$
52. The limit,  $\lim_{x \rightarrow 0} [x]$ ; where  $[x]$  is the greatest integer less than or equal to  $x$  is :
- 1) positive                      2) zero                      3) not unique                      4) negative





61. The value of  $\lim_{x \rightarrow a} \frac{\sin(a-x)}{a(a-x)}$  is :
- 1)  $a$                       2)  $\sin a$                       3)  $1/a$                       4) does not exist
62. If  $f: [a, b] \rightarrow \mathbb{R}$  is strictly increasing, then :
- 1)  $f^{-1}$  does not exist                      2)  $f^{-1}$  is strictly decreasing  
 3)  $f^{-1}$  is not monotonic                      4)  $f^{-1}$  exists on  $\text{Im}g, f$
63. Let  $f(x) = ax + b$  be a monotonic function on  $\mathbb{R}$  and satisfies condition  $f(x) = f^{-1}(x)$ , then values of  $a$  and  $b$  are :
- 1)  $a = 2, b = -1$                       2)  $a = -1, b \in \mathbb{R}$                       3)  $a = 1, b \in \mathbb{R}$                       4)  $a = 1, b = -1$
64. The function  $f(x) = \frac{|x|}{x}; x \neq 0$  may be made continuous at origin if :
- 1)  $f(0) = 0$                       2)  $f(0) = -1$   
 3)  $f(0) = \infty$                       4) cannot be made continuous for any value of  $f(0)$
65. Range of the function  $f(x) = x^2 + x - 6$  is :
- 1)  $[-6.25, \infty)$                       2)  $(-6.25, \infty)$                       3)  $(6.25, \infty)$                       4)  $(-\infty, 6.25)$
66. Which of the following series is absolutely convergent?
- 1)  $\sum \frac{(-1)^n}{n}$                       2)  $\sum \frac{1}{\sqrt{n}}$                       3)  $\sum \frac{1}{\log(n+1)}$                       4)  $\sum \frac{(-1)^n}{n^{3/2}}$
67. If  $f: A \rightarrow B, g: B \rightarrow C$  are two functions such that  $g \circ f: A \rightarrow C$  is onto then :
- 1)  $g: B \rightarrow C$  is onto                      2)  $f: A \rightarrow B$  is onto                      3) both  $f$  and  $g$  are onto                      4) none of these
68. The value of  $k$  for which the function  $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$  is continuous at  $x = 0$  is :
- 1) 0                      2) 1                      3) infinity                      4) none of these



69. The set of points where  $f(x) = |\sin x|$  is not differentiable is :
- 1) empty                      2)  $\{0\}$                       3)  $\{k \cdot \pi/2 : k \in \mathbb{Z}\}$                       4)  $\{k \cdot \pi : k \in \mathbb{Z}\}$
70. Area enclosed by the curve  $r = 2a \cos \theta$ ,  $0 \leq \theta \leq \frac{\pi}{2}$  is :
- 1)  $\pi a^2$                       2)  $2\pi a^2$                       3)  $\frac{\pi a^2}{2}$                       4) none of these
71. Integral  $\int_0^{\infty} \frac{dx}{\sqrt{x} e^{\sqrt{x}}}$  is equal to :
- 1) 1                      2) -2                      3) 2                      4)  $2/e$
72. If  $f(x) = \int_0^{x^2} \sqrt{\sin t + \cos t} dt$ , then derivative of  $f(x)$  w.r.t.  $x$  is :
- 1)  $2x \sqrt{\sin x^2 + \cos x^2}$     2)  $\sqrt{\sin x^2 + \cos x^2}$     3)  $x \sqrt{\sin x^2 + \cos x^2}$     4) none of these
73. Let  $I = \int_0^{\pi/6} \sin^2 t \cos^2 t dt$ , then :
- 1)  $I = \frac{\pi}{48}$                       2)  $I < \frac{\pi}{48}$                       3)  $I > \frac{\pi}{48}$                       4) none of these
74. The perimeter of the curve  $r = 2 \cos \theta$  is :
- 1)  $\pi/2$                       2)  $\pi$                       3)  $3\pi/2$                       4)  $2\pi$
75. Which of the following is correct?
- 1)  $\int_0^{\infty} e^{-x} dx$  is not convergent                      2)  $\int_0^{\infty} e^{-x^2} dx$  is not convergent
- 3)  $\int_{-\infty}^0 e^x dx$  is convergent                      4)  $\int_0^{\infty} e^{-x^2} dx$  is divergent
76. Consider the function  $f$  defined on  $[-1, 1]$  as  $f(x) = \begin{cases} k, & x \neq 0 \\ 0, & x = 0 \end{cases}$  then :
- 1)  $f$  is not Riemann integrable                      2)  $f$  is Riemann integrable
- 3)  $\int_0^1 f(x) dx = 0$                       4)  $\int_0^1 f(x) dx = 3k$





85. A bag has 8 red balls and 5 white balls. Three balls are drawn at random. Find the probability that the balls drawn are two red and one white.
- 1)  $\frac{C_2^8 \times C_1^5}{C_3^{13}}$       2)  $\frac{C_2^8 + C_1^5}{C_3^{13}}$       3)  $\frac{C_3^8 \times C_3^5}{C_3^{13}}$       4) none of these
86. If slope of the tangent to a curve  $y = f(x)$  at any point  $(x, y)$  is given by the equation  $\frac{dy}{dx} = (x-1)(x-2)^2(x-3)^3(x-4)^4$  then  $y$  will have a local maxima at :
- 1)  $x = 1$  and  $3$       2)  $x = 1$       3)  $x = 2$  and  $4$       4)  $x = 1, 2, 3$  and  $4$
87. Value of the integral  $\int_C (x dx + xy dy)$  over a line  $C$  from  $(1, 0)$  to  $(0, 1)$  is :
- 1)  $\frac{1}{6}$       2)  $\frac{1}{2}$       3)  $-\frac{1}{6}$       4)  $-\frac{3}{4}$
88. Value of the integral  $\iint_C x^2 y^2$  over the domain  $C = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$  is :
- 1)  $\frac{\pi}{96}$       2)  $\frac{\pi}{24}$       3)  $\frac{\pi}{16}$       4)  $\frac{\pi}{4}$
89. If  $f(x, y, z) = 3x^2y - y^3z^2$ , then the grad  $f$  at  $(1, -2, -1)$  is :
- 1)  $-12\hat{i} - 9\hat{j} - 16\hat{k}$       2)  $12\hat{i} - 9\hat{j} + 16\hat{k}$       3)  $12\hat{i} + 9\hat{j} + 16\hat{k}$       4) none of these
90. The directional derivative of  $f(x, y, z) = xy^2 + yz^3$  at point  $(2, -1, 1)$  in the direction of the vector  $\hat{i} + 2\hat{j} + 2\hat{k}$  is :
- 1)  $\frac{13}{3}$       2)  $\frac{11}{3}$       3)  $\frac{3}{11}$       4) none of these
91. Divergence of the 3-dimensional radial vector field  $\vec{r}$  is :
- 1)  $3\hat{i}$       2)  $3$       3)  $3\hat{i} + \hat{j} + \hat{k}$       4)  $1$
92. The vector field  $\vec{F} = x\hat{i} - y\hat{j}$  is :
- 1) divergence free but not irrotational      2) irrotational but not divergence free  
3) divergence free and irrotational      4) neither divergence free nor irrotational

93. Stoke's theorem connects :
- 1) line integral and surface integral      2) surface integral and volume integral  
3) line integral and volume integral      4) none of these
94. Area of a triangle formed by the tips of the vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  is :
- 1)  $\frac{1}{2}(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{c})$       2)  $\frac{1}{2}|(\vec{a}-\vec{b})\times(\vec{a}-\vec{c})|$       3)  $\frac{1}{2}|(\vec{a}\times\vec{b}\times\vec{c})|$       4) none of these
95. An integrating factor of  $x\frac{dy}{dx} + (3x+1)y = xe^{-2x}$  is :
- 1)  $xe^{3x}$       2)  $3xe^x$       3)  $xe^x$       4)  $x^3e^x$
96. General solution of the differential equation  $4x^2y'' - 8xy' + 9y = 0$  is :
- 1)  $c_1 e^{5x/2} + c_2 e^{-3x/2}$       2)  $c_1 e^{3x/2} + c_2 e^{-3x/2}$       3)  $(c_1 + c_2 \log x) x^{3/2}$       4) none of these
97. Solution of the differential equation  $y'' + 4y = 0$  subject to  $y(0) = 1, y'(0) = 2$  is :
- 1)  $\sin 2x + 1$       2)  $\cos 2x + 2x$       3)  $\cos 2x - \sin 2x$       4)  $\cos 2x + \sin 2x$
98. If  $\phi(x, y) = 0$  is a singular solution, then  $\phi(x, y)$  is a factor of :
- 1)  $p$ -discriminant only      2)  $c$ -discriminant only  
3) both  $p$  and  $c$ -discriminants      4) none of these
99. For a complex number  $z$ , the minimum value of  $|z| + |z - \cos\alpha - i \sin\alpha|$  is :
- 1) 2      2) 1      3) 0      4) 3
100. Let  $z$  be a complex number such that  $|z| = 4$  and  $\arg(z) = \frac{5\pi}{6}$ , then  $z$  is :
- 1)  $-\sqrt{3} + i$       2)  $2\sqrt{3} + 2i$       3)  $2\sqrt{3} - 2i$       4)  $-2\sqrt{3} + 2i$



**Key for TGT, Paper-II: Mathematics (21.2.2015(Evening) T-2/6  
'A' Series**

Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.
1	1	26	3	51	4	76	2
2	4	27	1	52	3	77	3
3	3	28	2	53	1	78	1
4	4	29	3	54	3	79	1
5	3	30	4	55	4	80	2
6	2	31	2	56	2	81	2
7	2	32	2	57	3	82	4
8	4	33	1	58	4	83	3
9	3	34	1	59	2	84	4
10	2	35	2	60	3	85	1
11	1	36	2	61	3	86	2
12	4	37	4	62	4	87	3
13	1	38	3	63	2	88	1
14	1	39	3	64	4	89	1
15	2	40	4	65	1	90	4
16	2	41	1	66	4	91	2
17	4	42	2	67	1	92	3
18	1	43	4	68	2	93	1
19	2	44	3	69	4	94	2
20	3	45	1	70	3	95	1
21	1	46	4	71	3	96	3
22	4	47	2	72	1	97	4
23	1	48	1	73	2	98	3
24	3	49	3	74	4	99	2
25	4	50	1	75	3	100	4

