A

22131 120 MINUTES

1. A non-decreasing sequence of real numbers is always
   A) Divergent B) Bounded above
   C) Convergent D) Bounded below

2. Which of the following statements are true?
   1. Countable union of countable sets is countable
   2. Set of all ordered pairs of integers is countable

   A) 1 only B) 2 only
   C) Both 1 and 2 D) Neither 1 nor 2

3. If a sequence $\{S_n\}$ of reals converges to a finite real s, then the sequence $\{S_n^2\}$

   A) Converges to s^2 B) Converges to s
   C) Converges to $\frac{s^2}{2}$ D) Diverges

4. If $\sum_{n=1}^{\infty} a_n < \infty$, then $\sum_{k=n}^{\infty} a_k$ tends to:

   A) zero B) a finite quantity
   C) infinity D) None of these

5. Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces and let $f : M_1 \to M_2$. Then f is continuous if and only if the inverse image of:

   A) every open set is open B) Every closed set is open
   C) every open set is closed D) Every closed set is closed

6. If $\langle M, \rho \rangle$ is a complete metric space and A is a closed subset of M. Then
   A) $\langle A, \rho \rangle$ is complete B) $\langle A, \rho \rangle$ need not be complete
   C) $\langle A, \rho \rangle$ is compact D) $\langle A, \rho \rangle$ is totally bounded

7. Let V and W be finite dimensional vector spaces over the field F such that dim(V)=dim(W). If T is a linear transformation from V into W, then

   A) T is invertible and singular
   B) T is not invertible and singular
   C) T is not invertible and non-singular
   D) T is invertible and non-singular

8. Consider the following set of vectors of R^3:
$S_1 = \{(3,0,-3),(4,2,-2),(-1,1,2)\}$ and
$S_2 = \{(1,0,0),(0,1,0),(0,0,1)\}$. Then which of the following statements are true?

1. S_1 contain linearly dependent vectors
2. S_2 contain linearly independent vectors.

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

9. If A and B are two n-rowed square matrices, then rank (AB)

A) $> \text{rank}(A) + \text{rank}(B)$ B) $\geq \text{rank}(A) + \text{rank}(B)$
C) $> \text{rank}(A) + \text{rank}(B) - n$ D) $\geq \text{rank}(A) + \text{rank}(B) - n$

10. Let $M = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}$ Then which of the following equation is correct?

A) $M^3-31M^2+13M+17I=0$ B) $M^3+31M^2-13M-17I=0$

C) $M^3-13M^2+31M-17I=0$ D) $M^3+13M^2+31M=0$

11. Which of the following is not a component of time series?
A) Secular Trend B) Seasonal Variation
C) Regular Trend D) Cyclical Variation

12. For any sequence $\{A_n\}$ of sets with $D_n = \bigcup_{k \geq n} A_k$, the value of lim D_n is:
A) lim A_n B) lim inf A_n

C) lim sup A_n D) empty set

13. Let I=(0,1), B be the borel field of subsets of I and μ be the Lebesgue Measure. For $n \geq 1$, let $A_n = (0, \frac{1}{n})$. Then $\mu(\liminf A_n)$ is equal to:

A) zero B) unity C) infinity D) $\frac{1}{n}$

14. Let $A_1=E_1$, $A_2 = E_1^C E_2$ *and* $A_3 = E_1^c E_2^c E_3$ be events on a probability space (Ω, A, P). Then $E_1 \cup E_2 \cup E_3$ is:
A) $A_1 \cup A_2 \cup A_3$ B) $A_1 \cap A_2 \cap A_3$
C) $A_1 \Delta A_2 \Delta A_3$ D) $A_1^C \cap A_2^C \cap A_3^C$

15. Pair-wise independent events A,B,C are mutually independent when:

A) A and $B \cup C$ are independent
B) A and $B \cap C$ are independent
C) A and $B^c \cap C^c$ are independent
D) A and $B^c \cup C^c$ are independent

16. For any three events A, B and C, the value of $P(A \cap B^C | C) + P(A \cap B | C)$ is:

A) P(A) B) $P(A|C)$ C) $P(A \Delta C)$ D) $P(A \cup B|C)$

17. For P(B) > 0,

A) $P(A|B) \leq P(A)$ B) $P(A|B) = P(A)$
C) $P(A|B) \geq P(A)$ D) $P(A|B^C) \leq P(A)$

18. For any two independent events A and B with P(A) < P(B), $P(AB) = \frac{6}{25}$ and $P(A|B) + P(B|A) = 1$, the value of P(B) is:

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$

19. If the joint distribution function of X and Yis,

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0 & , elsewhere \end{cases}$$

Then P(X<Y) is

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$

20. Let X and Y be jointly distributed with pdf,

$$f(x, y) = \begin{cases} \frac{1+xy}{4}, & |x| < 1, |y| < 1 \\ 0 & , elsewhere \end{cases}$$

Then

A) X and Y are independent B) X^2 and Y^2 are independent
C) X and X+Y are independent D) None of the above

21. If $E|X|^r = \lambda < \infty$, then $\lim_{n \to \infty} n^r p(X > n)$ is:

A) λ B) $\frac{\lambda}{1+\lambda}$ C) unity D) zero

22. If X and Y are independent random variables, Var(X-Y)=

A) Var(X)-Var(Y) B) Var(X)+Var(Y)

C) Var(XY) D) None of these

23. Let X be a non-negative random variable with distribution function F. Then

$$\int_0^\infty P(X > x) =$$

A) $E(X)$ B) $E(X^2)$ C) $E|X|^{\frac{1}{2}}$ D) 1

24. If the raw moments of a distribution are $\mu_r' = r!$ for $r \geq 1$, then its characteristic function is:

A) $(1+it)^{-1}$ B) $(1-it)^{-1}$

C) $(1+it)^{-1}(1-it)^{-1}$ D) None of these

25. For a distribution with pmf $p(x) = 2^{-x}, x = 1,2,\ldots$, the value of $P\{|X-2| \leq 2\}$ is:

A) $\frac{1}{2}$ B) $< \frac{1}{2}$ C) $> \frac{1}{2}$ D) $\leq \frac{1}{2}$

26. If $\{A_n\}$ is a sequence of independent events on a probability space (Ω, A, P) with

$$\sum_{k=1}^{\infty} P(A_k) = \infty$$

Then P(lim Sup A_n) is

A) zero B) unity C) $\frac{1}{2}$ D) $\frac{1}{4}$

27. Let $\{Y_n\}$ be a sequence of random variables defined on a probability space (Ω, A, P) and Y be a random variable defined on the same probability space. Then $Y_n \xrightarrow{d} Y \Rightarrow Y_n \xrightarrow{p} Y$ only when

A) $\{Y_n\}$ is a sequence of independent random variables.
B) Y is a continuous random variable
C) {Yn} is a sequence of non-negative random variables.
D) Y is a degenerate random variable.

28. If X follows binomial b(n,p), $E\left\{\left[\frac{X}{n} - p\right]^2\right\}$ is,

A) $p(1-p)$ B) $np(1-p)$ C) $\frac{p(1-p)}{n}$ D) $n^2 p(1-p)$

29. If X is a geometric variate with parameter p, then the value of E{X(X-1)} is

A) $\frac{(1-p)}{p}$ B) $\left(\frac{(1-p)}{p}\right)^2$ C) $\frac{(1-p)^2}{2p^2}$ D) $\frac{2(1-p)^2}{p^2}$

30. The cumulant generating function of power series distribution with pmf

$$p(x) = \begin{cases} \frac{a_x \theta^x}{g(\theta)}; & x = 0,1,2,\cdots, a_x \geq o \\ 0 & ; elsewhere \end{cases}$$

is:

A) $\log g(\theta e^t)$ B) $\log g(\theta e^t) + \log g(\theta)$

C) $\log g(\theta e^t) - \log g(\theta)$ D) $\log g(\theta e^t) - e^t \log g(\theta)$

31. Let X~$N(\mu,\sigma^2)$ and let $\phi(.)$ denote the cumulative distribution function of N(0,1). If $\mu^2 = \sigma^2$ $(\mu > 0)$, the value of $P(X < -\mu | X < \mu)$

A) $1-\phi(2)$ B) $2[1-\phi(2)]$ C) $\frac{1-\phi(2)}{2}$ D) $\frac{[1-\phi(2)]^2}{2}$

32. The harmonic mean of beta distribution of second kind with parameters p and q is:

A) $\frac{p}{1-q}$ B) $\frac{p}{1+q}$ C) $\frac{p}{q-1}$ D) $\frac{p-1}{q}$

33. If X and Y are independent gamma variates, then $\frac{X}{Y}$ follows

A) Gamma B) Beta type I C) Beta Type II D) Cauchy

34. If $X_1, X_2, \ldots, X_n$ are independent exponential variates, each with parameter θ, then Min{ $X_1, X_2, \ldots, X_n$} has

A) exponential distribution with parameter $n\theta$
B) exponential distribution with parameter θ^n
C) exponential distribution with parameter θ
D) gamma distribution with parameter $n \, and \, \theta$

35. Which of the following statement is true?

A) Standard Laplace distribution is leptokurtic, but standard logistics is platykurtic
B) Standard Laplace distribution is platykurtic, but standard logistics is leptokurtic
C) Both Standard Laplace distribution and standard logistics are leptokurtic
D) Both Standard Laplace distribution and standard logistics are platykurtic

36. Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential population with probability density function

$$f(x) = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & ; elsewhere \end{cases}$$

Then the cumulative distribution function of Max{ $X_1, X_2, \ldots, X_n$ } is

A) $1 - e^{-nx}$ B) $(1 - e^{-x})^n$ C) $1 - e^{-x}$ D) $1 - e^{\frac{-x}{n}}$

37. Variance of student's t distribution with n degrees of freedom exists when

A) $n > 1$ B) $n \geq 1$ C) $n > 2$ D) $n \geq 2$

38. Beta distribution of first kind with parameter m and n is bimodal if

A) m < 1,n < 1 B) m > 1, n > 1 C) m < 1, n > 1 D) m > 1, n < 1

39. In case of F distribution:

A) Mean < 1, Mode < 1 B) Mean > 1, Mode < 1

C) Mean < 1, Mode > 1 D) Mean > 1, Mode > 1

40. (X,Y) possess bivariate normal distribution if and only if:

A) X and Y are normal variates
B) X and Y are independent normal variates
C) X and Y are dependent normal variates
D) any linear combination of X and Y is a normal variate.

41. If X and Y are standard normal variates with correlation coefficient ρ, then the correlation coefficient between X_1^2 and X_2^2 is

A) ρ B) ρ^2 C) $2\rho^2$ D) $\frac{\rho^2}{2}$

42. A sufficient statistics is minimal if and only if it is a:
A) minimum sufficient statistics in a sequence of sufficient statistics
B) a function of every other sufficient statistics
C) a function of UMVU estimators
D) all the above

43. An estimator T, based on a sample of size n is considered to be the best estimator of θ if:
A) $P\{|T_n - \theta| < \varepsilon\} \geq P\{|T_n^* - \theta| < \varepsilon\}$
B) $P\{|T_n - \theta| > \varepsilon\} \geq P\{|T_n^* - \theta| > \varepsilon\}$
C) $P\{|T_n - \theta| < \varepsilon\} = P\{|T_n^* - \theta| < \varepsilon\}$ for all θ
D) None of the above

44. Let M be a sufficient statistic for θ and T be another statistic whose distribution is independent of θ then:
A) M and T are both sufficient for θ
B) M and T are independent if M is complete
C) M and T are independent if T is complete
D) M and T are independent if T is minimal

45. Area of the critical region depends on:
A) size of type 1 error B) size of type II error
C) value of the statistic D) number of observations

46. The decision criteria in SPRT depends on the functions of:
A) type I error B) type II error
C) type I and II errors D) none of the two types of errors

47. The Mann-Whitney U test is preferred to a t-test when:
A) The assumption of normality is not met
B) Sample sizes are small
C) Data are paired
D) Samples are dependent

48. Let X follows Bernoulli distribution with parameter θ, then which statistic is sufficient for θ based on the sample size 2?
A) $T = X_1+2X_2$ $T = 2X_1+2X_2$ C) $T=2X_1+ X_2$ D) $T=2X_1+4X_2$

49. Let $X_1, X_2,\dots,X_n$ be a random sample from $N(0,\sigma^2)$. Consider likelihood ratio test for which the critical region is given as $\Sigma X_i^2 > K$. The alternative hypothesis against $H_0:\sigma= \sigma_0$ which leads to an uniformly most powerful test is:

A) $\sigma \neq \sigma_0$ B) $\sigma^2 = \sigma_0$ C) $\sigma < \sigma_0$ D) $\sigma > \sigma_0$

50. Consider the model $Y_i = i\beta + \varepsilon_i$,i=1,2,3 where ε_1, ε_2 and ε_3 are independent with mean zero and variance σ^2,2 σ^2, 3 σ^2 respectively. Which of the following is the best linear unbiased estimate of β?

A) $\frac{11(y_1+y_2+y_3)}{6}$ B) $\frac{6}{11}(y_1+\frac{y_2}{2}+\frac{y_3}{3})$

C) $\frac{y_1+y_2+y_3}{6}$ D) $\frac{3y_1+2y_2+y_3}{10}$

51. Consider the problem of testing H_o: X~Normal with mean 0 and variance $\frac{1}{2}$ against H_1: X~Cauchy(0,1). Then for testing H_0 against H_1, the most powerful size αtest

A) does not exist
B) rejects Ho if and only if $|x|>c_2$ where c_2 is such that the test is of size α
C) rejects Ho if and only if $|x|<c_3$ where c_3 is such that the test is of size α
D) rejects Ho if and only if $|x|<c_4$or $|x|>c_5$, $c_4<c_5$where c_4 and $c_5$are such that the test is of size α

52. $X_1, X_2,\ldots,X_n$ are independently and identically distributed random variables, which follow b (1, p). To test H_0: p=$\frac{1}{2}$ vs H_1: p=$\frac{3}{4}$, with size α= 0.01, consider the test

$$\phi = \begin{cases} 1, & if \ \sum_{i=0}^{n} X_i > c_n \\ 0, & otherwise \end{cases}$$

Then, which of the following statement is true

A) As $n \to \infty$ power of the test converges to $\frac{1}{4}$
B) As $n \to \infty$ power of the test converges to $\frac{1}{2}$
C) As $n \to \infty$ power of the test converges to $\frac{3}{4}$
D) As $n \to \infty$ power of the test converges to 1

53. The value of statistic χ^2 is zero if and only if:

A) $O_i = E_i$ B) $\sum_i O_i = \sum_i E_i$

C) Ei is large D) all the above

54. In a clinical trial n randomly chosen persons were enrolled to examine whether two different skin creams, A and B, have different effects on the human body. Cream A was applied to one of the randomly chosen arms of each person, cream B to the other arm. Which statistical test is to be used to examine difference? Assume that the response measured is a continuous variable.

A) Two-sample t-test if normality can be assumed.
B) Paired t-test if normality can be assumed.
C) Two-sample Kolmogorov-Smirnov test.
D) Test for randomness.

55. Which of the following is true in statistical testing hypothesis problem?
A) P[type I error]+P[type II error]=1
B) level of significance of a test decreases as sample size increases
C) level of significance of a test increases as sample size increases
D) size of a test is always less than or equal to level of significance

56. Suppose we subdivide the population into at least two subgroups and then draw a random sample from each of the groups. This type of sampling scheme is called

A) Two stage sampling B) Cluster sampling
C) Stratified sampling D) Multistage sampling

57. In SRSWOR of n units from a population of N units which are numbered, the probability that the (N-1)thand Nth population units are included in the sample is

A) $\frac{N(N-1)}{N^2}$ B) $\frac{n(n-1)}{N(N-1)}$ C) $\frac{1}{\binom{n}{2}}$ D) $\frac{1}{\binom{N}{2}}$

58. A population of 60 units is split into 3 strata of equal sizes. The within stratum variances of the variable of interest Y are σ^2, $4\sigma^2$, $9\sigma^2$ for stratum 1, 2 and 3 respectively. A stratified sample of 18 units is to be drawn, the optimal allocation of the sample from strat 1,2,3 are respectively.

A) 1, 4, 9 B) 3, 6, 9 C) 3, 7, 9 D) 2, 5, 11

59. With usual notation finite population correction is:

A) $\frac{N-1}{n}$ B) $\frac{N-n}{n}$ C) $\frac{N-n}{N}$ D) 1- $\frac{1}{n}$

60. Ratio estimator of population mean is unbiased if the sampling is done according to

A) SRSWR B) PPSWR
C) PPSWOR D) Systematic Sampling

61. Variance of the regression estimate is smaller than that of the mean per unit if

A) Correlation between variables is 0
B) Correlation between variables is not equal to 0
C) Correlation between variables is equal to ratio of coefficient of variation
D) Correlation between variables is greater than ratio of coefficient of variation

62. Consider the following statements on PPS sampling

1. If the sample size n is drawn with probability proportional to sizes X_i and with replacement, then the probability of selecting the ith unit, i =1, 2, -----N at any draw is $\frac{X_i}{NX}$

2. An unbiased estimator of the population total under PPSWR is $\hat{Y} = \frac{X}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}$

Now state which of the following is correct?

A) 1 is true and 2 is false B) 2 is true and 1 is false
C) Both 1and 2 are true D) Both 1 and 2 are false

63. In a connected block design with v treatments and b blocks, the rank of the C matrix is:

A) v-1 B) < v C) 1 D) b-1

64. In the 2^4 factorial design, which of the factors or interactions are confounded with the following blocks.

Block 1: 0000 1001 1010 1100 0011 0101 0110 1111

Block 2: 1000 0001 0010 0100 1011 1101 1110 0111

A) AB B) AC C) ABC D) ABCD

65. A BIBD Design with parameters (v,b,k,r,λ) is:

A) Connected, balanced and orthogonal
B) Connected, not balanced and orthogonal
C) Connected, balanced and non-orthogonal
D) Not connected, balanced and non-orthogonal

66. In a RBD of 4 treatments and 3 blocks the degrees of freedom of the residual sum of squares (Error sum of squares) is

A) 3 B) 4 C) 5 D) 6

67. If the number of levels of each factor in an experiment is different then the experiment is called

A) Factorial B) Assymetrical
C) incomplete D) None of these

68. Partial confounding is defined as

A) The same set of treatments are confounded in all replications
B) The different set of treatments are confounded in different replications
C) Some treatments are confounded and some are not
D) None of the above.

69. If $X \sim N(\mathbf{0},\Sigma)$, then the distribution of $X'\Sigma^{-1}X$ is

A) $N_p(\mathbf{0},\Sigma)$
B) Chi-square with 1 degree of freedom
C) Chi-square with p degrees of freedom
D) Chi-square with p-1 degrees of freedom

70. Wishart distribution is a multivariate analogue of

A) Exponential distribution B) Chi-square distribution
C) t-distribution D) F-distribution

71. If $\mathrm{X} \sim N_p(\mathbf{0},\Sigma))$, then $\mathrm{X}'AX$ and $l'X$ are independent if and only if

A) $A\Sigma l \neq 0$ B) $A\Sigma^{-1}l = 0$ C) $A\Sigma^{-1}l \neq 0$ D) $A\Sigma l = 0$

72. If $X \sim N(\boldsymbol{\mu}, \Sigma)$, then $E(X'AX)$ is:

A) $tr(A\Sigma + \boldsymbol{\mu}' A\boldsymbol{\mu})$ B) $tr(A + \boldsymbol{\mu}'\boldsymbol{\mu})$

C) $tr(A + \boldsymbol{\mu}' A\boldsymbol{\mu})$ D) $tr(A\Sigma + \boldsymbol{\mu}'\boldsymbol{\mu})$

73. If $R_{1.23}=0$, then the partial correlations involving X_1 are always

A) $> \frac{1}{2}$ B) $< \frac{1}{2}$ C) zero D) unity

74. Which of the following statements are true?

1. Hotelling's T^2 statistic is not invariant under linear transformation.
2. The vector random variable $\underline{X}$ follows multivariate normal distribution if and only if every components of $\underline{X}$ are univariate normal

A) 1 only B) 2 only
C) both 1 and 2 D) neither 1 nor 2

75. Consider a random walk over {0,1,…,m} with absorbing barriers and let $P=((p_{ij}))$ be transition probability matrix. Then which of the following is true?

A) $p_{00} > p_{mm}$ B) $p_{00} < p_{mm}$ C) $p_{00} \neq p_{mm}$ D) None of these

76. If all the states of a Markov chain communicate with each other, then the chain is
A) irreducible B) not necessarily irreducible
C) reducible D) not necessarily reducible

77. Let $\{Z_j\}$ be a sequence of independent and identically distributed random variables with mean zero and let $X_n = \sum_{j=1}^{n} Z_j$ .Then $\{X_n\}$ is a

A) Markov process B) Weiner process
C) Stationary process D) Martingale process

78. Let $\{X(t), t \geq 0\}$ be a process with stationary independent increments, with X(0)=0 and $Var(X(1)) = \sigma^2 < \infty$. Then, for any t > s, Var(X(t)-X(s)) is

A) σ^2 B) $\sigma^2 t$ C) $\sigma^2(t-s)$ D) $\sigma^2 \min\{t, s\}$

79. Let T_n be the time of the n^{th} event of a Poisson process $\{X(t), t \geq 0\}$ with parameter λ and let one event has occurred in the time interval (0,t). Then, for any $s \leq t$,
$P\{T_1 \leq s | X(t) = 1\}$ is

A) $\frac{s}{t}$ B) st C) $\frac{st}{s+1}$ D) $\frac{st}{t+1}$

80. Fisher's ideal formula for index numbers does not satisfy
A) Time reversal test B) Factor reversal test
C) Unit test D) Circular test