A

23131 **120 MINUTES**

1. Which of the following sets are bounded?

   1. $\left\{\frac{(-1)^n}{n}, n \in N\right\}$ 2. $\left\{\frac{-(n+1)}{n}, n \in N\right\}$ 3. $\left\{\frac{1}{n}, n \in N\right\}$

   A) 1& 2 only B) 1 & 3 only C) 2 & 3 only D) 1, 2 & 3

2. Which of the following function is not differentiable at $x = 0$?

   A) $f(x) = x|x|, x \in R$

   B) $f(x) = |x| + |x-1|, x \in R$

   C) $f(x) = \frac{sinx}{x}$, if $x \neq 0$ and $f(0) = 1$

   D) $f(x) = x^2 sin\left(\frac{1}{x}\right)$, if $x \neq 0$ and $f(0) = 0$

3. $\lim_{x \to 0} \frac{3^x - 2^x}{x^2 - x} =$

   A) 0 B) $\frac{log3}{log2}$ C) $log2 - log3$ D) $\frac{log2}{log3}$

4. Which of the following statements are true?

   1. A constant function defined on the interval $[a, b]$ is Riemann integrable on $[a, b]$
   2. If $f: [a, b] \to R$ is continuous on $[a, b]$, then f is Riemann integrable on $[a, b]$
   3. The function $f(x) = [x]$, the greatest integer function is Riemann integrable on [0,3]
   4. If $|f|$ is Riemann integrable on $[a, b]$, then f is also Riemann integrable on $[a, b]$

   A) 1 & 2 only B) 3 & 4 only
   C) 1, 2 & 3 only D) 1, 2 & 4 only

5. For a metric space (X, d), which of the following statements are true?

   1. The union of an arbitrary family of open sets is open.
   2. The intersection of an arbitrary family of open sets is open.
   3. The union of an arbitrary family of closed sets is closed.
   4. The intersection of an arbitrary family of closed sets are closed.

   A) 1 & 3 only B) 2 & 3 only C) 1 & 4 only D) 3 & 4 only

6. The rank of the matrix $A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$ is:

   A) 0 B) 1 C) 2 D) 3

7. Which of the following statement(s) is/are true?
1. Products of triangular matrices are again triangular.
2. Triangular matrices whose diagonal entries are all zero are idempotent.

A) 1 only B) 2 only
C) Both 1 & 2 D) neither 1nor 2

8. Let A be an $n \times n$ non-singular matrix. Then the homogeneous system of equations $A\boldsymbol{x} = \boldsymbol{0}$ has:

A) a unique solution, which is $\boldsymbol{x} = \boldsymbol{0}$.
B) infinite number of solutions.
C) more than one, but finite number of solutions.
D) no solution

9. Let A_1 be an $m \times p$ matrix with p linearly independent columns and A_2 be any $n \times p$ matrix. Consider the matrix $A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$. Then

A) A has p linearly independent columns
B) A has less than p number of independent columns
C) A has exactly $p - 1$ linearly independent columns
D) A has linearly dependent columns

10. If r_1 and r_2 are respectively the ranks of the matrices $(A - I)$ and $(B - I)$, where A and B are square matrices of same order and I is the identity matrix, then

A) rank of $(AB - I)$ is equal to $r_1 + r_2$
B) rank of $(AB - I)$ is less than or equal to $r_1 + r_2$
C) rank of $(AB - I)$ is greater than or equal to $r_1 + r_2$
D) rank of $(AB - I)$ is equal to $r_1 r_2$

11. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two nonzero vectors in the Euclidean space R^m. If $(\boldsymbol{x}, \boldsymbol{y})$ is the inner product of $\boldsymbol{x}$ and $\boldsymbol{y}$ and $||\boldsymbol{x}||$ and $||\boldsymbol{y}||$ are the norms of $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, then the angle between $\boldsymbol{x}$ and $\boldsymbol{y}$ is such that:

A) $tan\theta = \frac{(\boldsymbol{x},\boldsymbol{y})}{||\boldsymbol{x}||.||\boldsymbol{y}||}$ B) $cos\theta = \frac{(\boldsymbol{x},\boldsymbol{y})}{||\boldsymbol{x}||.||\boldsymbol{y}||}$

C) $tan\theta = \frac{(\boldsymbol{x},\boldsymbol{y})}{||\boldsymbol{x}||+||\boldsymbol{y}||}$ D) $cos\theta = \frac{(\boldsymbol{x},\boldsymbol{y})}{||\boldsymbol{x}||+||\boldsymbol{y}||}$

12. Let A be an $m \times n$ matrix of real numbers and A^+ be it's Moore-Penrose g-inverse. Then which of the following is **not** true?

A) A^+A is symmetric and idempotent
B) A^+A is symmetric but not idempotent
C) $I - A^+A$ is symmetric and idempotent
D) A^+A is the orthogonal projector onto the row space of A

13. Let A be an $n \times n$ matrix and λ be its eigen value. Then the eigen vectors corresponding to λ are:

A) nonzero vectors in the null space of A
B) nonzero vectors in the column space of A
C) nonzero vectors in the null space of $A - \lambda I$
D) nonzero vectors in the column space of $A - \lambda I$

14. Which of the following statements are true?
1. Every bounded open set is measurable
2. Every bounded closed set is measurable
3. Union of two measurable sets is also measurable

A) 1 & 2 only B) 1& 3 only C) 2 & 3 only D) 1, 2 & 3

15. An urn contains ten balls of which three are black and seven are white. The following game is played: At each trial a ball is selected at random, its colour is noted, and it is replaced along with two additional balls of the same colour. Then the probability that a black ball is selected in each of the first three trial is equal to:

A) $\frac{1}{12}$ B) $\frac{1}{16}$ C) $\frac{3}{4}$ D) $\frac{5}{8}$

16. If A and B are two events in a probability space $(\Omega, \mathcal{A}, P)$, then which of the following statement is **wrong**?

A) If $P(A|B) \geq P(A)$, then $P(B|A) \geq P(B)$.
B) If $P(B|\overline{A}) = P(B|A)$, then A and B are independent.
C) If $P(\overline{A}) = a$, $P(\overline{B}) = b$, then $P(A \cap B) \leq 1 - a - b$.
D) If $P(A) = p$ and $P(B) = q$, then $P(A|B) \geq \frac{p+q-1}{q}$.

17. An elevator starts with 5 passengers and stops at 6 floors. The probability that no two passengers alight at the same floor is equal to

A) $\frac{1}{27}$ B) $\frac{5}{54}$ C) $\frac{5}{6}$ D) $\frac{7}{29}$

18. Let X be continuous random variable with distribution function $F(x)$. If X is symmetric about 0, then which of the following statement is **not** true?

A) $F(0) = 0.5$
B) For $a > 0$, $F(-a) + F(a) = 1$
C) For $a > 0$, $P(|X| > a) = 2F(a)$
D) For $a > 0$, $P(|X| \leq a) = 2F(a) - 1$

19. Let X be a continuous random variable with PDF $f(x) = \frac{e^{-x}x^{\theta}}{\theta!}, x > 0$, where θ is a nonnegative integer. The lower bound of $P[0 < X < 2(\theta + 1)]$ is:

A) $\frac{1}{\theta+1}$ B) $\frac{\theta}{\theta+1}$ C) $\frac{1}{\theta}$ D) $\frac{1}{\theta^2}$

20. For any random variable X with $E|X| < \infty$,

A) $E(X) = \int_0^{\infty} P(X > x)dx$

B) $E(X) = 2\int_0^{\infty} P(X > x)dx$

C) $E(X) = \int_0^{\infty} P(X \leq x)dx + \int_{-\infty}^{0} P(X > x)dx$

D) $E(X) = \int_0^{\infty} P(X > x)dx + \int_{-\infty}^{0} P(X \leq x)dx$

21. Let $\{A_n, n \in N\}$ be a sequence of events, then which of the following is true?

A) $(lim\ supA_n)^c = lim\ supA_n^c$

B) $(lim\ supA_n)^c = lim\ infA_n^c$

C) $lim\ supA_n \subset lim\ infA_n$

D) None of the above

22. Let $\{X_n, n \in N\}$ be a sequence of random variables with $P(X_n = 0) = 1 - \frac{1}{n}$ and $P\left(X_n = n^{\frac{2}{r}}\right) = \frac{1}{n}$, where r is a fixed positive integer. Also, let X be a random variable with $P(X = 0) = 1$. Which of the following statement(s) is/are true?

1. X_n converges in distribution to X
2. X_n converges in probability to X
3. X_n converges in the r^{th} mean to X

A) 1 only B) 1 & 2 only C) 1, 2 & 3 D) None of these

23. Let X be a continuous random variable with a probability distribution specified by

$$P(X > x) = \begin{cases} 1, & x < 0 \\ \left(1 + \frac{x}{\theta}\right)^{-\theta}, & x \geq 0 \end{cases}$$

where $\theta > 0$ is a constant. Then the probability density function of X is given by:

A) $f(x) = \begin{cases} \frac{\theta}{(1+\frac{x}{\theta})^{\theta}}, & x > 0 \\ 0, & otherwise \end{cases}$

B) $f(x) = \begin{cases} \frac{\theta}{(1+\frac{x}{\theta})^{\theta+1}}, & x > 0 \\ 0, & otherwise \end{cases}$

C) $f(x) = \begin{cases} \frac{1}{(1+\frac{x}{\theta})^{\theta+1}}, & x > 0 \\ 0, & otherwise \end{cases}$

D) $f(x) = \begin{cases} \frac{\theta+1}{(1+\frac{x}{\theta})^{\theta}}, & x > 0 \\ 0, & otherwise \end{cases}$

24. For which of the following probability density functions the random variable and its reciprocal are identically distributed:

1. $f(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{2}, & 0 < x \leq 1 \\ \frac{1}{2x^2}, & 1 < x < \infty \end{cases}$,

2. $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, -\infty < x < \infty$

3. $f(x) = \begin{cases} \frac{1}{4}, & |x| \leq 1 \\ \frac{1}{4x^2}, & |x| > 1 \end{cases}$

A) 2 only B) 1 & 2 only C) 1 & 3 only D) 1, 2 & 3

25. For the random variable with probability density function

$$f(x) = \frac{\mu}{\pi} \frac{1}{\mu^2 + (x-\theta)^2}, -\infty < x < \infty, \mu > 0, -\infty < \theta < \infty$$

A) median = 0, mode = 0 B) median = θ, mode = θ
C) median = μ, mode = μ D) median = θ, mode = μ

26. Let X be a random variable with probability density function $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$.
Let $A = \left\{x: \frac{1}{3} < x \le \frac{3}{4}\right\}$. Then the mean of the truncated random variable $(X|X \in A)$ is:
A) $\frac{1}{2}$ B) $\frac{5}{12}$ C) $\frac{13}{24}$ D) $\frac{14}{27}$

27. Let X and Y be independent random variables with probability mass functions $P(X = \pm 1) = 0.5$ and $P(Y = \pm 1) = 0.5$. Define $Z = XY$. Then which of the following is true?
A) X, Y, Z are independent
B) X, Y, Z are independent and identically distributed
C) X, Y, Z are pairwise independent but not mutually independent
D) X, Y, Z are not pairwise independent

28. If the joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x \ge 0,\ y \ge 0 \\ 0, & otherwise \end{cases}.$$

Then $P(X + Y \le 1) =$
A) $\frac{1}{e}$ B) $1 - \frac{1}{e}$ C) $\frac{1}{e^2}$ D) $1 - \frac{2}{e}$

29. Let X be a continuous random variable with probability density function

$f(x) = \begin{cases} \frac{(r+1)a^{r+1}}{(x+a)^{r+2}}; & x > 0, a > 0 \\ 0, & otherwise \end{cases}$. Then $E(X^{r+1})$

A) is equal to a^{r+1} B) is equal to $(r+1)a^{r+1}$
C) is equal to $(r+2)a^{r+1}$ D) does not exist

30. For a continuous random variable X, which of the following statement is **not** true?
A) $E(X^2) \ge [E(X)]^2$, provided $E(X^2)$ exists
B) If $X > 0$, then $E\left(\frac{1}{X}\right) \ge \frac{1}{E(X)}$, provided the expectations exist
C) If $X > 0$, then $E(X^{1/2}) \ge [E(X)]^{1/2}$, provided the expectations exist
D) If $X > 0$, then $E[log(X)] \le log[E(X)]$, provided the expectations exist

31. Suppose that X has uniform distribution on the interval $[0,1]$ and Y is a random variable defined on $(0, X]$ according to the probability density function $h(y|x) = \frac{1}{x}, 0 < y \le x$, and zero elsewhere. Then variance of Y is equal to:

A) $\frac{7}{144}$ B) $\frac{5}{36}$ C) $\frac{9}{400}$ D) $\frac{3}{4}$

32. The relationship between the fourth central moment μ_4 and the fourth cumulant κ_4 of a random variable is:

A) $\mu_4 = \kappa_4$

B) $\mu_4 = \kappa_4 + 2\kappa_2^3$, where κ_2 is the second cumulant

C) $\mu_4 = \kappa_4 + 3\kappa_2^2$, where κ_2 is the second cumulant

D) $\mu_4 = \kappa_4 + \kappa_2^2$, where κ_2 is the second cumulant

33. If X has F distribution with $(10,5)$ degrees of freedom, then the mean of $\frac{1}{X}$ is:

A) $\frac{5}{3}$ B) $\frac{5}{4}$ C) $\frac{10}{3}$ D) $\frac{3}{2}$

34. Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution over (0,1). Let $Y = (\prod_{i=1}^{n} X_i)^{\frac{1}{n}}$. Then the moment generating function of $logY$ is:

A) $M(t) = \left(1 + \frac{t}{n}\right)^{-n}$ B) $M(t) = \left(1 - \frac{t}{n}\right)^{-n}, t < n$

C) $M(t) = \left(1 + \frac{t}{n}\right)^{-n^2}$ D) $M(t) = \left(1 - \frac{t}{n}\right)^{-n^2}, t < n$

35. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ and $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ be the sample mean and sample variance. Let X_{n+1} follows $N(\mu, \sigma^2)$, and $X_1, X_2, \ldots, X_n, X_{n+1}$ are independent. Then the sampling distribution of $\left[\frac{X_{n+1}-\overline{X}}{S}\right]\sqrt{\frac{n}{n+1}}$ is:

A) $N\left(\mu, \left(1 + \frac{1}{n}\right)\sigma^2\right)$

B) $t -$ distribution with 1 degree of freedom

C) $t -$ distribution with $n - 2$ degree of freedom

D) $t -$ distribution with $n - 1$ degree of freedom

36. Let $X_1, X_2, \ldots, X_n$ be a random sample from discrete uniform distribution over $\{1,2, \ldots, N\}$. Let $M_n = min(X_1, X_2, \ldots, X_n)$. Then the probability mass function of M_n is given by:

A) $P(M_n = k) = \left(\frac{k}{N}\right)^n - \left(\frac{k-1}{N}\right)^n, k = 1,2, \ldots, N$

B) $P(M_n = k) = 1 - \left(\frac{N-k}{N}\right)^n, k = 1,2, \ldots, N$

C) $P(M_n = k) = \left(\frac{N-k}{N}\right)^n - \left(\frac{N-k-1}{N}\right)^n, k = 1,2, \ldots, N$

D) $P(M_n = k) = \left(\frac{N-k+1}{N}\right)^n - \left(\frac{N-k}{N}\right)^n, k = 1,2, \ldots, N$

37. A random sample of five observations is taken from a continuous distribution with probability density function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & otherwise \end{cases}$. Then the probability that three of these observations will be smaller than 0.5 is:

A) $\frac{5}{8}$ B) $\frac{5}{16}$ C) $\frac{2}{3}$ D) $\frac{3}{5}$

38. Consider a sequence of independent Bernoulli's trials with a constant probability of success p. Then the minimum value of n such that we want to be $(1-\alpha)100\%, 0 < \alpha < 1$ sure of achieving the first success in n trials is:

A) $\frac{log p}{log \alpha}$ B) $\frac{log(1-p)}{log(1-\alpha)}$ C) $\frac{log \alpha}{log(1-p)}$ D) $\frac{log(1-\alpha)}{log p}$

39. If X has Poisson distribution with parameter θ, then which of the following statement(s) is/are true?

1. For $n \geq 1, P(X = n) = \frac{\theta}{n} P(X = n-1)$
2. $E(X) < e^{\theta}$

A) 1 only B) 2 only
C) Both 1 & 2 D) Neither 1 nor 2

40. If (X, Y) has bivariate normal distribution $BN\left(\mu_1 = 0, \mu_2 = -1, \sigma_1^2 = 1, \sigma_2^2 = 4, \rho = \frac{-1}{2}\right)$, then the value of k such that $(kX + Y)$ and $(X + 2Y)$ are independent is:

A) 4 B) 7 C) -4 D) -7

41. An electronic device has a life length X (in 1000 hours) that can be assumed to have an exponential distribution with mean 1. The cost of manufacturing an item is Rs. 200 and the selling price is Rs. 500 per item. However, a full refund is guaranteed if $X < 0.8$. Then the manufacture's expected profit per item (if $e^{-0.8} = 0.4493$) is approximately

A) Rs. 25 B) Rs. 45 C) Rs. 115 D) Rs. 125

42. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. If $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ is the sample variance, then $E(S) =$

A) $\sigma\sqrt{\frac{n-1}{n}}$ B) $\sigma\frac{\sqrt{2}\,\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$ C) $\sigma\frac{\sqrt{\frac{2}{n}}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$ D) $\sigma\frac{\sqrt{\frac{2}{n-1}}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$

43. For the random sample size 20 from uniform distribution over (0,1), the joint probability density function of the first and the 20th order statistics is given by

A) $f(x, y) = 6840x(y - x)^{17}, 0 < x < y < 1$

B) $f(x, y) = 58140x^2(y - x)^{16}, 0 < x < y < 1$

C) $f(x, y) = 310080x^3(y - x)^{15}, 0 < x < y < 1$

D) $f(x, y) = 380(y - x)^{18}, 0 < x < y < 1$

44. Let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics for a random sample from a continuous distribution with distribution function $F(x)$. Let $U_{r:n} = F(X_{r:n}), r = 1,2, \ldots, n$. Then $E(U_{r:n})$ is equal to:

A) $\frac{r}{n}$ B) $\frac{r}{n-1}$ C) $\frac{r}{n+1}$ D) $\frac{r+1}{n}$

45. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed nonnegative integer valued random variables with common probability generating function $P(.)$ Suppose N is also a nonnegative integer valued random variable that is independent of X's , and let $S_N = \sum_{j=1}^{N} X_j$. If P_N is the probability generating function of N, then the probability generating function of S_N is:

A) $Q(s) = P_N[P(s)]$ B) $Q(s) = P_N(s)P(s)$

C) $Q(s) = [P_N(s)]^2 P(s)$ D) $Q(s) = NP(s)$

46. Let $X_1, X_2, \ldots, X_n$ be iid random variables with probability density function $f(x,\theta) = \begin{cases} \frac{\theta}{x^{\theta+1}}; & x \geq 1, \theta > 0 \\ 0, & otherwise \end{cases}$. Consider the statistics $T = \Pi_{i=1}^{n} X_i$ and $S = \Sigma_{i=1}^{n} log X_i$. Then:

A) T is sufficient for θ but S is not sufficient
B) T is not sufficient for θ but S is sufficient
C) T and S are sufficient for θ
D) T and S are not sufficient for θ

47. Suppose $X_1, X_2, \ldots, X_n$ is random sample from $N(\theta, 1)$. Then which of the following is **not** an ancillary statistic for θ?

A) $T = X_1 - X_2$ B) $T = X_1 + X_2 + \ldots + X_{n-1} - (n-1)X_n$

C) $T = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$ D) None of the above

48. Suppose the probability of success p in a sequence of n independent Bernoulli trials has a uniform prior distribution over $[0,1]$. If X is the number of successes in n trials, then Baye's estimator of $\theta = p^2$ under the quadratic loss function is given by:

A) $\hat{\theta} = \frac{X(X+1)}{(n+1)(n+2)}$ B) $\hat{\theta} = \frac{(X+1)(X+2)}{(n+1)(n+2)}$

C) $\hat{\theta} = \frac{X(X+2)}{n(n+2)}$ D) $\hat{\theta} = \frac{(X+2)(X+3)}{(n+1)(n+2)}$

49. If $\overline{X}$ is the sample mean of a random sample taken from a Poisson random variate X, then a consistent estimator of $P(X = 0)$ is:

A) $\overline{X}$ B) $e^{\overline{X}}$ C) $e^{-\overline{X}}$ D) $\overline{X}e^{-\overline{X}}$

50. Let $X_1, X_2, \ldots, X_n$ be iid Bernoulli random variables with probability of success $p, 0 < p < 1$. If $T_1 = X_1 X_2$, $T_2 = \sum_{i=1}^{n} X_i$ and $T = E(T_1|T_2)$, then which of the following statement(s) is/are true?
1. T is an unbiased estimator of p^2
2. $V(T_1) < V(T)$

A) 1 only
B) 2 only
C) Both 1 & 2
D) Neither 1 nor 2

51. Let $\{f_\theta\}$, $\theta \in \{\theta_0, \theta_1\}$ be a family of probability density functions. If α and β are the size and power of the most powerful test obtained using Neyman-Pearson Lemma for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ and $f_{\theta_0} \neq f_{\theta_1}$, then:

A) $\alpha < \beta$
B) $\alpha \leq \beta$
C) $\alpha > \beta$
D) $\alpha \geq \beta$

52. Let X be the lifetime of a light bulb manufactured by a certain company. Suppose X has probability density function $f(x) = \left(\frac{x}{\beta^2}\right) exp\left(-\frac{x}{\beta}\right), x > 0$ and zero elsewhere. Then the uniformly most powerful critical region for testing $H_0: \beta \leq \beta_0$ against $H_1: \beta > \beta_0$ based on the sample $X_1, X_2, \ldots, X_n$ is of the form:

A) $\sum_{i=1}^{n} X_i \geq k$
B) $\prod_{i=1}^{n} X_i \geq k$
C) $min(X_1, X_2, \ldots, X_n) \geq k$
D) $\sum_{i=1}^{n} X_i^2 > k$

53. A random sample of 15 infants of one month or older shows the following pulse rates (beats per minutes): 119, 120, 125, 122, 118, 117, 126, 114, 115, 123, 121, 120, 124, 127, 126. Assuming that the distribution of pulse rate is symmetric and continuous, to test the median pulse rate is 120 beats per minute, the values of the Wilcoxon's signed rank test statistics are:

A) $(W_-, W_+) = (30,61)$
B) $(W_-, W_+) = (30,90)$
C) $(W_-, W_+) = (32,59)$
D) $(W_-, W_+) = (32,88)$

54. The run test for randomness rejects the null hypothesis that an ordered sequence of two types of symbols is a random arrangement:
A) only when the number of runs is too small
B) only when the number of runs is too large
C) when the number runs is either too large or too small
D) when the number of runs is neither too large nor too small

55. Consider the problem of finding the confidence interval for θ in the case of uniform distribution over $(0, \theta)$. The following 10 observations have been taken from the population: 1.99, 1.15, 0.83, 0.16, 0.21, 1.71, 1.43, 0.31, 1.31, 0.76. Then the lower limit of the 95% shortest-length confidence interval based on the sufficient statistic of θ is:

A) 0.16
B) 1.89
C) 1.92
D) 1.99

56. Let Y_1, Y_2, Y_3 be uncorrelated observations with common variance σ^2 and $E(Y_1) = \theta_1$, $E(Y_2) = \theta_2$ and $E(Y_3) = \theta_1 + \theta_2$, where θ_1 and θ_2 are unknown parameters. The best linear unbiased estimator of $\theta_1 + \theta_2$ is:

A) Y_3

B) $Y_1 + Y_2$

C) $\frac{1}{3}(Y_1 + Y_2 + 2Y_3)$

D) $\frac{1}{2}(Y_1 + Y_2 + Y_3)$

57. Let the linear statistical model for a randomised block design be $y_{ij} = \mu + \tau_i + \beta_j + e_{ij}$, $i = 1,2,\ldots,a$; $j = 1,2,\ldots,b$, where μ , τ_i , β_j and e_{ij} represent the overall mean, effect of i-th treatment, effect of j-th block and error term respectively. Assuming the effects are fixed, the least square estimate of τ_i is:

A) $\hat{\tau_i} = \frac{1}{b}\sum_{j=1}^{b} y_{ij}$

B) $\hat{\tau_i} = \frac{1}{a}\sum_{i=1}^{a} y_{ij}$

C) $\hat{\tau_i} = \frac{1}{b}\sum_{j=1}^{b} y_{ij} - \frac{1}{ab}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}$

D) $\hat{\tau_i} = \frac{1}{a}\sum_{i=1}^{a} y_{ij} - \frac{1}{ab}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}$

58. Consider a 2^3 factorial experiment with factors A, B and C performed in single replicate. The following is the table for obtaining the estimate of the effects involved in the experiment using the Yate's procedure. The values of X1, X2, X3, X4 and X5 are:

Effects	Response	Column 1	Column 2	Column 3	Estimate of effect
Mean	22	54	144	327	40.875
A	32	90	X1	5	1.25
B	35	84	30	51	12.75
AB	55	99	X2	-7	-1.75
C	44	X3	36	39	9.75
AC	40	20	15	X4	-13.75
BC	60	-4	10	-21	X5
ABC	39	-21	-17	-27	-6.75

A) (X1, X2, X3, X4, X5) = (183, -25, 10, -55, -5.25)

B) (X1, X2, X3, X4, X5) = (183, 25, -10, -55, 5.25)

C) (X1, X2, X3, X4, X5) = (183, -25, -10, 55, -2.625)

D) (X1, X2, X3, X4, X5) = (183, 25, -10, -55, 2.625)

59. A 2^5 design with factors A, B, C, D and E conducted in 4 blocks, the effects ABC and CDE are chosen to generate the blocks. The number of additional effects, which will be confounded with blocks is:

A) 0

B) 1

C) 4

D) 15

60. Which of the following is/are used for pairwise comparisons between means of k populations?
1. Tukey's test
2. Fisher least significant difference
3. Duncan's multiple range test

A) 1 only B) 1 & 2 only C) 2 & 3 only D) 1, 2 & 3

61. A Markov chain has state space {0, 1, 2} and transition probability matrix $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$.

Which of the following is/ are true?
1. All the states are recurrent
2. Each state has period 2

A)	1 only	B)	2 only
C)	Both 1 & 2	D)	Neither 1 nor 2

62. Given that only one occurrence of a Poisson process {X(t)} has occurred by epoch T, the distribution of the time interval τ, in [0, T) in which it occurred is:

A)	Uniform in [0, T]	B)	Exponential with mean T
C)	Exponential with mean 1/T	D)	Gamma with mean T

63. A supermarket has two sales girls at the sales counters. The service time for each customer is exponential with a mean of 4 minutes and people arrive according to Poisson distribution at the rate of 10 in an hour. Then the expected percentage of idle time for each sales girl is:
A) 33.33% B) 66.67% C) 16.67% D) None of these

64. Assume that a device fails when the cumulative effect of 3 shocks occur. If the shocks happen according to a Poisson process with parameter θ, then the pdf for the life of the device is:
A) Gamma with parameters (4, θ)
B) Gamma with parameters $\left(3, \frac{1}{\theta}\right)$
C) Exponential with mean θ
D) Exponential with mean $\frac{1}{\theta}$

65. Which of the following statements is/ are true?
1. A stationary process that has finite second moments is covariance stationary
2. There are covariance stationary processes that are not stationary

A)	1 only	B)	2 only
C)	Both 1 & 2	D)	Neither 1 nor 2

66. Which of the following is an example for a pure birth process?

A)	Poisson process	B)	Brownian motion process
C)	Queueing process	D)	Yule process

67. Let $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ has a trivariate normal distribution with mean vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and covariance matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$. Then the joint distribution of $X_1 + X_2 + X_3$ and $X_1 - X_3$ is:

A) $N_2\left(\begin{bmatrix} 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 10 & 0 \\ 0 & 3 \end{bmatrix}\right)$ B) $N_2\left(\begin{bmatrix} 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix}\right)$

C) $N_3\left(\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\right)$ D) None of the above

68. Let $X \sim N_7(\mu, I)$ and P be an idempotent matrix which is non negative symmetric having rank 5. Then the distribution of $(X-\mu)'P(X-\mu)$ is:

A) Chi square with 7 d.f. B) Chi square with 5 d.f.
C) Chi square with 2 d.f. D) Standard normal

69. Let X_1, X_2, X_3, X_4, X_5 be a random sample from $N_3(\mu, \Sigma)$ with $\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$ and $S = \frac{1}{4}\sum_{i=1}^{5} (X_i - \bar{X})(X_i - \bar{X})'$. Then the distribution of $5(\bar{X}-\mu)'S^{-1}(\bar{X}-\mu)$ is:

A) $F_{2,3}$ B) $F_{3,2}$ C) $\frac{1}{6}F_{3,2}$ D) $6F_{3,2}$

70. If the covariance matrix Σ of multivariate normal distribution is not of full rank, then:
A) Multivariate normal distribution is degenerate
B) Density of multivariate Normal Distribution does not exist
C) Both A and B
D) None of the above

71. The secular trend is measured by the method of semi averages when:
A) Trend is curvilinear
B) Trend is linear
C) Time series consists of even number of values
D) Time series values are given on yearly basis

72. Which of the following method is commonly used to measure seasonal variation when the time series data do not contain any trend and cyclic components?
A) Method of simple averages
B) Ratio to trend method
C) Ratio to moving average method
D) Link relative method

73. Aggregate expenditure method of constructing cost of living index numbers uses
A) Laspeyres formula B) Fishers formula
C) Paasche's formula D) Marshall Edgeworth formula

74. Which of the following index number satisfy homogeneous test

A) Fishers index number B) Simple Index number

C) Chain index number D) All the above

75. The number of possible samples of size 3 out of 10 population size in SRSWR is equal to:

A) 120 B) 1000 C) 3^{10} D) 30

76. A sample of size 2 is drawn from a population of size 5 using probability proportional to size without replacement scheme. If the first selection is made with probabilities 0.4, 0.15, 0.25, 0.1, 0.1 associated with 5 units in the population and the second selection is made with equal probabilities from the remaining units, then what will be the probability that first unit is included in the sample?

A) 0.4 B) 0.55 C) 1 D) None of these

77. A sample of size 4 is drawn without replacement from a finite population of size N(> 4), using an arbitrary sampling scheme. Let π_i denote the inclusion probability of the i^{th} unit, $1 \leq i \leq N$. Which of the following statements is always true?

A) $\sum_{i=1}^{4} \pi_i = 1$ B) $\sum_{i=1}^{N} \pi_i = 1$ C) $\sum_{i=1}^{4} \pi_i = 4$ D) $\sum_{i=1}^{N} \pi_i = 4$

78. If a simple random sample of size n is drawn without replacement from a population of N units with mean $\bar{Y}$ and variance σ^2, then covariance between any two members of the sample is:

A) $-\frac{\sigma^2}{N}$ B) $-\frac{\sigma^2}{n}$ C) $-\frac{\sigma^2}{N-1}$ D) $-\frac{\sigma^2}{N(N-1)}$

79. If the correlation coefficient between pairs of units that are in the same systematic sample is 0, then:

A) Systematic sampling is better than stratified sampling

B) Stratified and systematic sampling are equally good

C) Stratified sampling is better than systematic sampling

D) None of the above

80. Let a population be divided into 3 strata having sizes 210, 91, 203. For a sample of size 72, determine the sample sizes in each stratum under proportional allocation:

A) 24, 24, 24 B) 30, 15, 27

C) 30, 13, 29 D) Data insufficient to determine sample sizes