MAT12

SUBJECT : **MATHEMATICS**

5242

Candidate's Roll No.

Time Allowed : 3 Hours	*Maximum Marks : 150*

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

1 There are eighteen (18) questions in all.

2 Candidate has to attempt any fifteen (15) questions in all.

3 Marks assigned to each question/part are given against it.

4 Word limit in questions, wherever specified should be adhered to.

5 Attempts of questions shall be counted sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the answer booklet must be clearly struck off.

6 No extra/additional sheet will be provided.

7 Answer must be written in the authorized medium. No marks will be given for answers written in a medium other than the authorized one.

1 Prove or disprove that if $a+b=c+d$ and $a^2+b^2=c^2+d^2$, then $a^n+b^n=c^n+d^n$ for $n \in \mathbb{N}$. **10**

2 If z_1 and z_2 both satisfy the relation $z+\overline{z}=2|z-1|$ and $\arg(z_1-z_2)=\frac{\pi}{4}$, then find the imaginary part of z_1+z_2. **10**

3 (i) A box contains 7 blue socks and 5 red socks. Find the number n of ways two socks can be drawn from the box if **$2\frac{1}{2}+2\frac{1}{2}=5$**

(a) they can be of any color

(b) they must be of the same color.

(ii) Consider the function $f:\mathbb{N}\times\mathbb{N}$ such that $f(x,y)=(2x+1)2^y-1$, where $\mathbb{N}$ is set of natural numbers including zero. Check whether function is bijective or not. **5**

4 (i) Find the coefficient of x^4 in the expansion of $\left(1+x+x^2\right)^{10}$. **5**

(ii) If x, y, z are positive real numbers, such that $x+y+z=a$, then prove that **5**

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\geq\frac{9}{a}$$

5 Let A and B be square matrices. If $AB = BA$, then prove by the method of mathematical induction that $(AB)^n = A^nB^n$ for $n \geq 1$. **10**

6 The value of xyz is 55 or 343/55 according as the series a, x, y, z, b is in A.P. or H.P., respectively. Find the values of a and b given that they are positive integers. **10**

7 Is the following system consistent? If consistent, check whether the solution is unique or infinite. **10**

$2y+z=3,\ 3x+y+4z=5,\ 2x+4y+6z=9$

8 Find the circle whose diameter is the common chord of the circles **10**

$x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$.

9 Find the value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b and c, **10**

respectively, are the p^{th}, q^{th}, and r^{th} terms of a harmonic progression.

10 Solve the differential equation; $\frac{dy}{dx}-\frac{3}{x}y=x^4 y^{\frac{1}{3}}$. **10**

11 Check whether the points $-6\vec{i}+3\vec{j}+2\vec{k}$, $-13\vec{i}+17\vec{j}-\vec{k}$, $3\vec{i}-2\vec{j}+4\vec{k}$, **10**

$5\vec{i}+7\vec{j}+3\vec{k}$ are coplanar or not.

12 The mean and variance of a Binomial variable X are 2 and 1, respectively. **10**
Find the probability that X takes values greater than 1.

13 (i) Find all the eigen values and eigen vectors of the following matrix. **5**

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Prove that the intersection of two subspaces W_1 and W_2 of a vector space $V(F)$ is also a subspace. **5**

14 Find the values of A and B, if $f(x)=\frac{\sin 2x + A\sin x + B\cos x}{x^3}$ is continuous **10**

at $x=0$.

15 If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then prove that $x^2+y^2+z^2+2xyz=1$. **10**

16 Find a matrix P such that $P^{-1}AP$ is a diagonal matrix where $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$. **10**

17 If each pair of the three equations $x^2+ax+b=0$, $x^2+cx+d=0$, and $x^2+ex+f=0$ has exactly one root in common, then show that $(a+c+e)^2=4(ac+ce+ea-b-d-f)$. **10**

18 Find the Laplace Transform of following: **5+5=10**

(i) $te^{-kt}\sin t$

(ii) t^3e^{-3t}