General Instructions :

Read the following instructions very carefully and follow them :

(i) *This Question Paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Nos. **1** to **18** are Multiple Choice Questions (MCQs) and Question Nos. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Nos. **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C** – Question Nos. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Nos. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E** – Question Nos. **36** to **38** are source based/case based/passage based/integrated units of assessment questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries 1 mark.

1. If $\frac{d}{dx} f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is **1**

   (A) $x^2 + 3 \log |x| + 1$

   (B) $x^2 + 3 \log |x|$

   (C) $2 - \frac{3}{x^2}$

   (D) $x^2 + 3 \log |x| - 4$

2. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is 1

(A) 2
(B) 1
(C) not defined
(D) 0

3. The integrating factor of the differential equation

$$(1 - y^2)\frac{dx}{dy} + yx = ay, \ (-1 < y < 1) \text{ is}$$ 1

(A) $\frac{1}{y^2 - 1}$
(B) $\frac{1}{\sqrt{y^2 - 1}}$
(C) $\frac{1}{1 - y^2}$
(D) $\frac{1}{\sqrt{1 - y^2}}$

4. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is 1

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$
(B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$
(D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

5. If in ΔABC, $\overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is 1

(A) $2\vec{a} + 3\vec{b}$
(B) $2\vec{a} - 3\vec{b}$
(C) $3\vec{b} - 2\vec{a}$
(D) $-2\vec{a} - 3\vec{b}$

6. If $|\vec{a} \times \vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = -3$, then angle between $\vec{a}$ and $\vec{b}$ is 1

(A) $\frac{2\pi}{3}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) $\frac{5\pi}{6}$

7. Equation of line passing through origin and making 30°, 60° and 90° with x, y, z axes respectively is 1

(A) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

(B) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

(C) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

(D) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

8. If A and B are two events such that P(A/B)=2 × P(B/A) and P(A) + P(B) = $\frac{2}{3}$, then P(B) is equal to 1

(A) $\frac{2}{9}$

(B) $\frac{7}{9}$

(C) $\frac{4}{9}$

(D) $\frac{5}{9}$

9. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is : 1

(A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$

(B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

(D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

10. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to 1

(A) $2\Delta^2$

(B) $4\Delta^2$

(C) 2Δ

(D) 4Δ

11. The function $f(x) = x|x|$ is 1

(A) continuous and differentiable at $x = 0$.

(B) continuous but not differentiable at $x = 0$.

(C) differentiable but not continuous at $x = 0$.

(D) neither differentiable nor continuous at $x = 0$.

12. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to 1

(A) $\frac{-y}{x}$

(B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$

(D) $-\sec^2\left(\frac{y}{x}\right)$

13. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? 1

(A) $a = 9, b = 1$

(B) $a = 5, b = 2$

(C) $a = 3, b = 5$

(D) $a = 5, b = 3$

14. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to 1

(A) 40

(B) 96

(C) 120

(D) 136

15. If A is a 2×3 matrix such that AB and AB' both are defined, then order of the matrix B is 1

(A) 2×2

(B) 2×1

(C) 3×2

(D) 3×3

16. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to 1

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

17. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a ∈ A, then the set A is **1**

(A) $\mathbb{R}$ (B) $\{0\}$

(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

18. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1 (B) -1

(C) 2 (D) 0

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **1**

Reason (R) : The sum of squares of the direction cosines of a line is 1.

20. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **1**

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

SECTION – B

This section comprises of Very Short Answer Type (VSA) questions, each of **2** marks.

21. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$. **2**

22. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

23. If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line. **2**

24. (a) If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \leq 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b. **2**

25. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius. **2**

SECTION – C

The section comprises Short Answer (SA) type questions of **3** marks each.

26. Evaluate $\displaystyle\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$ **3**

27. (a) Find the general solution of the differential equation : 3

$(xy - x^2)\, dy = y^2\, dx.$

OR

(b) Find the general solution of the differential equation : 3

$$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

28. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. 3

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first. 3

29. Solve the following linear programming problem graphically :

Maximize : $Z = x + 2y$

subject to constraints : $x + 2y \geq 100,$

$2x - y \leq 0,$

$2x + y \leq 200,$

$x \geq 0, y \geq 0.$ 3

30. (a) Evaluate $\displaystyle\int_{-1}^{1} |x^4 - x|\, dx.$ 3

OR

(b) Find $\displaystyle\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\, dx.$ 3

31. Find $\displaystyle\int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$ 3

SECTION - D

This section comprises Long Answer type (LA) questions of 5 marks each.

32. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. 5

OR

(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin. 5

33. Using Integration, find the area of triangle whose vertices are (–1, 1), (0, 5) and (3, 2). 5

34. A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. 5

35. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$

5

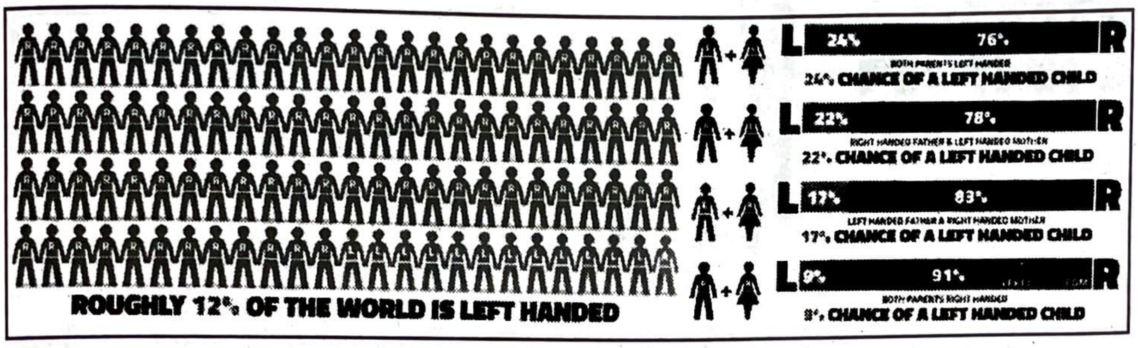
OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ 5

SECTION – E

This section comprises 3 source based case-based/passage based/integrated units of assessment questions of 4 marks each.

36. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$ **1**

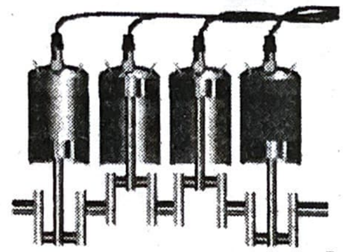
(ii) Find $P(\overline{L}/A)$ **1**

(iii) (a) Find $P(A/L)$ **2**

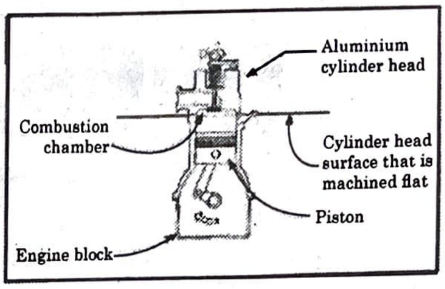
OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

37. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



One complete cycle of a four-cylinder four-stroke engine. The volume displaced is marked



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

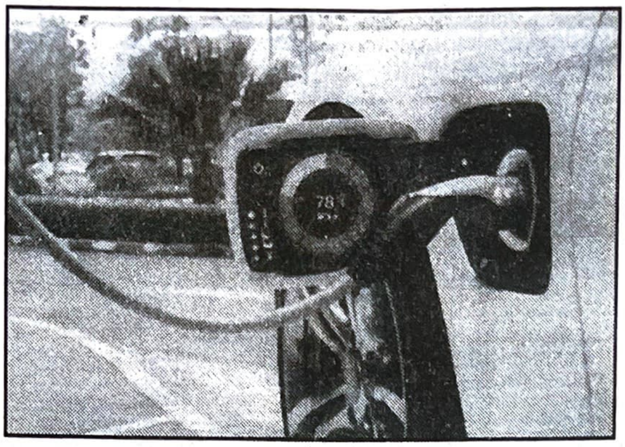
(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

38. The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**