MARKING SCHEME

Additional Practice Question Paper (2023-24)

CLASS-XII

MATHEMATICS (041)

	SECTION: A (Solution of MCQs of 1 Mark each)	
Q.No.	**Solution**	**Marks**
1.	$x+y+z=9$, $x+z=5$, $y+z=7$ On solving above equations, we get $x=2$, $y=4$, $z=3$ $\therefore x-y+z=1$ **Correct Answer is Option (c) 1**	**1**
2.	$\mathrm{A(Adj\,A)}=5\mathrm{I}$ $\mathrm{A(Adj\,A)}=\lvert A\rvert \mathrm{I}$ $\therefore\ \lvert A\rvert=5$ $\lvert Adj\ A\rvert=\lvert A\rvert^2=25$ **Correct Answer is Option (b) 25**	**1**
3.	Let $\left(AB^T-2BA^T\right)^T=C$ Consider $C^T=\left(AB^T-2BA^T\right)^T=BA^T-2AB^T\neq C\ or\ -C$ $\therefore\ C$ is neither Symmetric matrix nor Skew symmetric matrix **Correct Answer is Option (c) Neither Symmetric matrix nor Skew symmetric matrix**	**1**
4.	$If\ \ 1<x<2\ then\ f(x)=2(x-1)-3(x-2)=-x+4$ $\therefore\ f^{/}(x)=-1$ Hence $f(x)$ is Strictly decreasing function **Correct Answer is Option (b) Strictly Decreasing**	**1**
5.	Set A contains 5 elements and the set B contains 6 elements. For one-one function each element in set B is assigned to only one element in set A **Correct Answer is Option (d) 0**	**1**
6.	Order = 3, Degree = 1 **Correct Answer is Option (b) 4**	**1**
7.	**Correct Answer is Option (b) half plane not containing the origin**	**1**
8.	$\overrightarrow{AD}=\frac{3}{2}\hat{i}+\frac{5}{2}\hat{k}$ $AD=\frac{\sqrt{34}}{2}$ **Correct Answer is Option (a)** $\frac{\sqrt{34}}{2}$	**1**
9.	$f(x)=x^3\sin^4 x$ $f(-x)=(-x)^3[\sin(-x)]^4=-x^3[-\sin x]^4=-x^3\sin^4 x=-f(x)$ $\therefore f(x)\ is\ an\ odd\ \ function$ $\therefore\ \int_{-\pi/2}^{\pi/2}x^3\sin^4 x\ dx=0$ **Correct Answer is Option (a) 0**	**1**

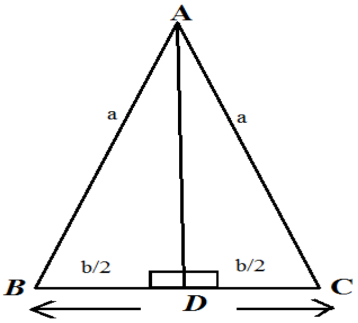
10.	$A^{-1} = kA$ $\frac{-1}{19}\begin{bmatrix}-2 & -3\\-5 & 2\end{bmatrix} = k\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ $\therefore k = \frac{1}{19}$ **Correct Answer is Option (b) 1/19**	**1**
11.	Corner Points Value of Z A (0, 2) Z = 0 + 12 = 12 B (3, 0) Z = 12 + 0 = 12 C (6, 0) Z = 24 + 0 = 24 D (6,8) Z = 24 + 48 = 72 E (0,5) Z = 0 + 30 = 30 Minimum value of Z = 12 **Correct Answer is Option (d)Any point on the line segment joining the points (0,2) and (3,0)**	**1**
12.	According to the Question $\frac{(\lambda\hat{i} + \hat{j} + 4\hat{k}).(2\hat{i} + 6\hat{j} + 3\hat{k})}{\left\|2\hat{i} + 6\hat{j} + 3\hat{k}\right\|} = 4$ $\frac{2\lambda + 6 + 12}{\sqrt{4 + 36 + 9}} = 4$ $\therefore \lambda = 5$ **Correct Answer is Option (c) 5**	**1**
13.	$\left(AB\right)^{-1} = B^{-1}A^{-1} = \begin{bmatrix}4 & -5\\-3 & 4\end{bmatrix}\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix} = \begin{bmatrix}37 & -14\\-29 & 11\end{bmatrix}$ **Correct Answer is Option (d)** $\begin{bmatrix}37 & -14\\-29 & 11\end{bmatrix}$	**1**
14.	B will win in second attempt or fourth attempt or sixth attempt or so on $\therefore\ P(B\,winning) = P(\overline{A}B) + P(\overline{A}\ \overline{B}\ \overline{A}\,B) + P(\overline{A}\ \overline{B}\ \overline{A}\,\overline{B}\ \overline{A}\,B) + \ldots\ldots\ldots..$ $= \frac{5}{6}.\frac{1}{6} + \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} + \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} + \ldots\ldots\ldots\ldots$ $= \frac{5/36}{1 - 25/36} = \frac{5}{11}$ **Correct Answer is Option (c) 5/11**	**1**
15.	According to the Question $\frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$ $\therefore \lambda = \frac{2}{3}$ **Correct Answer is Option (a) 2/3**	**1**
16	$Integrating\ Factor = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$ **Correct Answer is Option (b)** $\log x$	**1**
17.	$f'(x) = x^x(1 + \log x)$ For Stationary point $f'(x) = 0$ $x^x(1 + \log x) = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$	

	Correct Answer is Option (b) 1/e	**1**
18.	$3x+1=6y-2=1-z$ $3(x+1/3)=6(y-1/3)=-(z-1)$ $\frac{x+1/3}{1/3}=\frac{y-1/3}{1/6}=\frac{z-1}{-1}$ $\frac{x+1/3}{2}=\frac{y-1/3}{1}=\frac{z-1}{-6}$ **Correct Answer is Option (d) 2, 1, - 6.**	**1**
19.	**Correct Answer is Option (a)Both A and R true and R is the correct explanation for A.**	**1**
20.	**Correct Answer is Option (d)A is false but R is true.**	**1**
	Section –B **[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]**	
21.	$\cos^{-1}\alpha+\cos^{-1}\beta+\cos^{-1}\gamma=3\pi$ $\Rightarrow cos^{-1}\alpha = \pi, cos^{-1}\beta = \pi \;\&\; cos^{-1}\gamma = \pi$ $\therefore \alpha = \beta = \gamma = -1$ $\alpha(\beta+\gamma)-\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$ $=(-1)(-1-1)-(-1)(-1-1)+(-1)(-1-1)$ $=2-2+2=2$ **OR** $\cot^{-1}\left\{\frac{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}+\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}-\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\right\}$ $=\cot^{-1}\left\{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}\right\}$ $=\cot^{-1}\left(\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right)=\cot^{-1}\left(\tan\frac{x}{2}\right)$ $=\cot^{-1}\left(\cot(\frac{\pi}{2}-\frac{x}{2})\right)=\frac{\pi}{2}-\frac{x}{2}$	**1** **1** **1** **1/2** **1/2**

22.

In isosceles ΔABC, let AB = AC = a and BC = b (given),

$\frac{da}{dt} = -3$ cm/sec.



AD = $\sqrt{a^2 - \frac{b^2}{4}}$

A = Area of $\Delta\, ABC = \frac{1}{2}$ (BC) (AD) = $\left(\frac{1}{2}\right)(b)\sqrt{a^2 - \frac{b^2}{4}}$ **1**

$$\Rightarrow \frac{dA}{dt} = \left(\frac{b}{2}\right)\left(\frac{1}{2}\right)\left(a^2 - \frac{b^2}{4}\right)^{-1/2} . \left(2a\frac{da}{dt}\right)$$

$$= \frac{b(2a)(-3)}{4\sqrt{a^2 - \frac{b^2}{4}}} = \frac{-3ab}{2\sqrt{a^2 - \frac{b^2}{4}}}$$

$$\Rightarrow \left(\frac{dA}{dt}\right)_{at\,a=b} = \frac{-3b^2}{2\sqrt{b^2 - \frac{b^2}{4}}} = \frac{-3b^2}{\frac{2\sqrt{3}b}{2}} = -\sqrt{3}b$$

$\therefore$ Area is decreasing at the rate of $\sqrt{3}b$ cm/sec. **1**

OR

Let the side of a cube be x unit.

Volume of cube = V $= x^3$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt} = k\ (constant)$$

$$\frac{dx}{dt} = \frac{k}{3x^2}$$ **1**

Surface area = S = $6x^2$

$$\frac{dS}{dt} = 12\,x\frac{dx}{dt}$$

$$\frac{dS}{dt} = 12x\frac{k}{3x^2} = 4\left(\frac{k}{x}\right)$$

Hence, the surface area of the cube varies inversely as length of side **1**

23.

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\left(\vec{a} + \vec{b}\right).\left(\vec{a} + \vec{b}\right) = \left(-\vec{c}\right).\left(-\vec{c}\right)$$ **1/2**

$$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = \vec{c}.\vec{c}$$

$$\left|\vec{a}\right|^2 + 2\vec{a}.\vec{b} + \left|\vec{b}\right|^2 = \left|\vec{c}\right|^2$$

$$2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = \left|\vec{c}\right|^2 - \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2$$ **1/2**

$$2(3)(5)\cos\theta = 49 - 9 - 25$$ **1/2**

$$\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$ **1/2**

24.

Let $P(3\lambda-2, 2\lambda-1, 2\lambda+3)$ be any point on a line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ which is at a distance of 5 units from the point $Q(1,3,3)$.

According to the Question

$PQ=5$

$(PQ)^2=25$

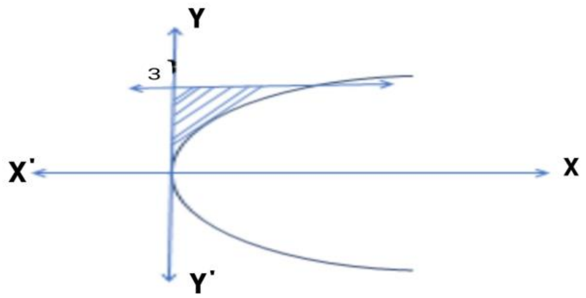
$(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2=25$ **1**

$17\lambda^2-34\lambda=0$

$\lambda=0\ or\ 2$

Required Point is (-2,-1,3) *or* (4,3,7) **1**

25.



1 Mark For Correct Figure

Required Area = $\int_0^3 \frac{y^2}{4}dy = \left.\frac{y^3}{12}\right]_0^3$ **1/2**

$= \frac{27}{12}-0=\frac{9}{4}$ square units **1/2**

Section –C

[This section comprises of solution short answer type questions (SA) of 3 marks each]

26.

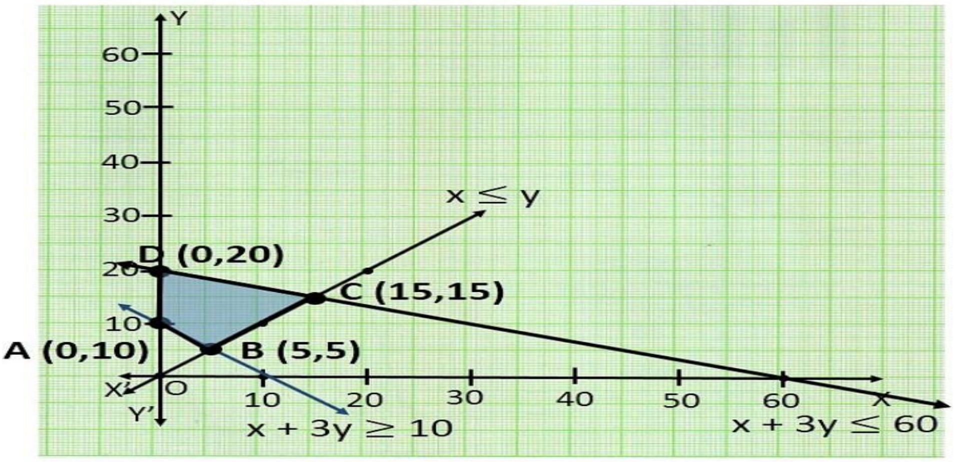
$x+3y\le 60$ $\quad$ $x+y\ge 10$ $\quad$ $x\le y$

$x+3y=60$ $\quad$ $x+y=10$ $\quad$ $x=y$

x	0	60
y	20	0

x	0	10
y	10	0

x	0	10
y	0	10



1.5 Marks For Correct Figure

Corner points are A (0, 10), B (5, 5), C (15, 15) and D (0, 20) **1/2**

	Corner Points / Value of Z A (0, 10) — Z = 0 + 90 = 90 B (5, 5) — Z = 15 + 45 = 60 C (15, 15) — Z = 45 + 135 = 180 D (0,20) — Z = 0 + 180 = 180 Minimum value of Z = 60	**1/2** **1/2**
27.	$n(S) = {}^6C_2 = \frac{6\times5}{2\times1} = 15$ Let X denote the larger of the two numbers obtained $\therefore$ X = 3, 4, 5, 6, 7 The Probability Distribution is X — P (X) 3 — 1/15 4 — 2/15 5 — 3/15 6 — 4/15 7 — 5/15 **OR** Let $P(A) = x$ *and* $P(B) = y$ According to the Question $P(A \cap B) = \frac{1}{6}$ *and* $P(A' \cap B') = \frac{1}{3}$ $P(A)P(B) = \frac{1}{6}$ *and* $P(A')P(B') = \frac{1}{3}$ $xy = \frac{1}{6}$ and $(1-x)(1-y) = \frac{1}{3}$ on solving we get $x = \frac{1}{2}$ *or* $\frac{1}{3}$	**1** **1/2** **1.5** **1** **1** **1**
28.	$I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$ $= \int \frac{\cos x}{(\sin x - 1)(\sin x - 2)} dx$ — put sin x = t cos x dx = dt $\Rightarrow I = \int \frac{dt}{(t-1)(t-2)}$ $= \int \left(\frac{-1}{t-1} + \frac{1}{t-2}\right) dt$ $= -\log\lvert t-1 \rvert + \log\lvert t-2 \rvert$ $= \log\left\lvert \frac{t-2}{t-1} \right\rvert + c$ $= \log\left\lvert \frac{\sin x - 2}{\sin x - 1} \right\rvert + c$	 **1** **1** **1**

	OR	
	$I=\int \frac{\sin\phi}{\sqrt{\sin^2\phi+2\cos\phi+3}}\,d\phi$ $=\int \frac{\sin\phi}{\sqrt{1-\cos^2\phi+2\cos\phi+3}}\,d\phi$ $=\int \frac{\sin\phi\;d\phi}{\sqrt{4+2\cos\phi-\cos^2\phi}}$ put cos ϕ = t - sin ϕ dϕ = dt	**1**
	$=\int \frac{-dt}{\sqrt{4+2t-t^2}} \quad =-\int \frac{dt}{\sqrt{-\left[t^2-2t-4\right]}}$ $=-\int \frac{dt}{\sqrt{-\left[t^2-2t+1-5\right]}} \quad =-\int \frac{dt}{\sqrt{\left(\sqrt{5}\right)^2-(t-1)^2}}$	**1**
	$=-\sin^{-1}\left(\frac{t-1}{\sqrt{5}}\right)+c \quad =-\sin^{-1}\left(\frac{\cos\phi-1}{\sqrt{5}}\right)+c$	**1**
29.	$2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ $2ye^{\frac{x}{y}}dx=\left(2xe^{\frac{x}{y}}-y\right)dy$ $\frac{dx}{dy}=\left(\frac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}\right)$ Put x = vy $\frac{dx}{dy}=v+y\frac{dv}{dy}$	**1**
	$v+y\frac{dv}{dy}=\frac{2vye^{v}-y}{2ye^{v}}$ $y\frac{dv}{dy}=\frac{2vye^{v}-y-2vye^{v}}{2ye^{v}}$ $\frac{dv}{dy}=\frac{-1}{2ye^{v}}$ $\int 2e^{v}dv=\int -\frac{1}{y}dy$	**1**
	$2e^{v}=-\log y+c$ $2e^{\frac{x}{y}}+\log y=c$	**1**
	OR $\frac{dy}{dx}-3y\cot x=\sin 2x$ Compare with $\frac{dy}{dx}+Py=Q$ $P=-3\cot x \quad , \qquad Q=\sin 2x$	**1/2**

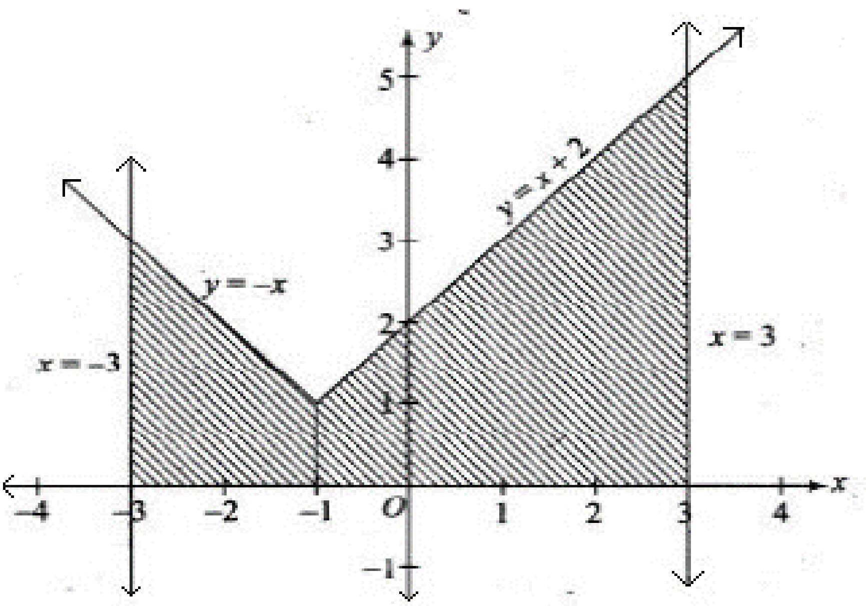
<table>
<tr><td></td><td>$I.F = e^{\int P dx} = e^{\int -3\cot x\, dx} = e^{-3\log\sin x}$

$= e^{log(sinx)^{-3}} = (sinx)^{-3} = \frac{1}{sin^3 x}$

The solution of given differential equation is
$y\left(I.F\right) = \int Q\left(I.F\right)dx$
$y\left(\frac{1}{\sin^3 x}\right) = \int \sin 2x.\frac{1}{\sin^3 x}dx$
$\frac{y}{\sin^3 x} = \int 2\sin x\cos x.\frac{1}{\sin^3 x}dx$
$\frac{y}{\sin^3 x} = \int 2\cot x\,\mathrm{cosecx}\,dx$
$\frac{y}{\sin^3 x} = -2\mathrm{cosec}\,x + C$
$y = -2\sin^2 x + c\sin^3 x$</td><td>

1

1

1/2</td></tr>
<tr><td>30.</td><td>

Required Area $= \int_{-3}^{-1} -x\,dx + \int_{-1}^{3}(x+2)dx$
$= \left[\frac{-x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$
$= \frac{-1}{2}(1-9) + \left[(\frac{9}{2}+6) - (\frac{1}{2}-2)\right]$
$= 4 + 12 = 16\, square\, units$</td><td>**1.5 Mark For Correct Figure**

1/2

1</td></tr>
<tr><td>31.</td><td>$x = a\sin t - b\cos t$ $y = a\cos t + b\sin t$
$\frac{dx}{dt} = a\cos t + b\sin t = y$ $\frac{dy}{dt} = -a\sin t + b\cos t = -x$
$\frac{dy}{dx} = \frac{-x}{y}$</td><td>
1
1/2</td></tr>
</table>

	$$\frac{d^2y}{dx^2}=\frac{y(-1)-(-x).\frac{dy}{dx}}{(y)^2}$$ $$=\frac{-y+x\left(-\frac{x}{y}\right)}{(y)^2}$$ $$=\frac{-y^2-x^2}{y^3}= \quad =\frac{-(y^2+x^2)}{y^3}$$	**1** **1/2**
	Section –D **[This section comprises of solution of long answer type questions (LA) of 5 marks each]**	
32.	$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & -3\\ -3 & 2 & -4\end{bmatrix}$$ $$\lvert A\rvert=1(-12+6)-2(-8-9)+3(4+9)$$ $$=-6+34+39=67\neq 0$$ $$\text{Adj. } A=\begin{bmatrix}-6 & 14 & -15\\ 17 & 5 & 9\\ 13 & -8 & -1\end{bmatrix}$$ $$A^{-1}=\frac{1}{67}\begin{bmatrix}-6 & 14 & -15\\ 17 & 5 & 9\\ 13 & -8 & -1\end{bmatrix}$$ The matrix form of the equations is $$\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-4\\ 14\\ -15\end{bmatrix}$$ $$A^tX=B$$ $$X=\left(A^t\right)^{-1}B$$ $$=\left(A^{-1}\right)^tB$$ $$=\frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}\begin{bmatrix}-4\\ 14\\ -15\end{bmatrix}$$ $$=\frac{1}{67}\begin{bmatrix}24+238-195\\ -56+70+120\\ 60+126+15\end{bmatrix}$$ $$=\frac{1}{67}\begin{bmatrix}67\\ 134\\ 201\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$ x = 1, y = 2, z = 3	**1/2** **2** **1/2** **1/2** **1/2** **1**

33.

Let $P(2\lambda+3,\lambda+3,\lambda)$ be any point on line $\dfrac{x-3}{2}=\dfrac{y-3}{1}=\dfrac{z}{1}$

Let the line through origin and making an angle of $\dfrac{\pi}{3}$ with the given line be along OP. Then direction ratios are proportional to $2\lambda+3-0, \lambda+3-0, \lambda-0$

i.e. $2\lambda+3, \lambda+3, \lambda$ **1**

Also, direction ratios of the given line are proportional to 2,1,1.

$$\therefore \quad \cos\frac{\pi}{3}=\frac{(2\lambda+3)(2)+(\lambda+3)(1)+(\lambda)(1)}{\sqrt{(2\lambda+3)^2+(\lambda+3)^2+(\lambda)^2}\sqrt{2^2+1^2+1^2}}$$ **1**

$$\Rightarrow \frac{1}{2}=\frac{6\lambda+9}{\sqrt{6\lambda^2+18\lambda+18}\sqrt{6}}$$

$$\Rightarrow \frac{1}{2}=\frac{3(2\lambda+3)}{6\sqrt{\lambda^2+3\lambda+3}}$$

$$\Rightarrow \sqrt{\lambda^2+3\lambda+3}=(2\lambda+3)$$

Squaring both sides, we get

$$\lambda^2+3\lambda+3=(2\lambda+3)^2$$

$$\Rightarrow \lambda^2+3\lambda+2=0$$

$$\Rightarrow \lambda=-1,-2$$

Therefore, the coordinates of point $P(1,2,-1)$ *or* $P(-1,1,-2)$ **2**

Hence Equations of required lines are

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{-1} \text{ and } \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$$ **1**

OR

The lines are

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \text{ and } \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

Let $P(\lambda-2, 2\lambda+3, 4\lambda-1)$ be any point on line (1) and $Q(2\mu+1, 3\mu+2, 4\mu+3)$ be any point on line (2). Also, the given point is $A(1,1,1)$. **1**

For some definite values of λ *and* μ, the required line passes through A, P and Q

The direction ratios of AP are $\lambda-3, 2\lambda+2, 4\lambda-2$

The direction ratios of AQ are $2\mu, 3\mu+1, 4\mu+2$

$$\therefore \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}$$ **1**

$$\Rightarrow \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{2\lambda-1}{2\mu+1}=k \ (let)$$

$$\Rightarrow \lambda-3=2\mu k \ , \ 2\lambda+2=3\mu k+k \ , \ 2\lambda-1=2\mu k+k$$ **1**

$$\Rightarrow \mu k=\frac{\lambda-3}{2} \ , \ 2\lambda+2=3\left(\frac{\lambda-3}{2}\right)+k \ , \ 2\lambda-1=2\left(\frac{\lambda-3}{2}\right)+k$$

$$\Rightarrow \mu k=\frac{\lambda-3}{2} \ , \ k=\frac{\lambda+13}{2} \ , \ k=\lambda+2$$

	$\therefore \ \dfrac{\lambda+13}{2}=\lambda+2 \Rightarrow \lambda=9$ $Also\ k=\lambda+2=11$ Hence The direction ratios of AP are $6,20,34$ i.e. $3,10,17$ Therefore, Equation of required line is $\dfrac{x-1}{3}=\dfrac{y-1}{10}=\dfrac{z-1}{17}$	**1** **1/2** **1/2**
34.	Let $\mathrm{I}=\int_0^{\pi}\dfrac{x\tan x}{\sec x+\tan x}dx$ ……… (1) Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $\mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$ $\Rightarrow \ \mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)(-\tan x)}{-\sec x-\tan x}dx$ $\Rightarrow \ \mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)\tan x}{\sec x+\tan x}dx$ ……… (2) $(1)+(2)\Rightarrow 2I=\int_0^{\pi}\dfrac{(x+\pi-x)\tan x}{\sec x+\tan x}dx$ $\Rightarrow \ \mathrm{I}=\dfrac{\pi}{2}\int_0^{\pi}\dfrac{\sin x}{1+\sin x}dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left(\dfrac{\sin x}{\cos^2 x}-\dfrac{\sin^2 x}{\cos^2 x}\right)dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left(\tan x\sec x-\tan^2 x\right)dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left[\sec x\tan x-\sec^2 x+1\right]dx$ $=\dfrac{\pi}{2}\left[\sec x-\tan x+x\right]_0^{\pi}$ $=\dfrac{\pi}{2}\left[(-1-0+\pi)-(1-0+0)\right]$ $\therefore \ \mathrm{I}=\dfrac{\pi}{2}[\pi-2]$ **OR** Let $\mathrm{I}=\int_0^{\pi}\log(1+\cos x)dx$ ...(1) $\Rightarrow \ \mathrm{I}=\int_0^{\pi}\log\left[1+\cos(\pi-x)\right]dx$	 **1** **1** **1** **1** **1**

$$\Rightarrow \quad I = \int_0^{\pi} \log(1-\cos x)\,dx \qquad \ldots(2)$$

Adding (1) and (2)

$$2I = \int_0^{\pi} \log\left[(1+\cos x)(1-\cos x)\right]dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi} \log\left(1-\cos^2 x\right)dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int_0^{\pi} \log\sin^2 x\,dx = \int_0^{\pi} \log\sin x\,dx$$

1

Since $\log\left[\sin(\pi - x)\right] = \log\sin x$

$$\therefore \qquad I = 2\int_0^{\pi/2} \log\sin x\,dx \qquad \ldots(3)$$

$$\Rightarrow \quad I = 2\int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx$$

$$\Rightarrow \quad I = 2\int_0^{\pi/2} \log\cos x\,dx \qquad \ldots(4)$$

Adding (3) and (4)

$$2I = 2\int_0^{\pi/2} \log\sin x\cos x\,dx$$

1

$$\Rightarrow \qquad I = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right)dx$$

$$= \int_0^{\pi/2} \log\sin 2x\,dx - \log 2\int_0^{\pi/2} 1.dx$$

$$\Rightarrow \qquad I = I_1 - \log 2\left[x\right]_0^{\pi/2} = I_1 - \frac{\pi}{2}\log 2 \qquad \ldots(5)$$

1

Now $\quad I_1 = \int_0^{\pi/2} \log\sin 2x\,dx$

Let $2x = t \Rightarrow 2dx = dt$

$$= \frac{1}{2}\int_0^{\pi} \log\sin t\,dt = \frac{1}{2}\int_0^{\pi} \log\sin x\,dx$$

(Changing t to x)

$$= \frac{1}{2}\times 2\int_0^{\pi/2} \log\sin x\,dx$$

$$\Rightarrow I_1 = \frac{1}{2}I$$

1

From (5) $\quad \Rightarrow \quad I = \frac{1}{2}I - \frac{\pi}{2}\log 2$

	$\Rightarrow \frac{1}{2} I = -\frac{\pi}{2} \log 2$ $\therefore \quad I = -\pi \log 2.$	**1**
35.	**One – one**: Let $x_1, x_2 \in R_+$ such that $f(x_1) = f(x_2)$ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$ $\Rightarrow 9\left(x_1^2 - x_2^2\right) + 6\left(x_1 - x_2\right) = 0$ $\Rightarrow \left(x_1 - x_2\right)\left\{9\left(x_1 + x_2\right) + 6\right\} = 0$ $\Rightarrow \quad x_1 - x_2 = 0 \quad or \quad 9x_1 + 9x_2 + 6 = 0$ which is not possible $\Rightarrow \quad x_1 = x_2$ $\therefore \quad f$ is one-one. **Onto**: Let $y = 9x^2 + 6x - 5$ $\Rightarrow 9x^2 + 6x - (5 + y) = 0$ $\Rightarrow \quad x = \frac{-6 \pm \sqrt{36 + 4(9)(5+y)}}{2(9)} = \frac{-6 \pm \sqrt{36}\sqrt{1+5+y}}{18}$ $x = \frac{6(-1 \pm \sqrt{y+6})}{6(3)} = \frac{-1 \pm \sqrt{y+6}}{3}$ Now, $x \in R_+ \Rightarrow x \geq 0$ and so $x = \frac{-1-\sqrt{y+6}}{3}$ is rejected $\therefore x = \frac{-1+\sqrt{y+6}}{3}$ Now $x \geq 0 \Rightarrow \frac{-1+\sqrt{y+6}}{3} \geq 0$ $\Rightarrow \sqrt{y+6} \geq 1 \Rightarrow y + 6 \geq 1$ $\Rightarrow y \geq -5$ $\therefore R_f = \left\{y : y \in [-5, \infty)\right\} =$ codomain of f. $\therefore f$ is onto. Hence f is one one and onto function.	**1** **1** **1/2** **1** **1** **1/2**
	Section –E **[This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)**	
36.	V(t) = $t^3 - 3t^2 + 3t - 100$ (i) No, the above function cannot be used to estimate number of vehicles in the year 2020 because for 2020 we have t = 0 and V(0) = $0 - 0 + 0 - 100 = -100$ Which is not possible	**1**

	(ii) V(20) = $(20)^3 - 3(20)^2 + 3(20) - 100$ Therefore, the estimated number of vehicles in the year 2040 are 6760. (iii) $V'(\text{t}) = 3t^2 - 6t + 3$ $= 3(t^2 - 2t + 1)$ $= 3(t-1)^2 \geq 0.$ Hence $V(t)$ is always increasing function.	**1** **1** **1**
37.	Let E_1 is the event that a student is regular E_2 is the event that a student is irregular A is the event that a student attains grade A $P(E_1) = \frac{30}{100}$, $P(E_2) = \frac{70}{100}$ $P(\text{A}/E_1) = \frac{80}{100}$, $P(\text{A}/E_2) = \frac{10}{100}$ (i) Required Probability = $P(\text{A}/E_2) = \frac{10}{100} = \frac{1}{10}$ (ii) $\textit{Required Probability} = P(\text{A})$ $= P(E_1)P(\text{A}/E_1) + P(E_2)P(\text{A}/E_2)$ $= \frac{30}{100} \cdot \frac{80}{100} + \frac{70}{100} \cdot \frac{10}{100} = \frac{31}{100}$ (iii) $\textit{Required Probability} = P(\text{E}_1/\text{A})$ $= \frac{P(E_1)P(\text{A}/E_1)}{P(E_1)P(\text{A}/E_1) + P(E_2)P(\text{A}/E_2)}$ $= \frac{\frac{30}{100} \cdot \frac{80}{100}}{\frac{30}{100} \cdot \frac{80}{100} + \frac{70}{100} \cdot \frac{10}{100}} = \frac{24}{31}$ **OR** (iii) $\textit{Required Probability} = P(\text{E}_2/\text{A})$ $= \frac{P(E_2)P(\text{A}/E_2)}{P(E_1)P(\text{A}/E_1) + P(E_2)P(\text{A}/E_2)}$ $= \frac{\frac{70}{100} \cdot \frac{10}{100}}{\frac{30}{100} \cdot \frac{80}{100} + \frac{70}{100} \cdot \frac{10}{100}} = \frac{7}{31}$	**1** **1** **1** **1** **1** **1**
38.	(i) Let length, breadth and height of the tank are x, x *and* y respectively According to the Question $\therefore x^2 y = 500 \Rightarrow y = \frac{500}{x^2}$ Surface Area $= S = x^2 + 4xy$ $S = x^2 + 4x(\frac{500}{x^2}) = x^2 + \frac{2000}{x}$ $\Rightarrow \frac{dS}{dx} = 2x - \frac{2000}{x^2}$	**1/2** **1/2**

	For maxima or minima, $\frac{dS}{dx}=0 \Rightarrow 2x-\frac{2000}{x^2}=0 \Rightarrow x=10\,m$	
	Now $\frac{d^2S}{dx^2}=2+\frac{4000}{x^3}$	**1/2**
	and $\left(\frac{d^2S}{dx^2}\right)_{at\,x=10}=2+\frac{4000}{(10)^3}>0$	
	$\therefore$ Surface Area is minimum when $x=10\,m$	
	$\therefore$ Minimum Surface Area $=100+\frac{2000}{10}=300\,m^2$	**1/2**
	(ii) $If\ \ x=10\,m\ \ then\ \ y=5\,m$	
	$and\ Volume\ of\ the$ tank $=x^2y=(10)^2(5)=500\,m^3$	**1/2**
	$New\ Volume=(2x)^2y=4x^2y=4(10)^2(5)=2000\,m^3$	**1/2**
	$\therefore\ Increase\ in\ Volume\ of\ the$ tank $=2000-500=1500\,m^3$	**1/2**
	$\therefore\ \%\,Increase\ in\ Volume\ of\ the$ tank $=300\%$	**1/2**