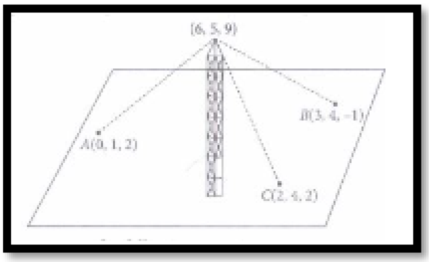
Q37. A mobile tower stands at the top of a hill. Consider the surface on which tower stand as a plane having points A(0, 1,2), B(3, 4, -1) and C(2, 4, 2) on it. The mobile tower is tied with 3 cables from the point A, Band C such that it stand vertically on the ground. The peak of the tower is at the point (6, 5, 9), as shown in the figure.



Based on the above information, answer the following questions
(i) The equation of plane passing through the points A, Band C is
(ii) The height of the tower from the ground is
(iii) The equation of line of perpendicular drawn from the peak of tower to the ground is.

Q38. The given Integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$

Consider $I = \int f(x)dx$
Put $x = g(t)$ so that $\frac{dx}{dt} = g'(t)$
we write $dx = g'(t)dt$
Thus $I = \int f(x)dx = \int f(g(t))g'(t)dt$

This change of variable formula is one of the important tools available to us in the name of integration by substitution.
(i) $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$ is equal to
(ii) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ is equal to
(iii) $\int \frac{\sin x}{(1+\cos x)^2}dx$ is equal to

SOLUTIONS:

SECTION A

S1.Ans.(a)
Sol.
Relation R is reflexive relation over set A as every element of A is related to itself in R.
Relation R is not symmetric as (1,2) is in R but (2,1) is not in R.
Relation R is also transitive as for a , b , c in A, if (a,b) is in R and (b,c) is in R, then (a,c) is also in R.
S2.Ans.(a)
Sol.
Given A is skew – symmetric matrix
$\therefore A^T = -A$
Or $A = -A^T$
Squaring both sides , we get , $A^2 = (-A^T)^2$
$\therefore A^2 = (A^T)^2$
$\therefore A^2 = (A^2)^T$
By definition of symmetric matrix , A^2 is symmetric matrix.
S3.Ans.(d)

Sol.

We know that
Number of arbitrary constants = Order of differential equation
Since order = 4
∴ Number of constants = 4

S4.Ans.(d)

Sol.

If $y = \log \frac{x}{(1+x)}$, then $\frac{dy}{dx} = \frac{1+x}{x}.\frac{d}{dx}\left(\frac{x}{1+x}\right)$

$= \frac{1+x}{x}.\left(\frac{(x+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x+1)}{(x+1)^2}\right)$

$= \frac{1+x}{x} \times \frac{x+1-x}{(x+1)^2}$

$= \frac{1}{x(x+1)}$

S5.Ans.(d)

Sol.

Corner points	Value of F = 4x + 6y
(0 ,2)	Z = 4(0) +6(2) = 12 (min)
(3,0)	Z= 4(3) +6(0) = 12 (min)
(6 ,0)	Z = 4(6) +6(0) = 24
(6 , 8)	Z= 4(6) +6(8) = 72 (max)
(0 ,5)	Z= 4(0) + 6(5) = 30

The minimum value of F occurs at any point on the line segment joining the points (0 ,2) and (3 , 0)

S6.Ans.(b)

Sol.

A stationary point of a function is a point where $f'(x) = 0$
For differentiating $f(x)$, we use logarithmic differentiation
$f(x) = x^x$
$= \log f(x) = x \log x$
Differentiating w.r.t x
$= \frac{1}{f(x)}.f'(x) = x.\frac{1}{x} + 1 . \log x$
$= f'(x) = x^x(1 + \log x)$
$= \ x^x(1 + \log x) = 0$
Either $x^x = 0$ or $1 + \log x = 0$
Since , x^x is exponential function ,It can never be zero.
Or $1 + \log x = 0 \Rightarrow x = \frac{1}{e}$

S7.Ans.(a)

Sol.

Given , $P(A|B) = P(B|A)$
$\Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(B)}$
$= \frac{1}{P(A)} = \frac{1}{P(B)}$
$\therefore P(A) = P(B)$

S8.Ans.(d)

Sol.

If $R = \{(x , y): x + 2y = 8 \}$ is a relation N , then the range of R is :
$R = \{(2 , 3)\}$

Range = { 3 }

S9.Ans.(a)

Sol.

Let $f : R \to R$ be defined as $f(x) = 3x$.

For one – one

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

∴ f is one – one

For onto

Let $y = 3x$ then there exist $x = \frac{y}{3}$ s.t $f(x) = 3x = 3 \times \frac{y}{3} = y$

∴ f is onto.

S10.Ans.(b)

Sol.

$\Rightarrow \left[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(\cot\left(\pi - \frac{\pi}{6}\right)\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(\cot\frac{5\pi}{6}\right)$

$\left[\because \tan^{-1}(\tan\theta) = \theta \ \forall\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \cot^{-1}(\cot\theta) = \theta \ \forall\ \theta \in (0, \pi)\right]$

$= \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$

S11.Ans.(b)

Sol.

$\Rightarrow \cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$

$\Rightarrow \sin^{-1}\frac{2}{5} + \cos^{1}x = \cos^{-1}0$

$= \sin^{-1}\frac{2}{5} + \cos^{-1}x = \frac{\pi}{2}$

$= \sin^{-1}\frac{2}{5} = \frac{\pi}{2} - \cos^{-1}x$

$= \sin\left(\sin^{-1}\frac{2}{5}\right) = \sin\left(\frac{\pi}{2} - \cos^{-1}x\right)$

$= \frac{2}{5} = \cos(\cos^{-1}x)$

$= x = \frac{2}{5}$

S12.Ans.(c)

Sol.

If $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath}$,

then value of $\vec{a}.\vec{b}$ = $(3\hat{\imath} - 2\hat{\jmath} + \hat{k}).(2\hat{\imath} + \hat{\jmath}) = 6 - 2 = 4$

S13.Ans.(a)

Sol.

$\Rightarrow I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx \qquad (\because \cos 2\theta = \cos^2\theta - \sin^2\theta)$

$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} dx$

$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

Let $\cos x + \sin x = t$

Differentiating w.r.t . x

$-\sin x + \cos x = \frac{dt}{dx}$

$dx = \frac{1}{\cos x - \sin x} dt$

$= \int \frac{\cos x - \sin x}{t} \times \frac{dt}{\cos x - \sin x}$

$= \int \frac{dt}{t}$

$= \log|t| + C$

$= \log|\cos\ x + \sin x| + C$

S14.Ans.(d)

Sol. $\frac{d}{dx}(3^x)$

$= \frac{d}{dx}(e^{x\log 3})$

$= e^{x\log 3}. \frac{d}{dx}(x \log 3)$

$= e^{x \log 3}. \log 3$

$= 3^x. \log 3$

S15.Ans.(d)

Sol.

$\Rightarrow adj(adj\ A) = |A|^{n-2}.A$

So, here $adj(adj\ A) = |A|^0.A$

$\Rightarrow adj(adj\ A) = A$

S16.Ans.(d)

Sol.

We know that

Number oof arbitrary constants = Order of differential equation

Since order = 4

∴ Number of constants = 4

S17.Ans.(a)

Sol.

If A and B are independent events.

It implies-

$P(A \cap B) = P(A).P(B)$

$P(A' \cap B) = P(A')P(B)$

And $P(A \cap B') = P(A)P(B')$

P(exactly one of A, B occurs) = $P(A' \cap B) + P(A \cap B')$

⇒ P(exactly one of A, B occurs) = P(A')P(B)+ P(A)P(B')

∴ Statement Is true.

S18.Ans.(a)

Sol.

Let $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$

Then $C_{31}.C_{23}$ = ?

$C_{31} = (-1)^{3+1}.M_{31}$

$= M_{31} = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$

$= C_{31} = (-1)^{3+1}.M_{31} = -1$

$C_{23} = (-1)^{2+3}.M_{23}$

$= M_{23} = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2 + 3 = 5$

$= C_{23} = (-1)^{2+3}.M_{23} = (-1)^{2+3}.5 = -5$

Then $C_{31}.C_{23} = -1 \times (-5) = 5$

S19.Ans.(a)

Sol.

Given points are (3 , 4 -7) and (1 , -1 , 6)

Position vector of the point are $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 7\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + 6\hat{k}$
Vector equation of the a line passing through the point $\vec{a}$ and $\vec{b}$ is given by :
$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
$\Rightarrow \vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 7\hat{k} + \lambda\left((\hat{\imath} - \hat{\jmath} + 6\hat{k}) - (3\hat{\imath} + 4\hat{\jmath} - 7\hat{k})\right)$
Substituting $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
$\Rightarrow x\hat{\imath} + y\hat{\jmath} + z\hat{k} = 3\hat{\imath} + 4\hat{\jmath} - 7\hat{k} + \lambda(-2\hat{\imath} - 5\hat{\jmath} + 13\hat{k})$
$\therefore (x-3)\hat{\imath} + (y-4)\hat{\jmath} + (z+7)\hat{k} = \lambda(-2\hat{\imath} - 5\hat{\jmath} + 13\hat{k})$
$\therefore$ Both A and R are true and R is the correct explanation of A.

S20.Ans.(a)

Sol. 0

It is given that $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$
$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$
It is known that , $P(A|B) = \frac{P(A \cap B)}{P(B)}$
$\Rightarrow P(A|B) = \frac{4}{5}$
$\therefore$ Both A and R are true and R is the correct explanation of A.

SECTION B

Sol 21.

$$I = \int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$$
$$= \int \frac{x\left(1 - \frac{1}{x^3}\right)^{\frac{1}{4}}}{x^5} dx$$
$$= \int \frac{\left(1 - \frac{1}{x^3}\right)^{\frac{1}{4}}}{x^4} dx$$
Let $\left(1 - \frac{1}{x^3}\right) = t^4$
Differentiate w.r.t x
$\Rightarrow \frac{3}{x^4} = 4t^3 \frac{dt}{dx}$
$= \frac{dx}{x^4} = \frac{4t^3}{3} dt$
Now ,
$\Rightarrow I = \int t.\frac{4t^3}{3} dt$
$$= \frac{4}{3}\int t^4 dt$$
$$= \frac{4}{3}\left[\frac{t^5}{5}\right] + C$$
$$= \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{\frac{5}{4}} + C$$

Sol 22.

Given differential equation of the family of circles having a centre on the y-axis is $x^2 + (y-b)^2 = r^2$
Given the radius of the circle is 3 units.
The differential; equation of the family of circles having a centre on the y-axis and radius 3 units is as below:
$x^2 + (y-b)^2 = 3^2$

$x^2 + (y-b)^2 = 9$……(i)
Differentiating (i) with respect to x,
2x + 2(y – b).y′ = 0
⇒ (y – b). y′ = -x
⇒ (y – b) = -x/y′ …….(ii)
Substituting (ii) in (i),

$\Rightarrow x^2 + \left(-\frac{x}{y'}\right)^2 = 9$

$= x^2\left(1 + \frac{1}{(y')^2}\right) = 9$

$= x^2((y')^2 + 1) = 9.(y')^2$

$= (x^2 - 9)(y')^2 + x^2 = 0$

Hence, this is the required differential equation.
Or

$\Rightarrow \frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$

$= \left\{\frac{d^2y}{dx^2} + (1+x)\right\}^3 = -\frac{dy}{dx}$

Thus , order is 2 and degree is 3 .So the sum is 5.

Sol 23.

If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath}$

$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 1 \\ 1 & -1 & 0 \end{vmatrix}$

$= (0+1)\hat{\imath} - (0-1)\hat{\jmath} + (-1-2)\hat{k}$

$= \hat{\imath} + \hat{\jmath} - 3\hat{k}$

then find the value of $|\vec{a} \times \vec{b}| = \sqrt{1^2 + 1^2 + (-3)^2} = \sqrt{2+9} = \sqrt{11}$

Or

Given that: $|\vec{a} + \vec{b}| = |\vec{a}|$

To prove: $(2\vec{a} + \vec{b}).\vec{b} = 0$

Since, $|\vec{a} + \vec{b}| = |\vec{a}|$

Squaring both sides ,we get

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a}|^2$

$\Rightarrow (\vec{a} + \vec{b})(\vec{a} + \vec{b}) = |\vec{a}|^2$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{a}|^2$

$\Rightarrow |\vec{b}|^2 + 2\vec{a}.\vec{b} = 0$

$\Rightarrow (2\vec{a} + \vec{b}).\vec{b} = 0$

Hence, $2\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$.

Sol 24.

If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$,

then $(A^2 - 5A)$ = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

= $\begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

Sol 25.

Let the distance covered with speed of 25kmph be 'x' km
and the distance covered with speed of 40kmph be 'y' km
Total distance covered be 'z' km
The L.P.P of the above problem is
Maximize z = x + y
Subject to constraints

$4x + 5y \leq 200$

$\frac{x}{50} + \frac{y}{40} \leq 1$

$x \geq 0 , y \geq 0$

SECTION C

Sol 26.

Let us take
A: car needs service
E_1: car is rented from x
E_2: car is rented from y
E_3: car is rented from z
We have to find the probability that car is chosen from z, if car needs service
$P(E_3/A)$
To find not chosen probability

$P(E_1) = 50\% = \frac{50}{100}$

$P(E_2) = 30\% = \frac{30}{100}$

$P(E_3) = 20\% = \frac{20}{100}$

$P(A/E_1) =$ Probability that car needs service = 9% = $\frac{9}{100}$

$P(A/E_2) = 12\% = \frac{12}{100}$

$P(A/E_3) = 10\% = \frac{10}{100}$

$$P(E_3/A) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{20}{100} \times \frac{10}{100}}{\frac{50}{100} \times \frac{9}{100} + \frac{30}{100} \times \frac{12}{100} + \frac{20}{100} \times \frac{10}{100}}$$

$$= \frac{200}{450 + 360 + 200}$$

$$= \frac{200}{1010}$$

$$= \frac{20}{101}$$

According to question we have to find car is not chosen from agency = 4 , if it needs service.

So , 1 - $P\left(\frac{E_3}{A}\right)$

$= 1 - \frac{20}{101}$

$= \frac{101-20}{101}$

$= \frac{81}{101}$

Sol 27.

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx$ ....(1)

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x+\sin x} dx$ ....(2)

(1) + (2)

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{sin^2 x+\cos^2 x}{\cos x+\sin x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{1}{\cos x+\sin x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\frac{1}{\sqrt{2}}}{\sin\frac{\pi}{4}.\cos x + \cos\frac{\pi}{4}.\sin x} dx$

$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(\frac{\pi}{4}+x\right)} dx$

$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} cosec \left(\frac{\pi}{4}+x\right) dx$

Put $\frac{\pi}{4}+x = t$

$dx = dt$

$= \frac{1}{\sqrt{2}} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} cosec\ t\ dt$

$= \frac{1}{\sqrt{2}} \left[\ln\left|\tan\frac{t}{2}\right|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= \frac{1}{\sqrt{2}} \left[\ln\left(\tan\frac{3\pi}{8}\right) - \ln\left(\tan\frac{\pi}{8}\right)\right]$

Now ,

$2I = \frac{1}{\sqrt{2}}\left[\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right]$

$I = \frac{1}{2\sqrt{2}} \log(\sqrt{2})^2$

$I = \frac{1}{\sqrt{2}} \log\sqrt{2}$

Sol 28.

The given function f(x) is continuous at x = 0

$\Rightarrow \lim_{x\to 0}\left(\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}\right) = f(0)$

$= \lim_{x\to 0}\left(\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}\right)\left(\frac{\sqrt{1+kx}+\sqrt{1-kx}}{\sqrt{1+kx}+\sqrt{1-kx}}\right) = \frac{0+1}{0-1}$

$= \lim_{x\to 0} \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}} = -1$

$= \frac{2k}{2} = -1$

$= k = -1$

Or

What values of λ, $f(x) = \begin{cases} \lambda(x^2-2x)\ ,\ if\ \ x \le 0 \\ 4x+1\ , if\ x > 0 \end{cases}$ is the function is continuous at x = 0?

L.H.L

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} f(-h)$

$= \lim_{h\to 0} \lambda((-h)^2 - 2(-h))$

$= 0$

R.H.L

$\lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h)$
$= \lim_{h\to0} f(h)$
$= \lim_{h\to0} 4h+1$
$= 1$
$\because L.H.L \neq R.H.L$
$\therefore f(x)$ is not continuous at x = 0
So , for any value of $\lambda \in \boldsymbol{R}$ **,** f is discontinuous at x = 0

Sol 29.

The given points are A(1 ,1 0) , B(1 , 2 , 1) and C(-2 , 2 , -1).

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = (-2-2)-(2+2) = -8 \neq 0$$

Therefore , a plane will pass through the points A , B and C
It is known that the equation of the plane through the points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$
$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$
$= (-2)(x-1) - 3(y-1) + 3z = 0$
$= -2x - 3y + 3z + 2 + 3 = 0$
$= -2x - 3y + 3z = -5$
$= 2x + 3y - 3z = 5$
This is the cartesian equation of the required plane.

Or

We know , the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is
$$d = \left|\frac{(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$$
On comparing the given equations $\vec{r} = (4\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ respectively .We have
$\vec{a_1} = 4\hat{\imath} - \hat{\jmath}$, $\vec{a_2} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\vec{b_1} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{b_2} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$
Now , $\vec{a_2} - \vec{a_1} = -3\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$
And $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{\imath} - \hat{\jmath} + 0\hat{k}$
$\therefore (\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = (-3\hat{\imath} + 0\hat{\jmath} + 2\hat{k}).(2\hat{\imath} - \hat{\jmath} + 0\hat{k}) = -6$
And $|\vec{b_1} \times \vec{b_2}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$
$\therefore$ Shortest distance , d = $\left|\frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$

Sol 30.

$$I = \int \frac{\sec^2 x}{cosec^2 x}\, dx$$
$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$
$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$
$$= \int \tan^2 x\, dx$$

$= \int (\sec^2 x - 1)dx$

$= \int \sec^2 x\,dx - \int dx$

$= \tan x - x + C$

Or

$I = \int_0^{\frac{\pi}{4}} \sin 4x \sin 3x\,dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} 2\sin 4x\,.\sin 3x\,dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} [\cos(4x - 3x) - \cos(4x + 3x)].\,dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos x\,dx - \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 7x\,dx$

$= \frac{1}{2}[\sin x]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[\frac{\sin 7x}{7}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\sin\frac{\pi}{4} - \sin 0\right] - \frac{1}{14}\left[\sin\frac{7\pi}{4} - \sin 0\right]$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{14} \times \left(\sin\left(2\pi - \frac{\pi}{4}\right)\right)$

$= \frac{1}{2\sqrt{2}} - \frac{1}{14}\left(-\sin\frac{\pi}{4}\right)$

$= \frac{1}{2\sqrt{2}} + \frac{1}{14\sqrt{2}}$

$= \frac{7+1}{14\sqrt{2}}$

$= \frac{4}{7\sqrt{2}}$

Sol 31.

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$ .Find

(i) $\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$

$= 0.4 = \frac{P(A \cap B)}{P(A)}$

$= 0.4 = \frac{P(A \cap B)}{0.8}$

$= P(A \cap B) = 0.4 \times 0.8 = 0.32$

(ii)$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$

(iii)$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.5 - 0.32$

$= 0.98$

SECTION D

Sol 32.

If $A = \begin{vmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{vmatrix}$, Find A^{-1}

$\Rightarrow A^{-1} = \frac{adj\,A}{|A|}$

$\Rightarrow |A| = 3\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 3 & -3 \\ 2 & -1 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix}$

$= 3(-2) - 1(-3 + 6) + 2(0 - 4)$

$= -6 - 3 - 8$

$= -17$

$\Rightarrow adj\ A = \begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = -\frac{1}{17}\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$

Or

Solve the linear system of equation :

x + 3y – 2z = 0

2x – 3y + z = 1

4x – 3y + z = 3

$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -3 & 1 \\ 4 & -3 & 1 \end{bmatrix}\begin{matrix} 0 \\ 1 \\ 3 \end{matrix}$

$R_2 - 2R_1 \rightarrow R_2$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -9 & 5 \\ 4 & -3 & 1 \end{bmatrix}\begin{matrix} 0 \\ 1 \\ 3 \end{matrix}$

$R_3 - 4R_1 \rightarrow R_3$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -9 & 5 \\ 0 & -15 & 9 \end{bmatrix}\begin{matrix} 0 \\ 1 \\ 3 \end{matrix}$

$R_3 - \frac{5}{3}R_2 \rightarrow R_3$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -9 & 5 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}\begin{matrix} 0 \\ 1 \\ \frac{4}{3} \end{matrix}$

$\Rightarrow \frac{2}{3}z = \frac{4}{3}$

$= z = 2$

$\Rightarrow -9y + 5z = 1$

$= -9y + 10 = 1$

$= -9y = -9$

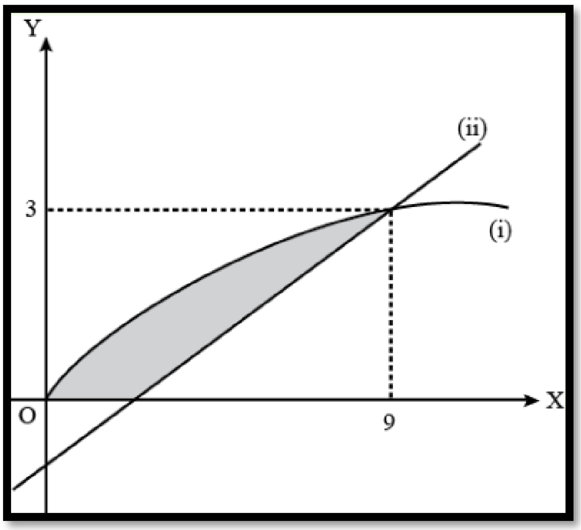
$= y = 1$

$\Rightarrow x + 3y - 2z = 0$

$= x + 3 - 4 = 0$

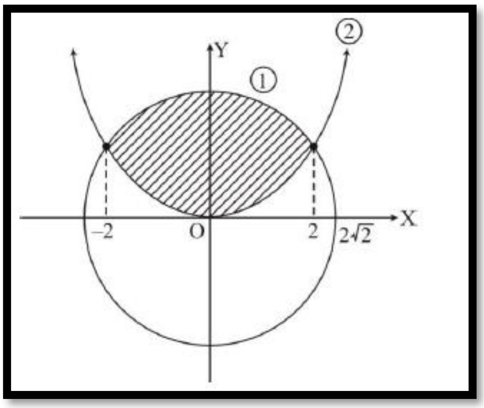
$= x = 1$

Sol 33.



We have $y = \sqrt{x}$ ...(i) , $2y + 3 = x$ ....(ii)
Solving (i) and (ii)
$\Rightarrow 2\sqrt{x} + 3 = x$
$= (x-3)^2 = 4x$
$= x^2 - 10x + 9 = 0$
$\therefore x = 1\ ,9\ \Rightarrow y = -1\ \ ,3$
But note that $y = \sqrt{x}$ so , $y > 0$
$\therefore$ the point of intersection is (9 , 3)
Now , required area = $\int_0^3 (2y+3)dy - \int_0^3 y^2 dy$
$= \left[\frac{(2y+3)^2}{2\times2} - \frac{y^3}{3}\right]_0^3$
$= \left[\frac{81}{2\times2} - \frac{27}{3}\right] - \left[\frac{9}{2\times2} - 0\right]$
$= \frac{72}{2\times2} - \frac{27}{3}$
$= 18 - 9$
$= 9$ sq, units

Or
$\{(x\ ,y\ ): x^2 + y^2 \leq 8\ , x^2 \leq 2y\}$
The given curves are $x^2 + y^2 = 8$ .....(1)
$x^2 = 2y$ .....(2)



Solving (1) and (2)

$\Rightarrow 8 - y^2 = 2y$
$= y = 2, -4$
$\Rightarrow y = 2\ (as\ y > 0)$
Substituting y = 2 in (2) , we get
$x^2 = 4 \Rightarrow x = -2\ or\ 2$
Required area = $\int_{-2}^{2} \sqrt{8 - x^2} dx - \int_{-2}^{2} \frac{x^2}{2} dx$
$= 2\left[\int_0^2 \sqrt{(2\sqrt{2})^2 - x^2}\, dx - \int_0^2 \frac{x^2}{2} dx\right]$
$= 2\left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\left(\frac{x}{2\sqrt{2}}\right)\right]_0^2 - \frac{1}{3}[x^3]_0^2$
$= 2\left[2 + 4 \times \frac{\pi}{4} - 0\right] - \frac{1}{3}[8 - 0]$
$= 4 + 2\pi - \frac{8}{3}$
$= 2\pi + \frac{4}{3}$ sq. units

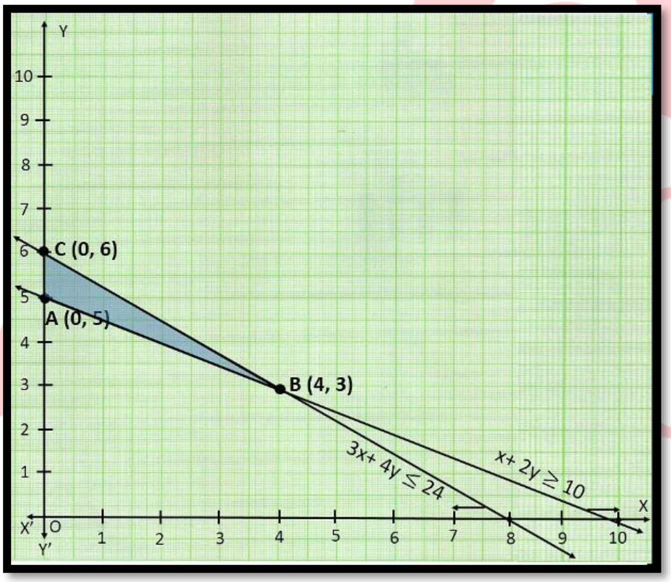
Sol 34.

$x + 2y \geq 10$

X	0	10
y	5	0

$3x + 4y \leq 24$

X	0	8
y	6	0



Corner points	Value of Z
(0 ,6)	3000
(0.5)	2500
(4,3)	2300 (min)

Hence Z is minimum at (4 , 3)

Sol 35.

Let A: team A is declared as a winner
Let B: team B is declared as a winner.
Let E: die shows six on the throw.

Clearly , $P(E)=\frac{1}{6}$

$P(\bar{E})=1-P(E)=1-\frac{1}{6}=\frac{5}{6}$

If captain of team A start then he may get a six in 1st throw or 3rd throw or 5th throw and so on.

$\therefore\ P(A)=P(E)+P\big((\bar{E})(\bar{E})(E)\big)+P\big((\bar{E})(\bar{E})(\bar{E})(\bar{E})E\big)+\cdots$

Using sum of infinite G.P. $S_{infinite}=\frac{a}{1-r}$

$=\frac{1}{6}+\left(\frac{5}{6}\right)^2.\frac{1}{6}+\left(\frac{5}{6}\right)^4.\frac{1}{6}+\cdots=\frac{\frac{1}{6}}{1-\frac{25}{36}}$

i.e., $P(A)=\frac{6}{11}$ and $P(B)=1-P(A)=1-\frac{6}{11}=\frac{5}{11}$

The decision of referee wasn't fair since team A has more chances of being declared a winner despite the fact that both the teams had secured same number of goals.

SECTION E

Sol 36.

(i)Total number of tickets = 50

Let event A = First ticket shows even number and B = Second ticket shows even number

Now , P(Both tickets show even number) = $P(A).P(B|A)=\frac{25}{50}.\frac{24}{49}=\frac{12}{49}$

(ii) Teacher ask Mivaan, what is the probability that both tickets drawn by Aadya shows odd number?

Let event A = First ticket shows odd number and B = Second ticket shows odd number

Now , P(Both tickets show odd number) = $P(A).P(B|A)=\frac{25}{50}.\frac{24}{49}=\frac{12}{49}$

(iii) Teacher ask Deepak, what is the probability that tickets drawn by Mivaan, shows a multiple of 4 on one ticket and a multiple 5 on other ticket

Required probability = P(One number is a multiple of 4 and other is a multiple of 5)

= P(multiple of 5 on first ticket and multiple of 4 on second ticket) +(P(multiple of 4 on first ticket and multiple of 5 on second ticket)

$=\frac{10}{50}.\frac{12}{49}+\frac{12}{50}.\frac{10}{49}=\frac{12}{245}+\frac{12}{245}=\frac{24}{245}$

Sol 37.

(i) The equation of plane passing through three non – collinear points is given by

$\Rightarrow\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_2 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$

$=\begin{vmatrix}x & y-1 & z-2\\ 3-0 & 4-1 & -1-2\\ 2-0 & 4-1 & 2-2\end{vmatrix}=0$

$=\begin{vmatrix}x & y-1 & z-2\\ 3 & 3 & -3\\ 2 & 3 & 0\end{vmatrix}$

$=x(0+9)-(y-1)(0+6)+(z-2)(9-6)=0$

$=9x-6y+3z-6=0$

$=3x-2y+z=0$

(ii) Height of tower = Perpendicular distance from the point (6 , 5 , 9) to the plane 3x -2y +z = 0

$=\left|\frac{18-10+9}{\sqrt{3^2+(-2)^2+1^2}}\right|=\frac{17}{\sqrt{14}}$ units

(iii) Direction ratio's of perpendicular are < 3 , -2 , 1 >

As we know that : Perpendicular is parallel to the normal to the plane

Since , perpendicular is passing through the point (6 , 5 , 9) , therefore its equation is $\frac{x-6}{3}=\frac{y-5}{-2}=\frac{z-9}{1}$

Sol 38.

(i)$I=\int\frac{e^{\tan^{-1}x}}{1+x^2}dx$

Let $\tan^{-1} x = t$

$\frac{dx}{1+x^2} = dt$

$I = \int e^t dt$

$= e^t + C$

$= e^{\tan^{-1} x} + C$

(ii)$I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

Let $\sin^{-1} x = t$

$= \frac{1}{\sqrt{1-x^2}} dx = dt$

$I = \int t dt$

$= \frac{t^2}{2} + C$

$= \frac{(\sin^{-1} x)^2}{2} + C$

(iii)$I = \int \frac{\sin x}{(1+\cos x)^2} dx$

Let 1 + cos x = t

= -sin x .dx = dt

$I = \int \frac{-dt}{t^2}$

$= \int -t^{-2} dt$

$= \frac{1}{t} + C$

$= \frac{1}{1+\cos x} + C$