SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Match the radiations listed in column-I with their uses listed in column-II correctly.

	Column-I		Column-II
(A)	UV rays	(P)	Physiotherapy
(B)	Infra red rays	(Q)	Treatment of cancer
(C)	X-rays	(R)	Lasic eye surgery
(D)	Microwave rays	(S)	Aircraft navigation

(1) A – S, B – P, C – R, D – Q
(2) A – R, B – P, C – Q, D – S
(3) A – Q, B – P, C – S, D – R
(4) A – R, B – P, C – S, D – Q

Answer (2)

Sol. UV rays are used for lasik eye surgery.
IR is used for physiotherapy.
X-rays are used for cancer treatment.
and Microwaves are used for aircraft navigation.

2. During an adiabatic process performed on a diatomic gas 725 J of work is done on the gas. The change in internal energy of the gas is equal to

(1) 495 J (2) 725 J
(3) 225 J (4) Zero

Answer (2)

Sol. For adiabatic process $Q = 0$

$\Delta U + W = 0$

$\Delta U - 725 = 0$

$\Delta U = 725$ J

3. Two balls are projected with equal speed (40 m/s), one at an angle of 30° and other at 60° with horizontal. Find the ratio of maximum heights of both the balls.

(1) $\frac{1}{4}$ (2) $\frac{3}{1}$
(3) $\frac{1}{3}$ (4) $\frac{4}{1}$

Answer (3)

Sol. $H_{\max} = \dfrac{v^2 \sin^2 \theta}{2g}$

$\Rightarrow \text{Ratio} = \dfrac{\sin^2 30^\circ}{\sin^2 60^\circ}$

$= \dfrac{1}{3}$

4. Find ionization energy of 2nd excited state of Li^{2+}. It is given that ionization energy of ground state of hydrogen atom is 13.6 eV.

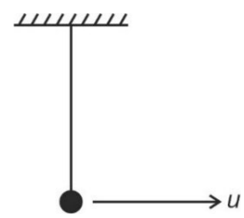
(1) 20.4 eV (2) 27.2 eV
(3) 6.8 eV (4) 13.6 eV

Answer (4)

Sol. $E = 13.6(3)^2\left[\dfrac{1}{3^2} - 0\right]$

= 13.6 eV

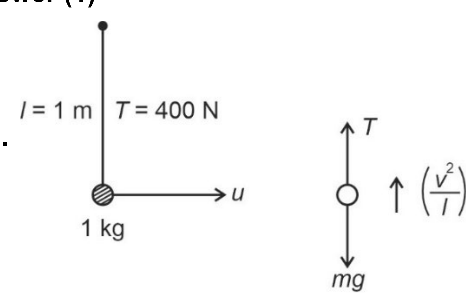
5. A ball of mass 1 kg is hanging from 1 m long inextensible string which can withstand maximum tension of 400 N. Find the maximum speed (u) that should be given to the ball.



(1) $\sqrt{390}$ m/s (2) $\sqrt{410}$ m/s
(3) 20 m/s (4) 22 m/s

Answer (1)

Sol.



$T = mg + \dfrac{mv^2}{l}$

$400\text{ N} = 10 + \dfrac{u^2}{l}$

$u = \sqrt{400 - 10} = \sqrt{390}$ m/s

6. Match the physical quantities given in column-I with the physical dimensions in column-II correctly.

	Column-I		Column-II
A.	Torque	(P)	$ML^{-1}T^{-2}$
B.	Stress	(Q)	ML^2T^{-2}
C.	Pressure gradient	(R)	$ML^{-2}T^{-2}$
D.	Angular momentum	(S)	ML^2T^{-1}

(1) A(S), B(P), C(R), D(Q)

(2) A(Q), B(P), C(R), D(S)

(3) A(P), B(S), C(R), D(Q)

(4) A(Q), B(P), C(S), D(R)

Answer (2)

Sol. $[\tau] = [r][F] = [L]\,[MLT^{-2}] = [ML^2T^{-2}]$

$$[\text{Stress}] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

$$[\text{Pressure gradient}] = \frac{[P]}{[Z]} = \frac{[ML^{-1}T^{-2}]}{[L^1]} = [ML^{-2}T^{-2}]$$

$[L] = [\tau][t] = [ML^2T^{-2}][T] = [ML^2T^{-1}]$

7. The equation of two simple harmonic motions are given by $y_1 = 10\sin(\omega t + \pi/3)$, and $y_2 = 5\left[\sin(\omega t) + \sqrt{3}\cos\omega t\right]$. The amplitude of resultant S.H.M is

(1) 10 m (2) 20 m

(3) 5 m (4) 15 m

Answer (2)

Sol. $y_1 = 5\left[\sin(\omega t) + \sqrt{3}\cos(\omega t)\right]$

$$= 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_1 + y_2 = y_{\text{resultant}} = 10\sin\left(\omega t + \frac{\pi}{3}\right) + 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_{\text{resultant}} = 20\sin\left(\omega t + \frac{\pi}{3}\right)$$

Amplitude = 20

8. Projectile 1 is thrown at angle 60° with horizontal with speed 40 m/s. Projectile 2 is thrown with same speed for same range as projectile 1 but at different angle. Sum of the maximum heights achieved by two projectiles is equal to (Take g = 10 m/sec^2)

(1) 40 m (2) 60 m

(3) 80 m (4) 160 m

Answer (3)

Sol. If angle of projection for projectile 1 is 60° then angle of projectile 2 is 30° for same range

$$H_1 = \frac{u^2\sin^2 60^\circ}{2g},\ H_2 = \frac{u^2\sin^2 30^\circ}{2g}$$

$$H_1 + H_2 = \frac{u^2}{2g} = \frac{40^2}{2\times 10} = 80 \text{ m}$$

9. A body has weight W on the surface of earth. Find the weight at a height 9 times the radius of earth.

(1) $\frac{W}{100}$ (2) $\frac{W}{81}$

(3) $\frac{W}{64}$ (4) $\frac{W}{121}$

Answer (1)

Sol. $g' = \dfrac{g_0}{\left(\frac{r}{R}\right)^2}$

$$= \frac{g_0}{10^2} = \frac{g_0}{100}$$

$$\Rightarrow\ W' = \frac{W}{100}$$

10. A wire is first coiled in n circular turns and current I is run through it. Now the same wire is again coiled in N circular turns and same current I is run through it. If B_1 and B_2 are the magnetic fields at centre of two coils respectively then $\frac{B_1}{B_2}$ is equal to

(1) $\sqrt{\frac{n}{N}}$ (2) $\left(\frac{n}{N}\right)^2$

(3) $\frac{n}{N}$ (4) $\frac{n^3}{N^3}$

Answer (2)

Sol. Let the length of wire is ℓ, thus the radius of first coil

$R_1 = \dfrac{\ell}{2\pi n}$ and the radius of second coil $R_2 = \dfrac{\ell}{2\pi N}$

So $B_1 = \dfrac{\mu_0 nI}{2\frac{\ell}{2\pi n}} = \dfrac{\mu_0\pi n^2 I}{\ell}$

And $B_2 = \dfrac{\mu_0 nI}{2\frac{\ell}{2\pi N}} = \dfrac{\mu_0\pi N^2 I}{\ell}$

So $\dfrac{B_1}{B_2} = \dfrac{n^2}{N^2}$

11. For a medium, it is given that

Young's modulus = 3.2×10^{10} N/m^2

Density = 8000 kg/m^3

Find speed of sound in this medium.

(1) 1000 m/s

(2) 2000 m/s

(3) 500 m/s

(4) 4000 m/s

Answer (2)

Sol. $v = \sqrt{\frac{Y}{\rho}}$

$= \sqrt{\frac{3.2 \times 10^{10}}{8000}}$

= 2000 m/s

12. When current of 4 amperes is made to run through a resistance of R ohms for 10 seconds, it produces heat energy of H units. Now if 16 amperes of current is made to flow through same resistance for 10 seconds than heat energy produced will be

(1) 16 H (2) 4 H

(3) 8 H (4) 2 H

Answer (1)

Sol. H = $i^2Rt = 4^2R \times 10 = 160R$

H′ = $I^2Rt = 16^2R \times 10 = 2560R$ = 16 H

13. Across an inductor of 5 mH an AC source with potential given as 268 sin(200 πt) Volts is used. The value of inductive reactance provided by inductor is equal to

(1) $2\pi\ \Omega$ (2) $\frac{\pi}{2}\ \Omega$

(3) $20\pi\ \Omega$ (4) $\pi\ \Omega$

Answer (4)

Sol. $X_L = \omega L = 200\pi \times 5 \times 10^{-3}$

$= \pi\ \Omega$

14. A lens of refractive index 1.5 and focal length 15 cm in air is submerged in water. Change in focal length of lens is $\left(r = \frac{4}{3}\right)$

(1) 45 cm (2) 60 cm

(3) 30 cm (4) 10 cm

Answer (1)

Sol. When lens is placed in air,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{15} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(1)$$

When submerged in water $\left(\mu = \frac{4}{3}\right)$

$$\Rightarrow \frac{1}{f'} = \left(\frac{1.5}{\left(\frac{4}{3}\right)} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(2)$$

$\frac{\text{Equation (1)}}{\text{Equation (2)}}$

$$\frac{f'}{15} = \left(\frac{0.5}{0.5} \times 4\right)$$

f' = 60 cm

$\Delta f = f' - f = 60 - 15 = 45$ cm

15. In a moving coil galvanometer, number of turns in the coil are increased to increase the current sensitivity by 50%. Find percentage change in voltage sensitivity.

(1) –50%

(2) 50%

(3) No change

(4) 25%

Answer (3)

Sol. Current sensitivity

$$\frac{\theta}{I} = \frac{nAB}{K}$$

Voltage sensitivity $= \left(\frac{nAB}{KR}\right)$

As current sensitivity increases by 50%

So number of turns increases by 50%

Resistance also increases by 50%

Therefore, voltage sensitivity remains constant.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In a series *RLC* circuit, R = 80 Ω, X_L = 100 Ω, X_C = 40 Ω. If the source voltage is 2500cos(628t) Volts, find peak current in the circuit (in Amperes)

Answer (25.00)

Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

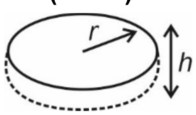
$= \sqrt{80^2 + (100-40)^2}$ = 100 Ω

$\Rightarrow I_o = \frac{V_o}{Z}$ = 25 A

22. Two discs of same mass, radii r_1 and r_2, thickness 1 mm and 0.5 mm have densities in the ratio 3 : 1. The ratio of their moment of inertia about diameter is 1 : x. Find x

Answer (06.00)

Sol.



Mass of both disc is equal

So ⇒ $M_1 = M_2$

$\Rightarrow (\pi r_1^2)h_1\rho_1 = (\pi r_2^2)h_2\rho_2$

$\Rightarrow r_1^2 \times \frac{h_1}{h_2} \times \frac{\rho_1}{\rho_2} = r_2^2$

$\Rightarrow r_1^2 \times 2 \times \frac{\rho_1}{\rho_2} = r_2^2$ $\qquad \frac{\rho_1}{\rho_2} = 3 \Rightarrow \frac{\rho_2}{\rho_1} = \frac{1}{3}$

$\Rightarrow \frac{r_1^2}{r_2^2} = \left(\frac{\rho_2}{2\rho_1}\right) = \left(\frac{1}{6}\right)$

Ratio of M.O.I = $\frac{\frac{1}{4}Mr_1^2}{\frac{1}{4}Mr_2^2} = \left(\frac{r_1^2}{r_2^2}\right) = \left(\frac{1}{6}\right)$

23. A body moving horizontally has an initial speed of 20 m/s. Due to friction, body stops after 5 seconds. If mass of body is 5 kg, co-efficient of friction is $\frac{x}{5}$. Find x. Take g = 10 m/s^2.

Answer (02.00)

Sol. $v = u + at$

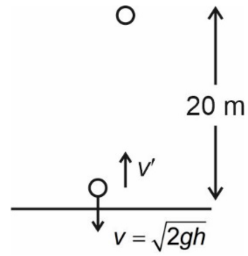
$\Rightarrow 0 = 20 + (-\mu g)(5)$

$\Rightarrow \mu = 0.4$

24. A ball was dropped from 20 m height from ground. Find the height (in m) upto which it rises after the collision. $\left(\text{use } e = \frac{1}{2}, g = 10 \text{ m/s}^2\right)$

Answer (05.00)

Sol.



$v' = ev$

$h' = \frac{(v')^2}{2g} = \frac{e^2v^2}{2g} = e^2h = (0.5)^2 \times 20$

$= \frac{20}{4} = 5$ m

h' = 5 m

25. A particle is in uniform circular motion with time period 4 s and radius $\sqrt{2}$ m. Find the magnitude of displacement (in m) in 3 s.

Answer (02.00)

Sol. $\theta = \frac{3}{4} \times 2\pi = \frac{3\pi}{2}$

$\Rightarrow |\text{Displacement}| = \sqrt{2}R$

= 2m

26. Two wavelengths λ_1 = 600 nm and λ_2 = 800 nm are used in a YDSE experiment. Their maximas coincide at certain locations on the screen. Find the minimum separation (in mm) between such a location and central maxima. It is given that d = 0.35 mm & D = 7 m

Answer (48.00)

Sol. $n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$

$\Rightarrow 6n_1 = 8n_2$

⇒ Minimum n_1 = 4

& n_2 = 3

⇒ Minimum separation = $\frac{4 \times 600 \text{ nm} \times 7 \text{ m}}{0.35 \text{ mm}}$

= 48

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which one of the following species is linear in shape?

(1) I_3^- (2) I_3^+

(3) ICl_3 (4) ICl_2^+

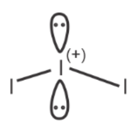
Answer (1)

Sol. The shapes of the given species are

I_3^- :

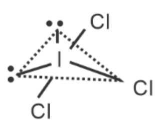
(Linear)

I_3^+ :



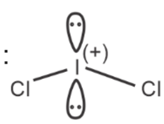
(Angular bent)

ICl_3 :



(T-shape)

ICl_2^+ :



(Angular bent)

2. For a given hydrocarbon, 11 moles of O_2 is used and produces 4 moles of H_2O. Then the formula of the hydrocarbon is

(1) $C_{11}H_8$ (2) C_9H_8

(3) $C_{11}H_{16}$ (4) C_6H_{14}

Answer (2)

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$\frac{y}{2} = 4 \quad \therefore \quad y = 8$

$x + \frac{8}{4} = 11$

$\therefore$ x = 9

$\therefore$ Hydrocarbon will be = C_9H_8

3. Which of the following plays an important role in neuromuscular functions?

(1) Ca (2) Mg

(3) Be (4) Li

Answer (1)

Sol. Calcium plays an important role in neuromuscular functions.

4. Which of the following compound contain maximum number of chlorine atoms?

(1) Chloropicrin

(2) Chloral

(3) Gammexane

(4) Freon-12

Answer (3)

Sol.

Compounds	**Number of Chlorine atoms**
Chloropicrin	3
Chloral	3
Gammexane	6
Freon-12	2

5. Decreasing order of Lewis acid character is

(1) $BF_3 > BCl_3 > BBr_3 > BI_3$

(2) $BI_3 > BBr_3 > BCl_3 > BF_3$

(3) $BF_3 > BCl_3 > BI_3 > BBr_3$

(4) $BI_3 > BCl_3 > BF_3 > BBr_3$

Answer (2)

Sol. Extent of back bonding

$$\frac{BF_3}{2p-2p} > \frac{BCl_3}{2p-3p} > \frac{BBr_3}{2p-4p} > \frac{BI_3}{2p-5p}$$

6. pH of acid rain is 5.6. Which of the following reaction is involved in acid rain?

(1) $H_2O + SO_2 + O_2 \rightarrow H_2SO_4$

(2) $N_2 + O_2 + H_2O \rightarrow HNO_3$

(3) $N_2O + O_2 + H_2O \rightarrow HNO_3$

(4) None of these

Answer (1)

Sol. The correct answer of this equation is option 1

7. Which of the following metals of F-block have half-filled *f*-subshell?

(a) Samarium (Sm)

(b) Gadolinium (Gd)

(c) Europium (Eu)

(d) Terbium (Tb)

(1) (a) and (b)

(2) (b) and (c)

(3) (c) and (d)

(4) (a) and (c)

[Atomic numbers : Sm = 62, Eu = 63, Gd = 64, Tb = 65]

Answer (2)

Sol. The valence shell electronic configuration of the given F-Block metals are

(a) Sm : $4f^6 6s^2$

(b) Gd : $4f^7 sd^1 6s^2$

(c) Eu : $4f^7 6s^2$

(d) Tb : : $4f^9 6s^2$

Therefore, Gd and Eu have half-filled f-subshell.

8. If Ionisation energy of H-atom is 13.6 eV. Find out Ionisation energy of Li^{2+} ions.

(1) 54.4 eV (2) 122.4 eV

(3) 13.6 eV (4) 3.4 eV

Answer (2)

Sol. IE $= 13.6 \times Z^2$

$= 13.6 \times (3)^2$

$= 13.6 \times 9$

$= 122.4$ eV

9. Which of the following compound is not a disinfectant?

(1) Chloroxylenol

(2) Bithionol

(3) Terpineol

(4) Peracetic acid

Answer (4)

Sol. Chloroxylenol, bithionol and terpineol are the disinfectants.

10. A reaction follows 1^{st} order kinetics with rate constant (k) = 20 min^{-1}. Calculate the time required to reach to concentration to $\frac{1}{32}$ times of initial concentration

(1) 0.17325 min (2) 1.7325 min

(3) 17.325 min (4) 173.25 min

Answer (1)

Sol. $C = \frac{C_O}{(2)^n} = \frac{C_O}{32}$

n = 5

$t = 5t_{1/2}$

$= \frac{5 \times 0.693}{20} = \frac{0.693}{4}$

= 0.17325 min

11. If solubility of AgCl in aqueous solution is 1.434×10^{-3} M, then find the value of [$-\log K_{sp}$], where K_{sp} is the solubility product of AgCl.

(1) 3.7 (2) 5.7

(3) 6.7 (4) 7.7

Answer (2)

Sol. Solubility of AgCl in water = 1.434×10^{-3} M

Solubility product (K_{sp}) of AgCl = $(1.434 \times 10^{-3})^2$

$\therefore$ $K_{sp} = 2 \times 10^{-6}$

$-\log K_{sp} = -\log 2 + 6$

$= 5.7$

12. Consider the following combination of n, l and m values.

(i) n = 3; l = 0; m = 0

(ii) n = 4; l = 0; m = 0

(iii) n = 3; l = 1; m = 0

(iv) n = 3; l = 2; m = 0

The correct order of energy of the corresponding orbitals for multielectron species is

(1) (ii) > (i) > (iv) > (iii)

(2) (iv) > (ii) > (iii) > (i)

(3) (i) > (iii) > (iv) > (ii)

(4) (iv) > (iii) > (i) > (ii)

Answer (2)

Sol. (i) n = 3; l = 0; m = 0 ⇒ 3*s* orbital

(ii) n = 4; l = 0; m = 0 ⇒ 4*s* orbital

(iii) n = 3; l = 1; m = 0 ⇒ 3*p* orbital

(iv) n = 3; l = 2; m = 0 ⇒ 3*d* orbital

The correct order of energy is 3*d* > 4*s* > 3*p* > 3*s*

Hence correct answer is (2)

13. Two metals are given,

Metal – 1 Work function = 4.8 ev

Metal – 2 Work function = 2.8 ev

Photons of wavelength 350 nm are incident on both metals separately which metal will eject electrons at this wavelength?

(1) Metal – 1 only

(2) Metal – 2 only

(3) Both metal – 1 and metal – 2

(4) None of metal – 1 and metal – 2

Answer (2)

Sol. $E_{photon} = \frac{12400}{3500} = 3.54$ ev

$W_{metal-1} > E_{photon} > W_{metal-2}$

⇒ Only metal – 2 will emit photons

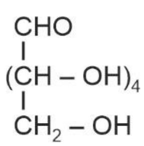
14. A biomolecule gives following observations

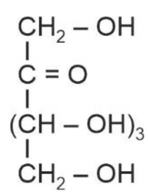
(i) With Br_2/H_2O it gives monocarboxylic acid.

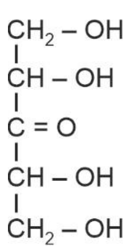
(ii) With acetate it gives tetraacetate.

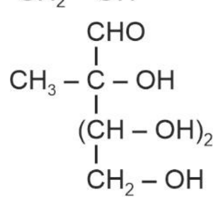
(iii) With HI/RedP it gives isopentane.

The correct structure of the biomolecule is

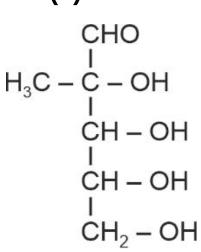
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

gives monocarboxylic acid with Br_2/H_2O and tetraacetate with acetate and isopentane with RedP/HI.

15. Which of the following has more relative lowering in vapour pressure at the same temperature.

(1) 0.1 M urea (2) 0.1 M NaCl

(3) 0.1 M sucrose (4) 0.1 M $CaCl_2$

Answer (4)

Sol. Relative lowering in vapour pressure is a colligative property and colligative property depends only on the amount of solute from the given, $CaCl_2$ will have maximum amount, hence its solution will show maximum relative lowering in vapour pressure hence the correct answer is option (4)

16. **Assertion :** First ionisation energy of 4d series elements is always greater than those of 3d series elements.

Reason : 4d series elements have much more nuclear charge than those of 3d series elements.

(1) Assertion is correct but Reason is incorrect.

(2) Assertion is incorrect but Reason is correct.

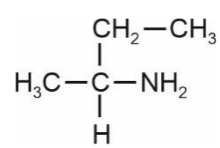
(3) Both the Assertion and Reason are correct.

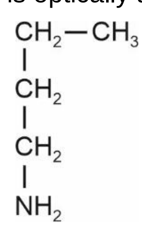
(4) Both the Assertion and Reason are incorrect.

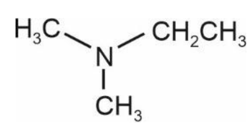
Answer (2)

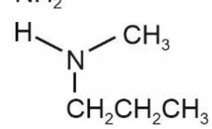
Sol. The first ionisation energy of 4d series elements is not always greater than those of 3d series elements. So Assertion is incorrect. The Reason is correct because 4d series elements have much more nuclear charge than those of 3d series elements.

17. What is the structural formula of compound $C_4H_{11}N$, which reacts with HNO_2 and is optically active?

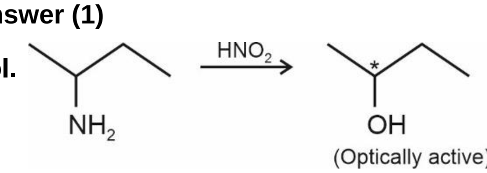
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

18. Energy of a radiation $\varepsilon = \frac{hc}{\lambda_{absorb}}$. If ε = +96 kJ/mole thus find $\lambda_{absorbed}$ (in Å)

(1) 12471Å (2) 124.71Å

(3) 1247.1Å (4) 1.2471Å

Answer (1)

Sol. $\frac{6.626\times10^{-34}\times3\times10^{8}}{96\times10^{3}}\times6.023\times10^{23}=\lambda$

λ = 1. 2471 × 10^{-6} M

= 12471 Å

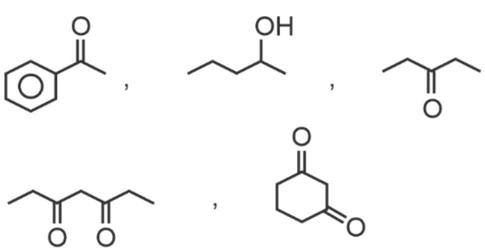
19.

20.

SECTION - B

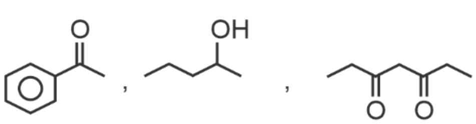
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

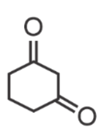
21. How many of the following compound(s) can give iodoform test?



Answer (4)

Sol.



and 

will give iodoform test

22. For the given reaction

$C + O_2 \longrightarrow CO_2(g)$

12 gm of C is reacted with 48 gm of O_2 to give CO_2. If volume of CO_2 gas produced at STP is t litre. Find out 2t (closest integer).

Given: Molar volume at STP = 22.4 L/mole

Answer (45)

Sol.

C	+	O_2	$\longrightarrow$	CO_2
12 gm		48 gm		—
1 mole		$\frac{3}{2}$ mole		—
↓		↓		↓
0				1 mole

Volume = t = 22.4 litre

2t = 44.8 litre

≈ 45 L

23. The non-stoichiometry compounds $M_{0.83}O$. M exists in 2 states +2 and +3. Calculate the % of M^{2+} ion in the compound. (Round off to nearest integer)

Answer (59)

Sol. Let M^{2+} is x.

Then M^{3+} will be 83 – x.

$\therefore$ x + 2 + (83 – x) × 3 = 200

x = 49

$\therefore \quad M^{2+} = \frac{49}{83}\times100 \simeq 59\%$

24. The resistivity of 0.8 M solution of an electrolyte is 5 × 10^{-3} Ω.cm. If λ_m is 2.5×10^x. Find out x.

Answer (05.00)

Sol. $k = \frac{10^3}{5}$ S cm^{-1}

$$\lambda_m = \frac{k\times1000}{m} = \frac{\frac{10^3}{5}\times1000}{0.8}$$

$$\Rightarrow \frac{200\times10^3}{0.8}$$

$$= \frac{2}{0.8}\times10^5 = 2.5\times10^5$$

x = 5

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$. There is a $\vec{u}$ such that $\vec{u} \times \vec{a} = \vec{b} \times \vec{c}$ & $\vec{u} \cdot \vec{a} = 0$. Find $25|\vec{u}|^2$

(1) 560 (2) $\frac{925}{7}$

(3) 446 (4) 330

Answer (2)

Sol. $(\vec{u} \times \vec{a})^2 + (\vec{u} \cdot \vec{a})^2 = |\vec{u}|^2 |\vec{a}|^2$

$|\vec{b} \times \vec{c}|^2 + 0 = |\vec{u}|^2 \cdot 14$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 1 & 2 & 2 \end{vmatrix}$$

$= \hat{i}(-8) - \hat{j}(-1) + \hat{k}(3)$

$= -8\hat{i} + \hat{j} + 3\hat{k}$

$|\vec{b} \times \vec{c}| = \sqrt{74}$

$74 + 0 = 14|u|^2$

$\Rightarrow \quad 25|u|^2 = \frac{74}{14} \cdot 25$

$= \frac{925}{7}$

2. The range of $y = \frac{x^2 + 2x + 1}{x^2 + 8x + 1}$ is ($x \in R$)

(1) $\left(-\infty, -\frac{2}{3}\right] \cup [2, \infty)$ (2) $(-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

(3) $(-\infty, \infty)$ (4) $\left(-\infty, \frac{-2}{5}\right] \cup [1, \infty)$

Answer (2)

Sol. $y = \frac{x^2 + 2x + 1}{x^2 + 8x + 1}$

$\Rightarrow \quad x^2(y - 1) + x(8y - 2) + y - 1 = 0, x \in R$

If $y \neq 1$

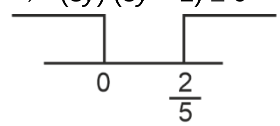
$D \geq 0$

$4(4y - 1)^2 - 4(y - 1)(y - 1) \geq 0$

$\Rightarrow \quad (4y - 1)^2 - (y - 1)^2 \geq 0$

$\Rightarrow \quad (4y - 1 - (y - 1))(4y - 1 + y - 1) \geq 0$

$\Rightarrow \quad (3y)(5y - 2) \geq 0$



$y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right) - \{1\}$

If $y = 1$

$6x = 0 \Rightarrow x = 0$

$\therefore \quad y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

3. If $a, b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$, then

(1) R_1 is symmetric but R_2 is not

(2) R_2 is symmetric but R_1 is not

(3) R_1 and R_2 are both symmetric

(4) R_1 and R_2 are both transitive

Answer (1)

Sol. $R_1 \rightarrow a^2 - b^2 \in I$

as $a, b \in Z$ if $a^2 - b^2 \in Z$ then $b^2 - a^2 \in Z$

Also $a^2 - b^2 \in Z$ & $b^2 - c^2 \in Z \Rightarrow a^2 - c^2 \in Z$

$\therefore \quad R_1$ is symmetric as well as transitive.

$R_2 \rightarrow 2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$

then $2 + \frac{b}{a} > 0$, then it is not necessary $\frac{a}{b} > -2$

$\therefore \quad R_2$ is not symmetric.

Now if $2 + \frac{a}{b} > 0$ & $2 + \frac{b}{c} > 0$

then $2 + \frac{a}{c}$ can be positive or negative.

4. If $\int \frac{xdx}{x^2+x+2}; Af(x)+Bg(x)+C$ where C is constant of integration, then A + 2B is equal to

(1) 1 (2) 0

(3) –1 (4) –2

Answer (2)

Sol. Let $x = \alpha(2x+1)+\beta$

$$\alpha = \frac{1}{2}, \beta = \frac{-1}{2}$$

$$I = \int \frac{xdx}{\sqrt{x^2+x+2}} = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+2}} - \frac{1}{2}\int \frac{1}{\sqrt{x^2+x+2}}$$

$$= \frac{1}{2} \times 2\sqrt{x^2+x+2} - \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}} + C$$

$$= \sqrt{x^2+x+2} - \frac{1}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+2}\right| + C$$

$$I = Af(x)+Bg(x)+C$$

$$\Rightarrow \quad A = 1, B = \frac{-1}{2}$$

5. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$ is equal to

(1) 27 (2) $\frac{27}{2}$

(3) 18 (4) 6

Answer (3)

Sol. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$

$$= \lim_{x\to\infty} \frac{2\cdot\left(\sqrt{3}\right)^6 x^6 + \text{.......(lower power of } x)}{2x^6 + \text{........(lower power of } x)}$$

= 27

6. Foot of perpendicular from origin to a plane which cuts the coordinate axes at A, B, C is (2, a, 4). Area of tetrahedron $OABC$ is 144 m^2. Which of the following points does not lie on plane?

(1) (2, 2, 4)

(2) (0, 3, 4)

(3) (1, 1, 5)

(4) (5, 5, 1)

Answer (2)

Sol. Equation of required plane:

$2(x – 2) + a(y – a) + 4(z – 4) = 0$

$\Rightarrow$ $2x + ay + 4z = 20 + a^2$

$$A\left(10+\frac{a^2}{2},0,0\right), B\left(0,\frac{20+a^2}{a},0\right), C\left(0,0,\frac{20+a^2}{4}\right)$$

Area of tetrahedron $= \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 144$

$$\Rightarrow \quad \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$$

$\Rightarrow$ $(20 + a^2)^3 = 144 \times 48a$

$\Rightarrow$ $a = 2$

$\therefore$ Equation of plane: $2x + 2y + 4z = 24$

$\Rightarrow$ $x + y + 2z = 12$

(0, 3, 4) does not lie on plane

7. If $z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$, then z is

(1) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(2) $\frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(4) $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

Answer (3)

Sol. $z = \dfrac{i-1}{\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}} = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = (i-1)\cdot e^{-\frac{\pi}{3}}$

$\Rightarrow \quad z = (i-1)\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$

$= (i-1)\left(\frac{1}{2} - \frac{\sqrt{3}\,i}{2}\right)$

$= \frac{1}{2}\left(i + \sqrt{3} - 1 + \sqrt{3}\,i\right)$

$\Rightarrow \quad \frac{\sqrt{3}-1}{2} + i\left(\frac{\sqrt{3}+1}{2}\right)$

$\therefore$ arg(z) = $\frac{5\pi}{12}$ & $|z| = \sqrt{2}$

$\therefore \quad z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

8. Given that $\theta \in [0, 2\pi]$, the largest interval of values of θ which satisfy the inequation $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) \geq 0$ is

(1) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(3) $[0, \pi]$ (4) $\left[\frac{\pi}{2}, \frac{5\pi}{4}\right]$

Answer (1)

Sol. $\sin^{-1}(\sin\theta) - \left(\frac{\pi}{2} - \sin^{-1}\sin\theta\right) \geq 0$

$\Rightarrow \quad \sin^{-1}\sin\theta \geq \frac{\pi}{4}$

$\frac{1}{\sqrt{2}} \leq \sin\theta \leq 1$

$\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

9. If $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, where $r \in \{p, q, \sim p, \sim q\}$, then the number of values of r is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (2)

Sol. $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$

$\Rightarrow \quad ((\sim p \vee \sim q) \vee (r \vee q)) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad ((\sim p \vee r \vee (q \vee \sim q)) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad T \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \quad \sim p \vee \sim r \vee q$

For the above statement to be tautology r can be $\sim p$ or q

$\therefore$ Two values of r are possible

10. If $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$, Given that $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$. Find $\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}}$

(1) 3 (2) $-\sqrt{3}$

(3) 1 (4) –3

Answer (2)

Sol. $\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{0}$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2\left(4|b|^2 + 9|c|^2 + 12\vec{b}\cdot\vec{c}\right)$

$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b} + 3\vec{c})$

$\frac{|\vec{a} \times \vec{c}|}{|\vec{a} \cdot \vec{b}|} = \frac{2|\vec{b} \times \vec{c}|}{2\vec{b}\cdot\vec{b} + 3\vec{c}\cdot\vec{b}}$

$|\vec{b} \times \vec{c}|^2 = |b|^2|c|^2 - (\vec{b}\cdot\vec{c})^2$

$= \frac{1}{4}\cdot 4 - \left(1\left(-\frac{1}{2}\right)\right)^2$

$= 1 - \frac{1}{4} = \frac{3}{4}$

$\frac{|\vec{a} \times \vec{c}|}{|\vec{a} \cdot \vec{b}|} = \frac{\sqrt{3}}{2\cdot\frac{1}{4} - \frac{3}{2}} = \frac{\sqrt{3}}{-1}$

11. Number of 7 digit odd numbers formed using 7 digits 1, 2, 2, 2, 3, 3, 5 will be

(1) 80 (2) 420

(3) 240 (4) 140

Answer (3)

Sol. Even numbers formed

_ _ _ _ _ _ _ 2

Number of ways $= \frac{6!}{2!2!} = 180$

Total numbers $= \frac{7!}{3!2!} = \frac{720 \times 7}{12} = 420$

Odd numbers = 420 – 180

= 240

12. The minimum value of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[x]$ denotes greatest integer function, is

(1) $\frac{3}{4}$ (2) $\frac{5}{4}$

(3) $\frac{1}{4}$ (4) 0

Answer (1)

Sol. $x^2 - x + 1 = g(x)$ attains minimum value

when $x = \frac{1}{2}$

So, minimum value of $f(x)$ will be at $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{3}{4} + 0$

$= \frac{3}{4}$

13. If for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $(\pm 4, 0)$ are foci and $e = \sqrt{3}$. Then length of latus rectum is

(1) 8 (2) $\frac{16}{\sqrt{3}}$

(3) 4 (4) $2\sqrt{3}$

Answer (2)

Sol. $ae = 4$

$a = \frac{4}{\sqrt{3}}$

$LR = \frac{2b^2}{a}$

$= \frac{2}{a} a^2(e^2 - 1)$

$= 2a(e^2 - 1)$

$= \frac{8}{\sqrt{3}}(3 - 1)$

$= \frac{16}{\sqrt{3}}$

14. If $[\alpha\ \beta\ \gamma] \begin{bmatrix} 5 & 6 & 8 \\ 6 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix} = [0\ 0\ 0]$

Where (α, β, γ) be a point on $2x + 5y + 3z = 5$ then $6\alpha + 5\beta + 9\gamma = ?$

(1) 20 (2) $\frac{20}{3}$

(3) 21 (4) 7

Answer (2)

Sol. $5\alpha + 6\beta - \gamma = 0$

$6\alpha + 3\beta + 3\gamma = 0$

$8\alpha + 8\beta = 0 \Rightarrow \boxed{\alpha = -\beta}$

& $\boxed{\beta = \gamma}$

$\alpha = k, \beta = k, \gamma = -k$

$2(k) + 5(-k) + 3(-k) = 5$

$k = -\frac{5}{6}$

$\alpha = -\frac{5}{6}, \beta = \frac{5}{6}, \gamma = \frac{5}{6}$

$6\alpha + 5\beta + 9\gamma = -5 + \frac{25}{6} + \frac{45}{6}$

$= \frac{40}{6} = \frac{20}{3}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Coefficient of x^{-6} in expansion of $\left(\frac{4x}{5} - \frac{5}{2x^2}\right)^9$ is

Answer (–5040)

Sol. $T_{r+1} = {}^9C_r \left(\frac{4x}{5}\right)^{9-r} \left(\frac{-5}{2x^2}\right)^r$

$9 - 3r = -6$

$\boxed{r = 5}$

Coefficient of $x^{-6} = {}^9C_5 \left(\frac{4}{5}\right)^4 \left(-\frac{5}{2}\right)^5$

$= \frac{9!}{5!4!} \frac{4^4}{5^4} \left(\frac{-5^5}{2^5}\right)$

$= 6 \cdot 7 \cdot 3 \cdot 8(-5)$

$= -5040$

22. The value of sum

$1.1^2 - 2.3^2 + 3.5^2 - 4.7^2 \ldots + 15.(29)^2$ is

Answer (6952)

Sol. Separating odd placed and even placed terms we get

$S = (1.1^2 + 3.5^2 + \ldots 15.(29)^2) - (2.3^2 + 4.7^2 + \ldots + 14.(27)^2)$

$$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2$$

Applying summation formula we get

= 29856 – 22904 = 6952

23. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2 × *2* matrix such that *a*, *b*, *c*, *d* $\in$ {0, 1, 2, 3, 4, }. The number of matrices *A* such that sum of elements of *A* is a prime number lying between 2 and 13 is

Answer (204)

Sol. As given $a + b + c + d = 3$ or 5 or 7 or 11

if sum = 3

$(1 + x + x^2 + \ldots + x^4)^4 \longrightarrow x^3$

$(1 - x^5)^4 (1 - x)^{-4} \longrightarrow x^3$

$\therefore \quad 4 + {}^{3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1 - 4x^5)(1 - x)^{-4} \longrightarrow x^5$

$\Rightarrow \quad {}^{4+5-1}C_5 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1 - 4x^5)(1 - x)^{-4} \to x^7$

$\Rightarrow \quad {}^{4+7-1}C_7 - 4.\,{}^{4+2-1}C_2 = {}^{10}C_7 - 4.\,{}^5C_2 = 80$

If sum = 11

$(1 - 4x^5 + 6x^{10})(1 - x)^{-4} \to x^{11}$

$\Rightarrow \quad {}^{4+11-1}C_{11} - 4.{}^{4+6-1}C_6 + 6.{}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4.{}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

24. If $\dfrac{{}^{2n+1}P_{n-1}}{{}^{2n+1}P_n} = \dfrac{11}{21}$, then $n^2 + n + 15$ equals

Answer (45)

Sol. $\dfrac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \dfrac{11}{21}$

$\Rightarrow \quad \dfrac{(2n+1)(2n)}{(n+2)(n+1)n} = \dfrac{11}{21}$

$\Rightarrow \quad \dfrac{2n+1}{(n+1)(n+2)} = \dfrac{11}{42}$

$\Rightarrow \quad n = 5$

$\Rightarrow \quad n^2 + n + 15 = 25 + 5 + 15 = 45$

25. $\displaystyle\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha} - \sqrt{x}}\,dx = \frac{16 + 20\sqrt{2}}{15}$

then α is equal to

Answer (02.00)

Sol. $\displaystyle\int_0^{\alpha} \frac{x}{\alpha}\left(\sqrt{x+\alpha} + \sqrt{x}\right)$

$\displaystyle\int_0^{\alpha} \frac{1}{\alpha}\left[(x+\alpha)^{3/2} - \alpha(x+\alpha)^{1/2} + x^{3/2}\right]$

$\displaystyle\frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{5/2} - \alpha\frac{2}{3}(x+\alpha)^{3/2} + \frac{2}{5}x^{5/2}\right]_0^{\alpha}$

$= \dfrac{1}{\alpha}\left(\dfrac{2}{5}(2\alpha)^{5/2} - \dfrac{2\alpha}{3}(2\alpha)^{3/2} + \dfrac{2}{5}\alpha^{5/2} - \dfrac{2}{5}\alpha^{5/2} + \dfrac{2}{3}\alpha^{5/2}\right)$

$= \dfrac{1}{\alpha}\left(\dfrac{2^{7/2}\alpha^{5/2}}{5} - \dfrac{2^{5/2}\alpha^{5/2}}{3} + \dfrac{2}{3}\alpha^{5/2}\right)$

$= \alpha^{3/2}\left(\dfrac{2^{7/2}}{5} - \dfrac{2^{5/2}}{3} + \dfrac{2}{3}\right)$

$= \dfrac{\alpha^{3/2}}{15}\left(3 \cdot 2^{7/2} - 5 \cdot 2^{5/2} + 10\right)$

$= \dfrac{\alpha^{3/2}}{15}\left(24\sqrt{2} - 20\sqrt{2} + 10\right) = \dfrac{\alpha^{3/2}}{15}\left(4\sqrt{2} + 10\right)$

$= \dfrac{16 + 20\sqrt{2}}{15}$

$\Rightarrow \quad \alpha = 2$