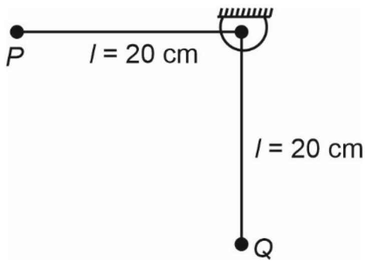
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Bob P is released from the position of rest at the moment shown. If it collides elastically with an identical bob Q hanging freely then velocity of Q, just after collision is (g = 10 m/s^2)



(1) 1 m/s (2) 4 m/s

(3) 2 m/s (4) 8 m/s

Answer (3)

Sol. Velocity of P just before collision is $= \sqrt{2gl}$

= 2 m/sec

As collision is elastic and the mass of P and Q is equal therefore just after collision velocity of P is 0 and that of Q is 2 m/sec.

2. Choose the option showing the correct relation between Poisson's ratio (σ), Bulk modulus (B) and modulus of rigidity (G).

(1) $\sigma = \dfrac{3B-2G}{2G+6B}$ (2) $\sigma = \dfrac{6B+2G}{3B-2G}$

(3) $\sigma = \dfrac{9BG}{3B+G}$ (4) $B = \dfrac{3\sigma-3G}{6\sigma+2G}$

Answer (1)

Sol. $E = 2G(1+\sigma)$ ....(1)

$E = 3B(1-2\sigma)$ ....(2)

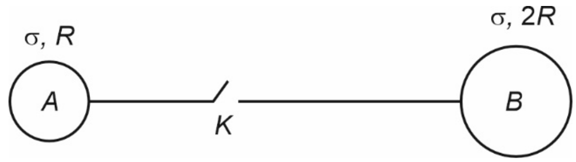
$$1 = \frac{2G}{3B}\left(\frac{1+\sigma}{1-2\sigma}\right)$$

$\Rightarrow 3B - 6B\sigma = 2G + 2G\sigma$

$\Rightarrow 3B - 2G = \sigma\,(2G + 6B)$

$$\sigma = \left(\frac{3B-2G}{2G+6B}\right)$$

3. Two conducting solid spheres (A & B) are placed at a very large distance with charge densities and radii as shown:



When the key K is closed, find the ratio of final charge densities.

(1) 4 : 1 (2) 1 : 2

(3) 2 : 1 (4) 1 : 4

Answer (3)

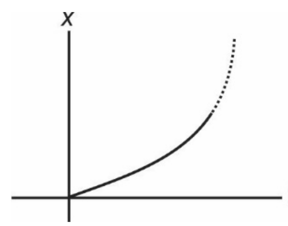
Sol. Final potential is same

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{2R} \quad ....(1)$$

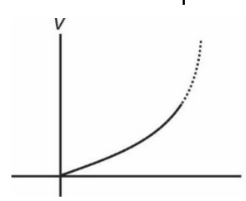
Also, $Q_1 + Q_2 = \sigma\cdot 4\pi R^2 + \sigma\cdot 4\pi(2R)^2$ .....(2)

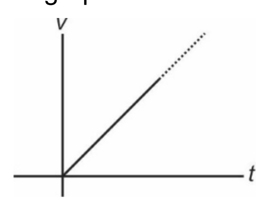
$$\Rightarrow \frac{\sigma_1}{\sigma_2} = 2.$$

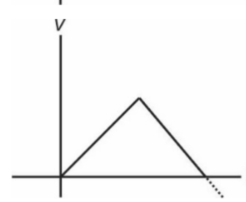
4. Position-time graph for a particle is parabolic and is as shown:

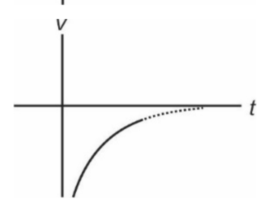


Choose the corresponding $v - t$ graph

(1)


(2)


(3)


(4)


Answer (2)

Sol. Since $x \propto t^2$

$$\Rightarrow \quad v = \frac{dx}{dt} \propto t'$$

$\Rightarrow$ Option 2 is correct

5. For a system undergoing isothermal process, heat energy is supplied to the system. Choose the option showing correct statements

(a) Internal energy will increase

(b) Internal energy will decrease

(c) Work done by system is positive

(d) Work done by system is negative

(e) Internal energy remains constant

(1) (a), (c), (e) (2) (b), (d)

(3) (c), (e) (4) (a), (d), (e)

Answer (3)

Sol. For isothermal process,

$dT = 0$

so, $dU = 0 \Rightarrow$ Internal energy remains same

$dQ = dW$

as dQ is positive,

so dW is positive

6. The heat passing through the cross-section of a conductor, varies with time 't' as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. ($\alpha$, β and γ are positive constants.) The minimum heat current through the conductor is

(1) $\alpha - \frac{\beta^2}{2\gamma}$ (2) $\alpha - \frac{\beta^2}{3\gamma}$

(3) $\alpha - \frac{\beta^2}{\gamma}$ (4) $\alpha - \frac{3\beta^2}{\gamma}$

Answer (2)

Sol. Heat through cross section of rod

$Q = \alpha t - \beta t^2 + \gamma t^3$

so heat current $= \frac{dQ}{dt}$

heat current $= \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$

for heat current to be minimum

$$\frac{d^2Q}{dt^2} = -2\beta + 6\gamma t = 0$$

$$t = \frac{2\beta}{6\gamma} = \left(\frac{\beta}{3\gamma}\right)$$

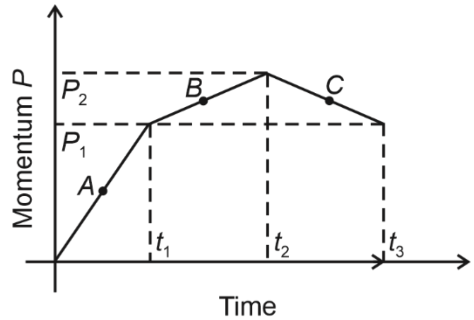
so minimum heat current

$$\left.\frac{dQ}{dt}\right|_{\text{minimum}} = \alpha - 2\beta \times \frac{\beta}{3\gamma} + 3\gamma \times \frac{\beta^2}{9\gamma^2}$$

$$= \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma}$$

$$= \left(\alpha - \frac{\beta^2}{3\gamma}\right)$$

7. Momentum-time graph of an object moving along a straight line is as shown in figure. If $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1 < (t_3 - t_2)$ then at which points among A, B and C the magnitude of force experienced by the object is maximum and minimum respectively.

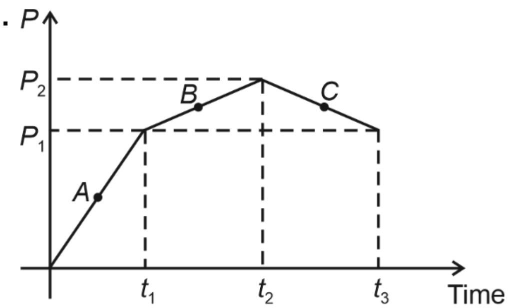


(1) A, B (2) A, C

(3) B, C (4) B, A

Answer (2)

Sol.



$$F_A = \frac{P_1}{t_1}$$

$$F_B = \frac{P_2 - P_1}{t_2 - t_1}$$

$$F_C = \frac{P_2 - P_1}{t_3 - t_2}$$

Therefore the maximum force is at A and minimum force is at C.

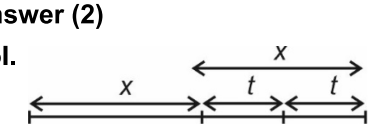
8. A particle moving in unidirectional motion travels half of the total distance with a constant speed of 15 m/s. Now first half of the journey time it travels at 10 m/s and second half of the remaining journey time it travels at 5 m/s. Average speed of the particle is

(1) 12 m/s (2) 10 m/s

(3) 7 m/s (4) 9 m/s

Answer (2)

Sol.



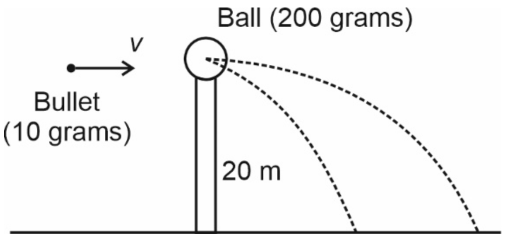
$$v_{av} = \frac{2x}{\frac{x}{15} + 2t}$$

$$= \frac{2x}{\frac{x}{15} + \frac{2x}{10+5}}$$

= 10 m/sec

9. A bullet strikes a stationary ball kept at a height as shown. After collision, range of bullet is 120 m and that of ball is 30 m. Find initial speed of bullet. Collision is along horizontal direction.

Take g = 10 m/s^2



(1) 150 m/s (2) 90 m/s

(3) 240 m/s (4) 360 m/s

Answer (4)

Sol. $m_1V + m_2(O) = m_1v_1' + m_2V_2'$ ...(1)

$$\Delta t = \sqrt{\frac{2h}{g}} = 2s \quad ...(2)$$

$$\Rightarrow v_1' = \frac{120\text{ m}}{2s} = 60\text{ m/s}$$

$$\& \; v_2' = \frac{30\text{ m}}{2s} = 15\text{ m/s}$$

$\Rightarrow v$ = 360 m/s

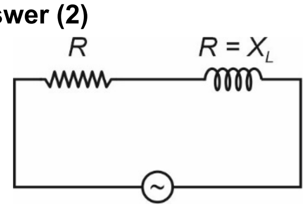
10. If an inductor with inductive reactance, $X_L = R$ is connected in series with resistor R across an A.C voltage, power factor comes out to be P_1. Now, if a capacitor with capacitive reactance, $X_C = R$ is also connected in series with inductor and resistor in the same circuit, power factor becomes P_2. Find $\frac{P_1}{P_2}$

(1) $\sqrt{2}:1$ (2) $1:\sqrt{2}$

(3) 1 : 1 (4) 1 : 2

Answer (2)

Sol.

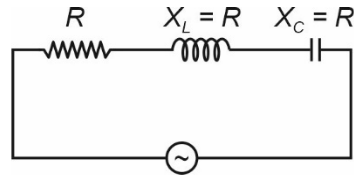


$$Z = \sqrt{R^2 + R^2}$$

$$= \sqrt{2}R$$

P_1 = cosϕ = power factor = $\frac{R}{Z} = \left(\frac{1}{\sqrt{2}}\right)$

When capacitor is also connected in series



The LCR circuit is in resonance stage

So, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = R$

P_2 = cosϕ = power factor $= \frac{R}{Z} = \frac{R}{R} = 1$

So, $\frac{P_1}{P_2} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{1} = \frac{1}{\sqrt{2}}$

11. Electromagnetic wave beam of power 20 mW is incident on a perfectly absorbing body for 300 ns. The total momentum transferred by the beam to the body is equal to

(1) 2 × 10^{-17} Ns (2) 1 × 10^{-17} Ns

(3) 3 × 10^{-17} Ns (4) 5 × 10^{-17} Ns

Answer (1)

Sol. Total energy incident = Pt

So total initial momentum = $\frac{Pt}{c}$

Total final momentum = 0

Total momentum transferred $= \frac{Pt}{c}$

$$= \frac{20 \times 10^{-3} \times 300 \times 10^{-9}}{3 \times 10^8}$$

= 2 × 10^{-17} Ns

12. The velocity of an electron in the seventh orbit of hydrogen-like atom is 3.6 × 10^6 m/s. Find the velocity of the electron in the 3rd orbit.

(1) 4.2 × 10^6 m/s (2) 8.4 × 10^6 m/s

(3) 2.1 × 10^6 m/s (4) 3.6 × 10^6 m/s

Answer (2)

Sol. For hydrogen like atom,

$$v \propto \frac{1}{n}$$

$$\left(\frac{v_1}{v_2}\right) = \left(\frac{n_2}{n_1}\right)$$

$$\Rightarrow \frac{3.6 \times 10^6}{v_2} = \frac{3}{7}$$

$$\Rightarrow v_2 = \frac{7}{3} \times 3.6 \times 10^6$$

= 8.4 × 10^6 m/s

13. Electric field in a region is given by $\vec{E} = \frac{a}{x^2}\hat{i} + \frac{b}{y^3}\hat{j}$, where x & y are co-ordinates. Find SI units of a & b.

(1) a – Nm^2C^{-1}
b – Nm^3C^{-1}

(2) a – Nm^3C^{-1}
b – Nm^2C^{-1}

(3) a – NmC^{-1}
b – Nm^2C^{-1}

(4) a – Nm^2C^{-1}
b – Nm^2C^{-1}

Answer (1)

Sol. E – NC^{-1}

x^2 – m^2

y^3 – m^3

$\Rightarrow$ a – Nm^2C^{-1}

& b – Nm^3C^{-1}

14. Coil A of radius 10 cm has N_A number of turns and I_A current is flowing through it. Coil B of radius 20 cm has N_B number of turns and I_B current is flowing through it. If magnetic dipole moment of both the coils is same then

(1) $I_AN_A = 4I_BN_B$

(2) $I_AN_A = \frac{1}{4}I_BN_B$

(3) $I_AN_A = 2I_BN_B$

(4) $I_AN_A = \frac{1}{2}I_BN_B$

Answer (1)

Sol. Magnetic dipole moment $\mu = NIA = NI\pi R^2$

So $\frac{\mu_A}{\mu_B} = \frac{N_AI_AR_A^2}{N_BI_BR_B^2} = 1$

$\frac{N_AI_A\,(10^2)}{N_BI_B\,(20^2)} = 1$

$N_AI_A = 4N_BI_B$

15. An ideal gas undergoes a thermodynamic process following the relation PT^2 = constant. Assuming symbols have their usual meaning then volume expansion coefficient of the gas is equal to

(1) $\frac{2}{T}$

(2) $\frac{3}{T}$

(3) $\frac{1}{2T}$

(4) $\frac{1}{T}$

Answer (2)

Sol. Volume expansion coefficient $= \frac{dV}{VdT}$

For PT^2 = constant

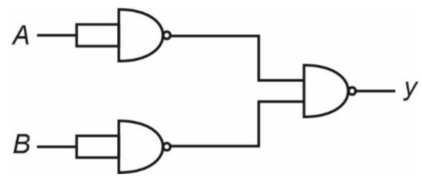
Or $\frac{T^3}{V} = \text{constant}$

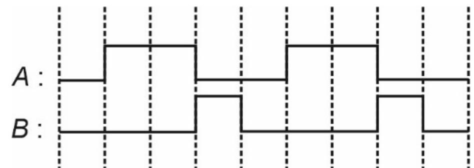
Or $\frac{dV}{dT} = (C)\,3T^2$

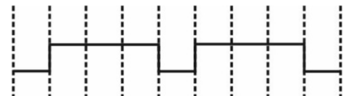
Or $\frac{dV}{VdT} = \frac{3T^2}{T^3}$

$\frac{dV}{VdT} = \frac{3}{T}$

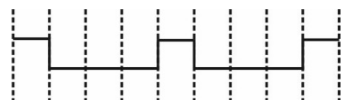
16. Consider a combination of gates as shown :

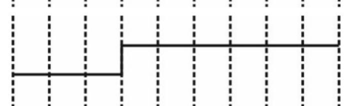




(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. $y = (A'B') = A + B$

$\Rightarrow$ OR gate

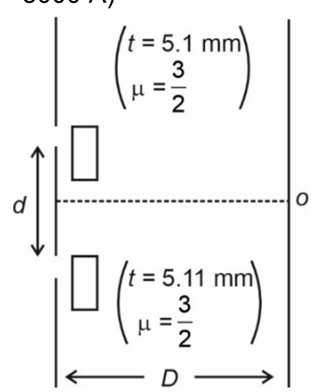
$\Rightarrow$ Option 1

17. For the given YDSE setup. Find the number of fringes by which the central maxima gets shifted from point O.

(Given d = 1 mm

D = 1 m

λ = 5000 Å)



(1) 10

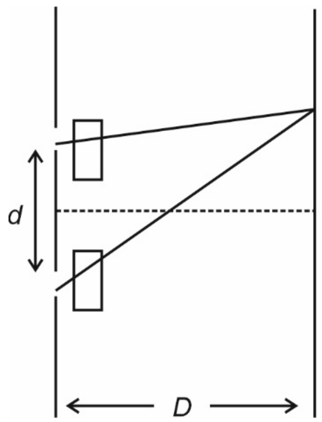
(2) 15

(3) 8

(4) 12

Answer (1)

Sol.



at central position, path difference, is,

$(\mu - 1)t_1 - (\mu - 1)t_2$

$\Delta x = (\mu - 1)(t_1 - t_2)$

$$\Delta x = \left(\frac{3}{2} - 1\right)(5.11 - 5.10)\text{ mm}$$

$$= \frac{1}{2} \times (0.01)\text{ mm}$$

= 0.005 mm

= 5×10^{-6} m

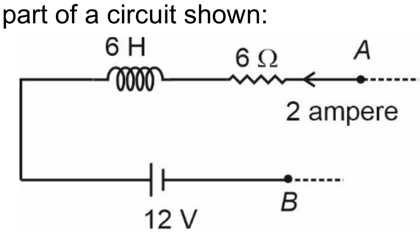
No. of fringes shifted = $\dfrac{\Delta x}{\lambda} = \dfrac{5 \times 10^{-6}\text{ m}}{5 \times 10^{-7}\text{ m}}$

= 10

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In a part of a circuit shown:



Find $V_A - V_B$ in volts. It is given that current is decreasing at a rate of 1 ampere/s.

Answer (18)

Sol. $V_A - iR - L\dfrac{di}{dt} - 12 = V_B$

$\Rightarrow$ $V_A - V_B = +18$ volts

22. A particle undergoing SHM follows the position-time equation given as $x = A\sin\left(\omega t + \dfrac{\pi}{3}\right)$. If the SHM motion has a time period of T, then velocity will be maximum at time $t = \dfrac{T}{\beta}$ for first time after $t = 0$. Value of β is equal to

Answer (03.00)

Sol. $x = A\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$\Rightarrow v = A\omega\cos\left(\omega t + \dfrac{\pi}{3}\right)$

For maximum value of v

$\cos\left(\omega t + \dfrac{\pi}{3}\right) = \pm 1$

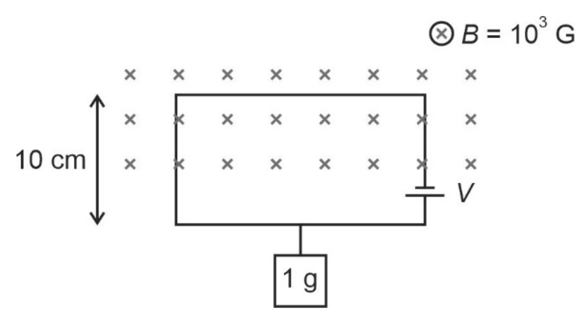
$\Rightarrow \omega t + \dfrac{\pi}{3} = \pi$ (for nearest value of t)

$\omega t = \dfrac{2\pi}{3}$

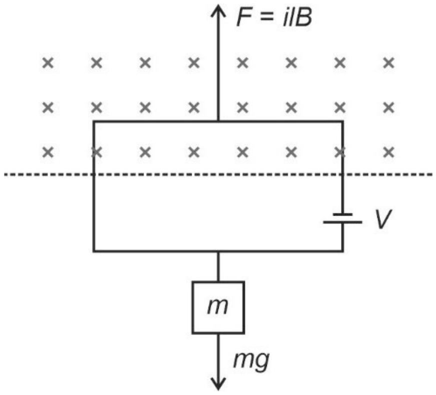
$t = \dfrac{T}{3}$

So $\beta = 3$

23. A block of mass 1 g is equilibrium with the help of a current carrying square loop which is partially lying in constant magnetic field (B) as shown. Resistance of the loop is 10 Ω. Find the voltage (V) (in volts) of the battery in the loop.



Answer (10.00)

Sol. 

$ilB = mg$

$$i = \left(\frac{mg}{lB}\right) = \frac{(1\times10^{-3}\ \text{kg})\times(10\ \text{m/s}^2)}{(0.1\ \text{m})\times(0.1\ \text{T})}$$

$= 1 \times 10^{-3} \times 10^{3}$

$i = 1$ A

As resistance of loop = 10 Ω

$$i = \frac{V}{R} = 1\ \text{A}$$

$V = (1 \times 10)$ V

= 10 V

24. Initial volume of 1 mole of a monoatomic gas is 2 litres. It is expanded isothermally to a volume of 6 litres. Change in internal energy is xR. Find x.

Answer (00)

Sol. $\Delta U = nC_V\Delta T$

$= nC_V(0)$ (∵ isothermal)

$\Rightarrow \Delta U = 0$

25. An object is placed at a distance of 40 cm from the pole of a converging mirror. The image is formed at a distance of 120 cm from the mirror on the same side. If the focal length is measured with a scale where each 1 cm has 20 equal divisions. If the fractional error in the measurement of focal length is $\frac{1}{10k}$ Find k.

Answer (60.00)

Sol. $u = -40$ cm

$v = -120$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow -\frac{1}{120} - \frac{1}{40} = \frac{1}{f}$$

$$\frac{1}{f} = \left(\frac{-1-3}{120}\right) = -\frac{4}{120}$$

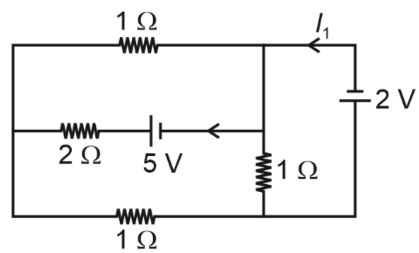
$f = -30$ cm

Least count of scale = $\left(\frac{1}{20}\right)$ cm

Fractional error = $\left(\frac{\frac{1}{20}}{30}\right) = \left(\frac{1}{600}\right)$

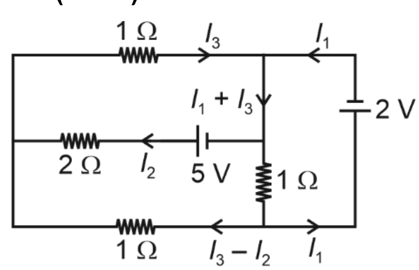
as $\frac{1}{10k} = \frac{1}{600}$

$k = 60$

26. 

In two circuit shown above the value of current I_1 (in amperes) is equal to $-\frac{y}{5}$ A . Value of y is equal to

Answer (11.00)

Sol. 

Using Kirchoff's law.

$I_1 + I_3 - I_2 = -2$ …(i)

$I_3 + 2I_2 = 5$ …(ii)

$2I_2 - (I_3 - I_2) - (I_1 + I_3 - I_2) = 5$ …(iii)

$$\Rightarrow I_1 = -\frac{11}{5}\ \text{A}$$

$\Rightarrow y = 11$

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Caprolactam when heated at high temperature, gives

   (1) Nylon 6, 6

   (2) Dacron

   (3) Teflon

   (4) Nylon 6

Answer (4)

Sol. Caprolactam on heating at high temperature gives Nylon-6 polymer.

2. Molarity of CO_2 in soft drink is 0.01 M. The volume of soft drink is 300 mL. Mass of CO_2 in soft drink is

   (1) 0.132 g

   (2) 0.481 g

   (3) 0.312 g

   (4) 0.190 g

Answer (1)

Sol. Moles = 0.01 × 0.3 = 0.003

Mass = 0.003 × 44 = 0.132 gm

3. During the qualitative analysis of SO_3^{-2} using dilute H_2SO_4, SO_2 gas evolved which turns $K_2Cr_2O_7$ solution (acidified H_2SO_4)

   (1) Green (2) Black

   (3) Blue (4) Red

Answer (1)

Sol. Orange colour of dichromate solution ($K_2Cr_2O_7$) converts to green (Cr^{3+}).

4. Number of lone pair of electrons on central atom?

	Column-I		Column-II
(A)	IF_7	(P)	0
(B)	ICl_4^-	(Q)	1
(C)	XeF_2	(R)	2
(D)	XeF_6	(S)	3

Match the following

(1) (A)→(P); (B)→(Q); (C)→(R); (D)→(S)

(2) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)

(3) (A)→(R); (B)→(S); (C)→(P); (D)→(Q)

(4) (A)→(S); (B)→(R); (C)→(Q); (D)→(P)

Answer (2)

Sol. Molecule/species No. of lone pair

IF_7 → 0

ICl_4 → 2

XeF_2 → 3

XeF_6 → 1

5. Which one of the following is water soluble?

   (a) $BeSO_4$

   (b) $MgSO_4$

   (c) $CaSO_4$

   (d) $SrSO_4$

   (e) $BaSO_4$

   (1) Only a and b (2) Only a, b, c

   (3) Only d and e (4) Only a and e

Answer (1)

Sol. Solubility of sulphates of group-2 elements decreases down the group. $BeSO_4$ and $MgSO_4$ are appreciably soluble in water. $CaSO_4$, $SrSO_4$ and $BaSO_4$ are practically insoluble in water.

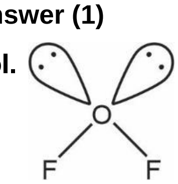
6. Shape of OF_2 molecule is?

   (1) Bent (2) Linear

   (3) Tetrahedral (4) T-shaped

Answer (1)

Sol.



It is *sp*^{3} hybridised therefore its shape will be bent or V-shaped.

7. Inhibitor of cancer growth

(1) Cisplatin

(2) EDTA

(3) Cobalt

(4) Ethane 1, 2 - diamine

Answer (1)

Sol. Cisplatin acts as an anticancer agent.

8. Speed of e^- in 7^{th} orbit is 3.6×10^6 m/s then find the speed in 3^{rd} orbit

(1) 3.6×10^6 m/s

(2) 8.4×10^6 m/s

(3) 7.5×10^6 m/s

(4) 1.8×10^6 m/s

Answer (2)

Sol. Speed of electron in n^{th} orbit of a Bohr atom is given by

$$v_n = (v_1)_H \frac{Z}{n}$$

If n = 7

$$v_7 = (v_1)_H \frac{Z}{7} = 3.6 \times 10^6 \text{ m/s}$$

If n = 3

$$v_3 = (v_1)_H \frac{Z}{3}$$

$$= \frac{7 \times 3.6 \times 10^6}{3}$$

= 8.4×10^6 m/s

9. Match the following :

Atomic Number

(i) 52	(p) s-block
(ii) 37	(q) p-block
(iii) 65	(r) d-block
(iv) 74	(s) f-block

(1) (i) → (q); (ii) → (p); (iii) → (r); (iv) → (s)

(2) (i) → (q); (ii) → (p); (iii) → (s); (iv) → (r)

(3) (i) → (s); (ii) → (r); (iii) → (p); (iv) → (q)

(4) (i) → (r); (ii) → (p); (iii) → (q); (iv) → (s)

Answer (2)

Sol. 37 is Rubidium belonging to 1^{st} group of s-block.

10. Consider the following reactions

$$NO_2 \xrightarrow{UV} A + B$$

$$A + O_2 \longrightarrow C$$

$$B + C \longrightarrow NO_2 + O_2$$

A, B and C are respectively

(1) O, NO, O_3 (2) NO, O, O_3

(3) NO, O_3, O (4) O_3, O, NO

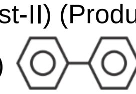
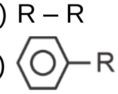
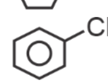
Answer (1)

Sol. $NO_2 \xrightarrow{UV} \underset{(B)}{NO} + \underset{(A)}{O}$

$$O + O_2 \longrightarrow O_3 (C)$$

$$NO + O_3 \longrightarrow NO_2 + O_2$$

11. Which of the following option contains the correct match:

(List-I) (Reactions)	(List-II) (Products)
(A) Wurtz	(P) 
(B) Fittig	(Q) R – R
(C) Wurtz Fittig	(R) 
(D) Sandmeyer	(S) 

(1) A → Q; B → P; C → R; D → S

(2) A → P; B → Q; C → R; D → S

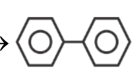
(3) A → S; B → R; C → Q; D → P

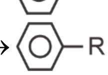
(4) A → R; B → S; C → P; D → Q

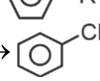
Answer (1)

Sol. The correct matches are

(A) Wurtz → R – R

(B) Fittig → 

(C) Wurtz fittig → 

(D) Sandmeyer → 

12. If volume of ideal gas is increased isothermally, then its internal energy

(1) Increased

(2) Remains constant

(3) Is decreased

(4) Can be increased or decreased

Answer (2)

Sol. Internal energy of ideal gas depends only upon temperature.

13. Arrange the following ligands according to their increasing order of field strength

$S^{2-}, C_2O_4^{2-}, NH_3, en, CO$

(1) $S^{2-} < CO < NH_3 < en < C_2O_4^{2-}$

(2) $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$

(3) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

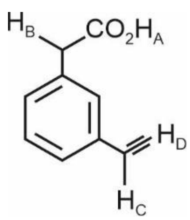
(4) $CO < en < NH_3 < C_2O_4^{2-} < S^{2-}$

Answer (3)

Sol. The correct order of field strength is

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

14. Consider the following molecule



Select the correct order of acidic strength

(1) $H_A > H_D > H_B > H_C$ (2) $H_B > H_A > H_D > H_C$

(3) $H_A > H_B > H_C > H_D$ (4) $H_C > H_B > H_D > H_A$

Answer (1)

Sol. The correct order of acidic strength is

$H_A > H_D > H_B > H_C$

15. Which of the following compound is used as the antacid?

(1) Ranitidine

(2) Prontosil

(3) Norethindrone

(4) Codeine

Answer (1)

Sol. Ranitidine is used as the antacid.

16. The role of SiO_2 in Cu extraction is

(1) Converts FeO to $FeSiO_3$

(2) Converts CaO to $CaSiO_3$

(3) Reduces Cu_2S to Cu

(4) None of these

Answer (1)

Sol. It converts FeO to $FeSiO_3$

17. Assertion: Ketoses gives seliwanoff test.

Reason : Ketoses undergo β- elimination to form furfural.

(1) Assertion and reason both are correct and reason is the correct explanation of assertion

(2) Assertion and reason both are correct but reason is not the correct explanation of assertion.

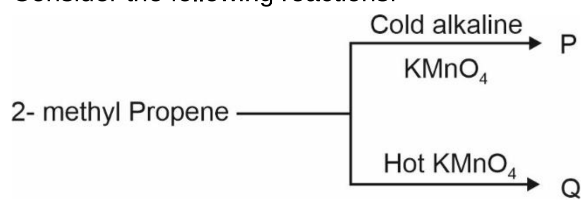
(3) Assertion is correct and reason is incorrect

(4) Assertion is incorrect but reason is correct.

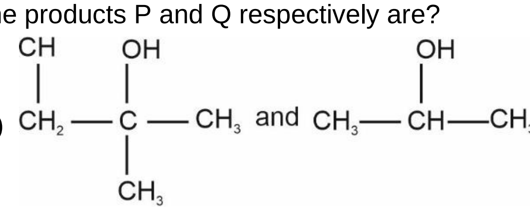
Answer (1)

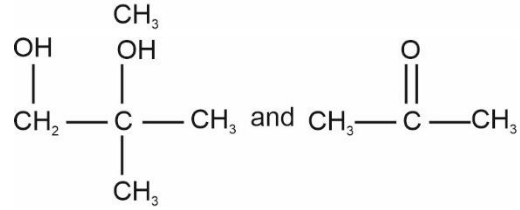
Sol. Assertion and reason both are correct and reason is the correct explanation of assertion.

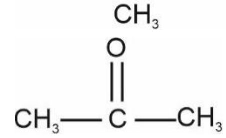
18. Consider the following reactions:



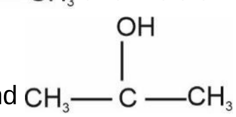
The products P and Q respectively are?

(1) 

(2) 

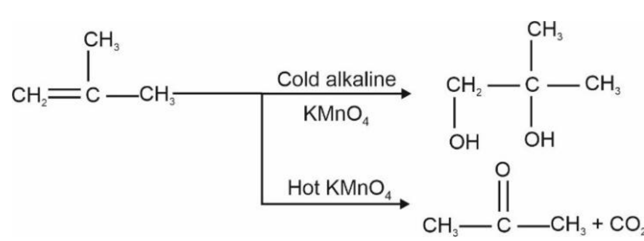
(3) 

and HCOOH

(4) HCOOH and 

Answer (2)

Sol.



SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For given cell, at T K

$$\underset{(1\,bar)}{Pt\,|\,H_2(g)}\,|\,\underset{(1\,M)}{H^+}\,||\,Fe^{3+};Fe^{2+}\,|\,Pt$$

E_{cell} = .712 V

$E^\circ_{cell} = .770$ V

if $\frac{[Fe^{2+}]}{[Fe^{3+}]}$ is t $\left(\frac{2.303\,RT}{F} = .058\right)$

then find $\left(\frac{t}{5}\right)$

Answer (2)

Sol. $.712 = .770 - \frac{.058}{2}\log\left[\frac{Fe^{2+}}{Fe^{3+}}\right]^2$

$$-.058 = -.058\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

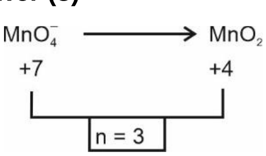
$$\frac{Fe^{2+}}{Fe^{3+}} = 10 = t$$

$$\frac{t}{5} = 2$$

22. How many moles of electrons are required to reduce 1 mole of permanganate ions into manganese dioxide

Answer (3)

Sol.



3 mole of e^- are required

23. 600 mL of 0.04 M HCl is mixed with 400 mL of 0.02 M H_2SO_4. Find out the pH of resulting solution (Nearest integer).

Answer (01.00)

Sol. m moles of H^+ from HCl = 0.04 × 600

= 24

m moles of H^+ from H_2SO_4 = 0.02 ×2 × 400

= 16

Total m moles of H^+ = 24 + 16 = 40

Final volume of solution = 1000 mL

$$[H^+] = \frac{40}{1000} = 0.04\text{ M}$$

pH = – log 0.04 = 1.4

24. A solution of 2 g of a solute and 20 g water has boiling point 373.52 K. Then find the molar mass of solute in grams? [Given : K_b = 0.52 K kg/mole and solute is non-electrolyte].

Answer (100)

Sol. $\Delta T_b = K_b.m$

$$0.52 = 0.52 \times \frac{2/M}{.02}$$

M = 100 g

25. When first order kinetic, rate constant is 2.011 × 10^{-3} sec^{-1}, the time taken in decomposition of substance from 7 g to 2 g will be. [Use log7 = 0.845 and log2 = 0.301]

Answer (623)

Sol. A → Products

Initial moles of $A = \frac{7}{M}$ (M is molar mass of A)

Final moles of $A = \frac{2}{M}$

Rate constant K = 2.011 × 10^{-3} s^{-1}

$$t = \frac{2.303}{k}\log\frac{7}{2}$$

$$= \frac{2.303}{2.011\times10^{-3}}[0.845 - 0.301]$$

= 623 s

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Coefficient of x^{301} in $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \ldots + x^{500}$ is equal to

(1) $^{506}C_{306}$

(2) $^{501}C_{300}$

(3) $^{501}C_{301}$

(4) $^{500}C_{300}$

Answer (3)

Sol. Coeff of $x^{301} = {}^{500}C_{301} + {}^{499}C_{300} + {}^{498}C_{299} + \ldots + {}^{199}C_0$

$= {}^{500}C_{199} + {}^{499}C_{199} + {}^{498}C_{199} + \ldots + {}^{199}C_{199}$

$= {}^{501}C_{200}$

$= {}^{501}C_{301}$

2. $\tan 15^\circ + \dfrac{1}{\tan 165^\circ} + \dfrac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then value of $\left(a + \dfrac{1}{a}\right)$ is

(1) $4 - 2\sqrt{3}$

(2) $\dfrac{-4}{\sqrt{3}}$

(3) 2

(4) $5 - \dfrac{3}{2}\sqrt{3}$

Answer (2)

Sol. tan15° + cot165° + cot105° + tan195°

= tan15° – cot15° – tan15° + tan15°

= tan15° – cot15°

$= -2\sqrt{3}$

$\Rightarrow \quad a = -\sqrt{3}$

$$a + \frac{1}{a} = -\sqrt{3} - \frac{1}{\sqrt{3}} = \frac{-4}{\sqrt{3}}$$

3. If set $A = \{a, b, c\}$

$R : A \to A$

$R = \{(a, b), (b, c)\}$

How many elements should be added for making it symmetric and transitive.

(1) 2

(2) 3

(3) 4

(4) 7

Answer (4)

Sol. For symmetric

$(a, b), (b, c) \in R$

$\Rightarrow (b, a), (c, b) \in R$

For transitive.

$(a, b), (b, c) \in R$

$\Rightarrow (a, c) \in R$

Now,

$(a, c) \in R$

$\Rightarrow (c, a) \in R$ {For symmetric}

$(a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R$

$(b, c), (c, b) \in R$

$\Rightarrow (b, b) \in R$

$(c, b), (b, c) \in R$

$\Rightarrow (c, c) \in R$

$\therefore$ elements to be added

$\{(b, a)\ (c, b)\ (b, b)\ (a, a)\ (a, c)\ (c, a)\ (c, c)\}$

Total 7 elements

4. Let $P(h, k)$ be two points on $x^2 = 4y$ which is at shortest distance from $Q(0, 33)$ then difference of distances of $P(h, k)$ from directrix of $y^2 = 4(x + y)$ is

(1) 2

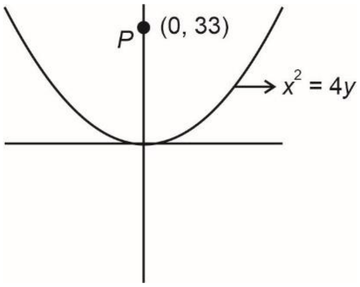
(2) 4

(3) 6

(4) 8

Answer (2)

Sol. For normal through (0, 33)



Normal at point $(2t, t^2)$

$x = -ty + 2at + at^3$

$0 = -t \cdot 33 + 2t + t^3$

$\Rightarrow \quad t = 0 \text{ OR } \pm\sqrt{31}$

Points at which normal are drawn are

$A(0, 0), B\left(2\sqrt{31}, 31\right), C(-2\sqrt{31}, 31)$

Shortest distance

$$= PB = PC = \sqrt{124+4} = 8\sqrt{2} \text{ units}$$

Given parabola $(y-2)^2 = 4(x+1)$

Directrix is $x = -2$, that is line L

$B_L - C_L = \left|\left(-2+2\sqrt{31}\right) - \left(2+2\sqrt{31}\right)\right|$

$= 4$

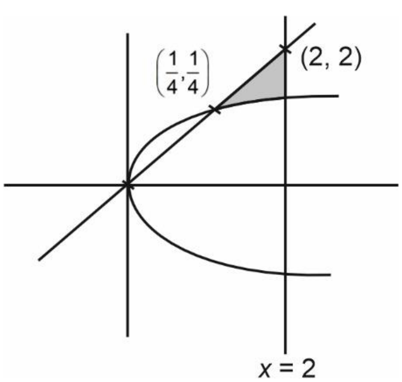
5. Area bounded by larger part in I quadrant by $x = 4y^2$, $x = 2$ and $y = x$ is A then $3A$ equals

(1) $6+\frac{1}{32}-2\sqrt{2}$ (2) $2+\frac{1}{96}-\frac{2\sqrt{2}}{3}$

(3) $\frac{2\sqrt{2}}{3}$ (4) 96

Answer (1)

Sol.



Shaded area is the required area

$$A = \int_{1/4}^{2}\left(x - \frac{\sqrt{x}}{2}\right)dx$$

$$= \frac{x^2}{2} - \frac{x^{3/2}}{3}\Bigg|_{1/4}^{2}$$

$$= \left(2 - \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{32} - \frac{1}{24}\right)$$

$$= 2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \quad 3A = 6 + \frac{1}{32} - 2\sqrt{2} \text{ sq. units.}$$

6. A die with points (2, 1, 0, –1, –2, 3) is thrown 5 times. The probability that the product of outcomes on all throws is positive is

(1) $\frac{521}{2592}$

(2) $\frac{16}{81}$

(3) $\frac{41}{288}$

(4) $\frac{28}{81}$

Answer (1)

Sol. Either all outcomes are positive or any two are negative.

The required probability $= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3$

$+{}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{162} + \frac{1}{32} + \frac{5}{36} = \frac{521}{2592}$

7. Let $S = \{1, 2, 3, 4, 5\}$

if $f : S \to P(S)$, where $P(S)$ is power set of S. Then number of one-one functions f can be made is

(1) $(32)^5$

(2) $\frac{32!}{27!}$

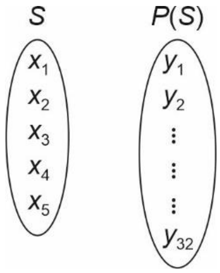
(3) ${}^{32}C_{27}$

(4) ${}^{32}P_{27}$

Answer (2)

Sol. $n(S) = 5$

$n(P(S)) = 2^5 = 32$



$\therefore$ No. of one-one function= $32 \times 31 \times 30 \times 24 \times 28$

$$= \frac{32!}{27!}$$

8. A line is cutting x axis and y axis at two points A and B, respectively, where $OA = a$, $OB = b$. A perpendicular is drawn from O (origin) to AB at an angle of $\frac{\pi}{6}$ from positive x-axis. If area of triangle $OAB = \frac{98\sqrt{3}}{3}$ sq. units, then $\sqrt{3}\,a + b$ is equal to

(1) 28 (2) 14

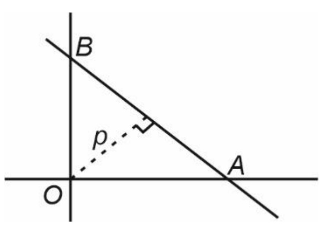
(3) 12 (4) 7

Answer (1)

Sol. Let the perpendicular distance of line from origin is p.

$\Rightarrow$ Equation of $AB : \frac{x\sqrt{3}}{2} + \frac{y}{2} = p$

$\Rightarrow \frac{x}{\frac{2p}{\sqrt{3}}} + \frac{y}{2p} = 1$



$OA = \frac{2p}{\sqrt{3}}, OB = 2p$

$\frac{1}{2} \cdot \frac{2p}{\sqrt{3}} \cdot 2p = \frac{98}{\sqrt{3}}$

$\Rightarrow p = 7$

$OA = a = \frac{14}{\sqrt{3}}$

$OB = b = 14$

$\sqrt{3}a + b$

$\Rightarrow 14 + 14 = 28$

9. For solution of differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}\left(x^3\right)}{\left(1+x^6\right)^{\frac{3}{2}}} y = -\frac{x^3 \tan^{-1} x^3}{\sqrt{1+x^6}}$$

given that $y(0) = 0$ then $y(1)$ is

(1) $1 - e^{\frac{\pi}{4\sqrt{2}}}$

(2) $1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$

(3) $e^{\frac{1}{\sqrt{2}}} - e^{\frac{\pi}{4\sqrt{2}}}$

(4) $e^{\frac{\pi}{4\sqrt{2}}}$

Answer (2)

Sol. IF = $\int \frac{-3x^5 \tan^{-1}\left(x^3\right)}{\left(1+x^6\right)^{\frac{3}{2}}} dx$

Let $\tan^{-1}\left(x^3\right) = t$

IF = $e^{-\int t \sin t} = e^{(t\cos t - \sin t)}$

Solution of Differential equation

$y \cdot e^{(t\cos t - \sin t)} = \int e^{(t\cos t - \sin t)}(-t\sin t)dt$

$y \cdot e^{(t\cos t - \sin t)} = e^{(t\cos t - \sin t)} + c$

$t = 0 \rightarrow y = 0$

$\therefore c = -1$

When $x = 1,\ t = \frac{\pi}{4}$

$$y \cdot e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} = e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} - 1$$

$$y = 1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$$

10. $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx$ equals

(1) $e^9 - e$ (2) $e^8 - 1$

(3) $e^8 - e$ (4) $e^9 - 1$

Answer (3)

Sol. $I=\int_1^2 x^2 e^{[x]+[x^3]}dx = e\int_1^2 x^2\cdot e^{[x^3]}dx$

Let $x^3=t$

$I=e\int_1^8 \frac{dt}{3}e^{[t]}=\frac{e}{3}\left(e+e^2+...+e^7\right)$

$=\frac{e^2}{3}\left(\frac{e^7-1}{e-1}\right)$

So, $\frac{3(e-1)}{e}\cdot\frac{e^2}{3}\cdot\frac{e^7-1}{e-1}=e^8-e$

11. $\hat{n}$ is a vector, $\vec{a}\neq 0, \vec{b}\neq 0$. If $\vec{n}\perp\vec{c}, \vec{a}=\alpha\vec{b}-\hat{n}$ and $\vec{b}\cdot\vec{c}=12$ then the value of $|\vec{c}\times(\vec{a}\times\vec{b})|$ equals (where $\hat{n}$ represents unit vector in the direction of $\vec{n}$)

(1) 144

(2) $\sqrt{12}$

(3) 12

(4) 24

Answer (3)

Sol. $\vec{a}=\alpha\overrightarrow{b}-\hat{n}$

$\Rightarrow\ \vec{a}\times\vec{b}=-\hat{n}\times\vec{b}$

Now,

$|\vec{c}\times(\vec{a}\times\vec{b})|$

$=|\vec{c}\times\left(-\hat{n}\times\vec{b}\right)|$

$=|\hat{n}(12)-\vec{b}(0)|$

= 12

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. $\lim_{x\to 0}\frac{\int_0^x \frac{t^3}{1+t^6}dt}{x^4}$ equals

Answer (12)

Sol. $\lim_{x\to 0}\frac{48\int_0^x \frac{t^3}{t^6+1}dt}{x^4}$

As $\frac{0}{0}$ form, applying L' hospital rule we get

$\lim_{x\to 0} 48\frac{x^3}{\left(x^6+1\right)\cdot 4x^3}=48\cdot\frac{1}{4}=12$

22. If $a_n=\frac{-2}{4n^2-16n+15}$ and $a_1+a_2+....+a_{25}=\frac{m}{n}$ where m and n are coprime, then the value of $m+n$ is

Answer (191)

Sol. $a_n=\frac{-2}{4n^2-16n+15}=\frac{-2}{(2n-3)(2n-5)}$

$=\frac{1}{2n-3}-\frac{1}{2n-5}$

$a_1+a_2+....+a_{25}=\left(\frac{1}{-1}-\frac{1}{-3}\right)+...\left(\frac{1}{47}-\frac{1}{45}\right)$

$=\frac{1}{47}+\frac{1}{3}=\frac{50}{141}$

$\therefore\ m+n=191$

23. If $z = 1 + i$ and $z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)} = z_1$, then find the value of $\frac{12}{\pi}\arg(z_1)$.

Answer (3)

Sol. $z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)} = \frac{i+(1-i)(1-i)}{(1-i)(-i)} = \frac{1}{1-i}$

$\arg z_1 = \arg\left(\frac{1}{1-i}\right) = -\arg(1-i) = \frac{\pi}{4}$

$\frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{\pi}{4} = 3$

24. Mean & Variance of 7 observations are 8 & 16 respectively, if number 14 is omitted then *a* & *b* are new mean & variance. The value of $a + b$ is

Answer (19)

Sol. Let $x_1, \ldots x_7$ are observation

New mean $= \frac{8\times 7 - 14}{6} = 7$

$\therefore \quad \frac{\sum_{i-1}^{n} x_i^2}{7} - 64 = 16 \Rightarrow \sum x_i^2 = 560$

$\sum x_{i(\text{new})}^2 = 560 - 14^2$

$\therefore \quad b = \frac{364}{6} - 7^2 = \frac{70}{6} = \frac{35}{3}$

$\therefore \quad a + b = 7 + \frac{35}{3} = \frac{56}{3} = 18.67$

Rounding off gives 19

25. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of $\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15}$ then $|ab - 5|$ is equal to

Answer (04.00)

Sol. $a_n\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r}\left(x^3\right)^{15-r} b^{-r} x^{\frac{-r}{3}}$

$45 - 3r - \frac{r}{3} = 15 \Rightarrow \frac{10r}{3} = 30$

$\boxed{r = 9}$

$a_n\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r} x^{\frac{15-r}{3}} b^{-r} x^{-3r}$

$\frac{15-r}{3} - 3r = -15$

$15 - r - 9r = -45$

$\Rightarrow r = 6$

So, ${}^{15}C_9 a^6 b^{-9} = {}^{15}C_6 a^9 b^{-6}$

$\Rightarrow \quad a^{-3}b^{-3} = 1$

or $\boxed{ab = 1}$

$|ab - 5| = 4$

26. Using 1, 2, 3, 5, 4-digit numbers are formed, where repetition is allowed. How many of them is divisible by 15?

Answer (21)

Sol. Units digit will be 5

$\underline{a}\ \ \underline{b}\ \ \underline{c}\ \ \underline{5}$

$a + b + c = (3\lambda + 1)$ type

For (a, b, c) possibilities are

(2, 2, 3) (1, 1, 5) (1, 1, 2)

(3, 3, 1) (5, 5, 3) (2, 3, 5)

For (2, 2, 3) $\Rightarrow \frac{3!}{2!} = 3$

For (1, 1, 5) $\Rightarrow \frac{3!}{2!} = 3$

For (1, 1, 2) $\Rightarrow \frac{3!}{2!} = 3$

For (3, 3, 1) $\Rightarrow \frac{3!}{2!} = 3$

For (5, 5, 3) $\Rightarrow \frac{3!}{2!} = 3$

For (2, 3, 5) $\Rightarrow 3! = 6$

Total = 21

27. If $5f(x+y)=f(x)\cdot f(y)$ and $f(3)=320$, then the value of $f(1)$ is

Answer (20)

Sol. $5f(x+y)=f(x)\cdot f(y)$ ...(i) $f(3) = 320$

Put $x = 1$, $y = 2$ in (i)

$5f(3)=f(1)\cdot f(2)$

$\Rightarrow f(1)\cdot f(2)=5\times 320=1600$ ...(ii)

Put $x = y = 1$ in (i)

$5f(2)=(f(1))^2$

$\Rightarrow f(2)=\dfrac{(f(1))^2}{5}$ ...(iii)

Using (iii) in (ii),

$f(1)\cdot\dfrac{(f(1))^2}{5}=1600$

$(f(1))^3=8000$

$f(1)=20$

28. If for $\log_{\cos x}(\cot x)-4\log_{(\sin x)}\cot x=1$,

$x=\sin^{-1}\left(\dfrac{\alpha+\sqrt{\beta}}{2}\right)$. Find $(\alpha+\beta)$, given $x\in\left(0,\ \dfrac{\pi}{2}\right)$

Answer (04.00)

Sol. $\log_{\cos x}\cot x-4\log_{\sin x}\cot x=1$

$1-\log_{\cos x}\sin x-4(\log_{\sin x}\cos x-1)=1$

Let $\log_{\cos x}\sin x=t$

$-t-4\left(\dfrac{1}{t}-1\right)=0$

$\Rightarrow t+\dfrac{4}{t}=4$

$\Rightarrow t=2$

$\log_{\cos x}\sin x=2$

$\Rightarrow \cos^2 x=\sin x$

$\Rightarrow 1-\sin^2 x-\sin x=0$

$\Rightarrow \sin^2 x+\sin x-1=0$

So, $\sin x=\dfrac{-1\pm\sqrt{5}}{2}$

$\alpha=-1$, $\beta=5$

$\alpha+\beta=4$