SECTION – A

1. The area of the region $\left\{(x,y): x^2 \le y \le 8-x^2, y \le 7\right\}$ is.

(1) 24 (2) 21 (3) 20 (4) 18

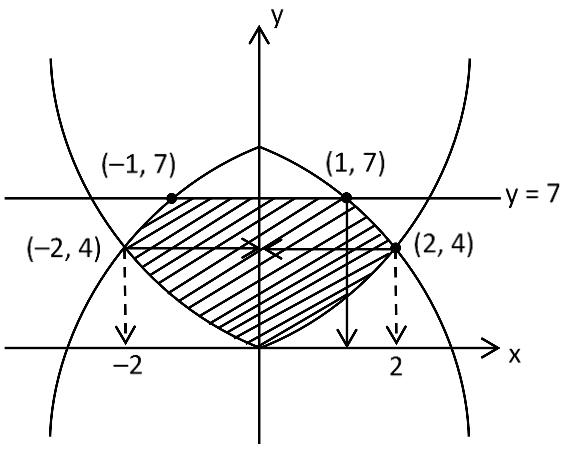
Sol. **(3)**

$y \ge x^2 \quad y \le 8-x^2 \qquad y \le 7$

$x^2 = 8 - x^2$

$x^2 = 4$

$x = \pm 2$



$$2\left(1.7+\int_1^2\left(8-2x^2\right)dx\right)-2\int_0^1\left(x^2\right)dx$$

$$=2\left[7+\left(8x-\frac{2x^3}{3}\right)_1^2\right]-2\left(\frac{x^3}{3}\right)_0^1$$

$$=2\left[7+\left(16-\frac{16}{3}\right)-\left(8-\frac{2}{3}\right)\right]-2\left(\frac{1}{3}\right)$$

$$=2\left[7+\frac{32}{3}-\frac{22}{3}\right]=2\left[7+\frac{10}{3}\right]\frac{-2}{3}$$

$$=\frac{60}{3}=20$$

2. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then 2a+ b -3c – 4d equal to

(1) 2004 (2) 2007 (3) 2005 (4) 2006

Sol. **(3)**

$Q = PAP^T$

$P^T. Q^{2007}. P = P^T. Q.Q \dots Q.P$

$= P^T(PAP^T)\,(P.AP^T) \dots (PAP^T)P.$

$\Rightarrow (P^TP)A(P^TP)A \dots A(P^TP)$

$$P^T.P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore\ P^T.\ Q^{2007}.\ P = A^{2007}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$

3. Negation of $(p \rightarrow q) \rightarrow (q \rightarrow p)$ is

(1) $(-q) \wedge p$ (2) $p \vee (\sim q)$ (3) $(\sim p) \vee q$ (4) $q \wedge (\sim p)$

Sol. **(4)**

$(p \rightarrow q) \rightarrow (q \rightarrow p)$

$\sim[\sim\ p \rightarrow q\ \wedge q \rightarrow p]$

$\Rightarrow\ p \rightarrow q\ \wedge \sim\ q \rightarrow p$

$\Rightarrow\ \sim p \vee q\ \wedge\ q \wedge \sim p$

$\Rightarrow q \wedge \sim p.$

4. Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines

$4x + 3y = 69$,

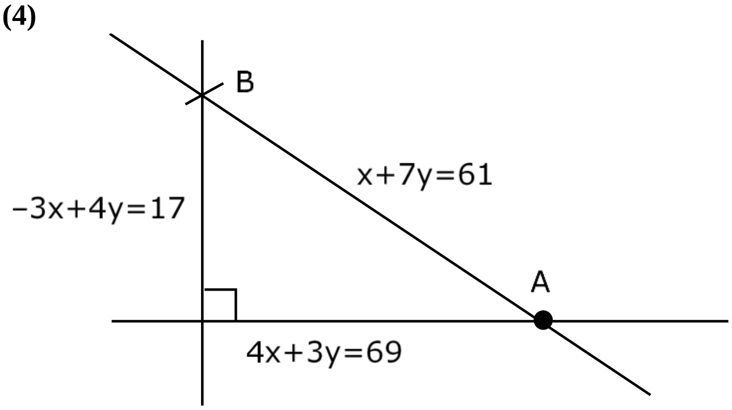
$4y - 3x = 17$ and

$x + 7y = 61$.

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(1) 18 (2) 15 (3) 16 (4) 17

Sol. **(4)**



$4x + 28y = 244$

$4x + 3y = 69$

$-\quad -\quad -$

$25y = 175$

$y = 7, x = 12$

$A(12, 7)$

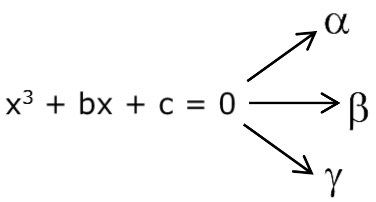
$-3x + 4y = 17$
$3x + 21y = 183$

$25y = 200$
$y = 8, x = 5$
$B(5, 8)$
$\therefore$ Circumcenter
$\alpha = \frac{17}{2} \beta = \frac{15}{2}$
$\left(\frac{17}{2}, \frac{15}{2}\right)$
$(\alpha - \beta)^2 + \alpha + \beta$
$1+16 = 17$

5. Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(1) $\frac{155}{8}$ (2) 21 (3) 19 (4) $\frac{169}{8}$

Sol. **(3)**



$\beta\gamma = 1$
$\alpha = -1$
Put $\alpha = -1$
$-1 - b + c = 0$
$c - b = 1$

also
$\alpha.\beta.\gamma = -c$
$-1 = -c \Rightarrow c = 1$
$\therefore b = 0$
$x^3 + 1 = 0$
$\alpha = -1, \beta = -w, \gamma = -w^2$
$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$
$0 + 2 + 3 + 6 + 8 = 19$

6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

(1) 752 (2) 772 (3) 782 (4) 792

Sol. **(4)**

$n(A\times B)=10$

${}^{10}C_3+{}^{10}C_4+{}^{10}C_5+{}^{10}C_6=792$

7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(1) 5481 (2) 3654 (3) 2436 (4) 1817

Sol. **(2)**

$\frac{{}^nC_r}{{}^nC_{r-1}}=5 \qquad \frac{{}^nC_{r+1}}{{}^nC_r}=4$

$\frac{n-r+1}{r}=5 \qquad n=5r+4 \ \ldots(2)$

n = 6r – 1 … (1)

$\therefore n=29, r=5$

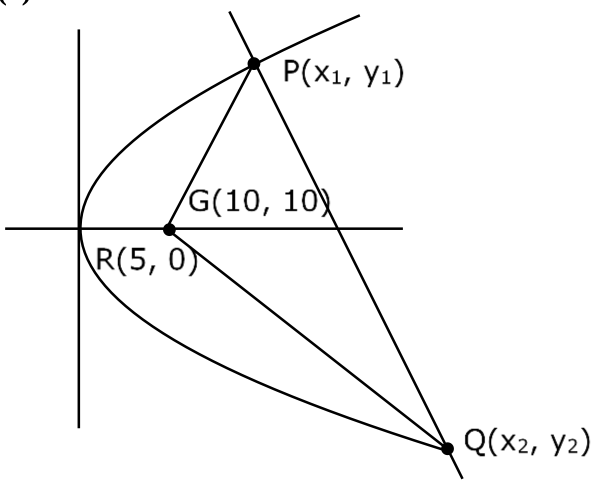
Coeff of 4^{th} term = ${}^{29}C_3$

= 3654

8. Let R be the focus of the parabola $y^2 = 20x$ and the line y = mx + c intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If c – m = 6, then $(PQ)^2$ is

(1) 325 (2) 346 (3) 296 (4) 317

Sol. **(1)**



$y^2 = 20x$, y = mx + c

$y^2 = 20\left(\frac{y-c}{m}\right)$

$y^2-\frac{20y}{m}+\frac{20c}{m}=0 \qquad \frac{y_1+y_2+y_3}{3}=10$

$\frac{20}{m}=30$

$m=2/3$

and c – m = 6

$c = \frac{2}{3} + 6 \Rightarrow \frac{20}{3} = c$

$y^2 - 30y + \frac{20 \times 20/3}{2/3} = 0 \Rightarrow$ $y^2 - 30y + 200 = 0$

y = 10, y = 20

y = 20, x = 20 P(5, 10); (20, 20)Q

$\frac{20+5+x}{3} = 10 \Rightarrow$ x = 5 $PQ^2 = 15^2 + 10^2 = 225 + 100 =$ 325

9. Let $S_K \frac{1+2+...+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}$ $(Bn^2 + Cn + D)$, where A, B, C, D $\in$N and A has least value. Then

(1) A + B is divisible by D

(2) A + B = 5 (D – C)

(3) A + C + D is not divisible by B

(4) A + B + D is divisible by 5

Sol. **(1)**

$S_k = \frac{k+1}{2}$

$S_k^2 = \frac{k^2+1+2k}{4}$

$\therefore \sum_{j-1}^{n} S_j^2 = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + n(n+1)\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{4}\left[\frac{2n^2+3n+1}{6} + n + 2\right]$

$= \frac{n}{4}\left[\frac{2n^2+9n+13}{6}\right] = \frac{n}{24}\left[2n^2+9n+13\right]$

A = 24, B = 2, C = 9, D = 13

10. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(1) $2\sqrt{6}$ (2) $3\sqrt{6}$ (3) $6\sqrt{3}$ (4) $6\sqrt{2}$

Sol. **(2)**

$S_d = \left|\frac{(\vec{a}-\vec{b}) \times (\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|}\right|$

$\bar{a} = (4,-2,-3)$

$\bar{b} = (1,3,4)$

$\bar{n}_1 = (4,5,3)$

$\bar{n}_2 = (3,4,2)$

$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2,1,1)$

$S_d = \frac{(3,-5,-7)\cdot(-2,1,1)}{\sqrt{6}} = \left|\frac{-6-5-7}{\sqrt{6}}\right| = 3\sqrt{6}$

11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.

(1) 16800 (2) 14800 (3) 18000 (4) 33600

Sol. **(1)**

IEEEE,

NNN, DD, P, C

$$\frac{8!}{3!2!}\times\frac{6!}{41}=16800$$

12. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}, 6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(1) 49 (2) 36 (3) 25 (4) 16

Sol. **(2)**

$$(\alpha,10,13);(6,11,11),\left(\frac{9}{2},\beta,-8\right)$$

$$\frac{\alpha-6}{3/2}=\frac{-1}{11-\beta}=\frac{2}{19}$$

$$\alpha-6=\frac{3}{19} \qquad -19=22-2\beta$$

$$\alpha=6+\frac{3}{19}=\frac{117}{19} \qquad 2\beta=41$$

$$\therefore(19\alpha-6\beta)^2=(117-123)^2=36$$

13. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.

(1) $\frac{5}{14}$ (2) $\frac{3}{7}$ (3) $\frac{9}{28}$ (4) $\frac{2}{7}$

Sol. **(1)**

$P(A) = \frac{2}{10}$ $P(B) = \frac{3}{10}$ $P(C) = \frac{5}{10}$

$P(\text{Defective}/A) = \frac{3}{100}$, $P\left(\text{Defective}/B\right)=\frac{4}{100}$, $P\left(\text{Defective}/C\right)=\frac{2}{100}$

$$P(E) = \frac{5/10\times 2/100}{\frac{2}{10}\times\frac{3}{100}+\frac{3}{10}\times\frac{4}{100}+\frac{5}{10}\times\frac{2}{100}}=\frac{10}{6+12+10}$$

$$=\frac{10}{28}$$

$$=\frac{5}{14}$$

14. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 +i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

(1) $x^2 + 3x -4 = 0$ (2) $x^2 + 7x + 12= 0$

(3) $x^2 + x -12 = 0$ (4) $x^2 + 2x -3 = 0$

Sol. **(2)**

$|z + 2| = |\alpha + i\beta + 2|$

$= \alpha + i\beta + 4 + 4i$

$\sqrt{(\alpha+2)^2+\beta^2} = (\alpha+4)+i(\beta+4)$ $\qquad \beta + 4 = 0$

$(\alpha+2)^2+16=(\alpha+4)^2$ $\qquad \beta = -4$

$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$

$4 = 4\alpha$

$\alpha = 1$

$\alpha = 1, \beta = -4$

$\alpha + \beta = -3, \alpha\beta = -4$

Sum of roots = – 7

Product of roots = 12

$x^2 + 7x + 12 = 0$

15. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to ______

(1) 24 (2) 9 (3) 18 (4) 15

Sol. **(3)**

$$\lim_{x\to 0}\left[\frac{1-\cos^2 3x}{9x^2}\right]\frac{9x^2}{\cos^3 4x}\cdot\frac{\left(\frac{\sin 4x}{4x}\right)^3\times 64x^3}{\left[\frac{\ln(1+2x)}{2x}\right]^5\times 32x^5}$$

$$\lim_{x\to 0}2\left(\frac{1}{2}\times\frac{9}{1}\times\frac{1\times 64}{1\times 32}\right)=18$$

16. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(1) $7(720)^2$ (2) 720 (3) $7(360)^2$ (4) $126(5!)^2$

Sol. **(4)**

$6!\times {}^7C_5 \times 5!$

$\Rightarrow 720 \times 21 \times 120$

$\Rightarrow 2 \times 360 \times 7 \times 3 \times 120$

$\Rightarrow 126 \times (5!)^2$

17. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0.\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to

(1) $\frac{-2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{-1}{3\sqrt{3}}$ (4) $\frac{2}{3\sqrt{3}}$

Sol. **(2)**

$$f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f'(x) = -\frac{1}{2}\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f''(x) = -\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right) \cdot \tan\left(\frac{x}{2}\frac{-\pi}{8}\right) \cdot \frac{1}{2}$$

$$f\left(\frac{7\pi}{12}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$f''\left(\frac{7\pi}{12}\right) = -\frac{1}{2}\sec^2\frac{\pi}{6}.\tan\frac{\pi}{6} = \frac{-1}{2} \cdot \frac{4}{3} \times \frac{1}{\sqrt{3}} = \frac{-2}{3\sqrt{3}}$$

$$f\left(\frac{7\pi}{12}\right) \cdot f''\left(\frac{7\pi}{12}\right) = \frac{2}{9}$$

18. If the eqation of the plane containing the line x + 2y + 3z – 4 = 0 2x + y –z + 5 and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is ax+by +cz = 4, then (a-b+c) is equal to

(1) 22 (2) 24 (3) 20 (4) 18

Sol. **(1)**

D.R's of line $\vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$

D.R's of normal of second plane

$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$

A point on the required plane is $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$

The equation of required plane is

27x + 30y + 25z = 4

$\therefore$ a – b + c = 22

19. Let A = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If |adj(adj(adj2A))| =$(16)^n$, then n is equal to

(1) 8 (2) 9 (3) 12 (4) 10

Sol. **(4)**

|A| = 2[3] – 1[2] = 4

$\therefore \left|adj\left(adj\left(adj2A\right)\right)\right|$

$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$

$\Rightarrow \left(2^3|A|\right)^8 = 16^n$

$\Rightarrow \left(2^3 \times 2^2\right)^8 = 16^n$

$= 2^{40} = 16^n$
$= 16^{10} = 16^n \Rightarrow n = 10$

20. Let $I(x) = \int \frac{(x+1)}{x(1+x\,e^x)^2}\,dx$, x > 0. If $\lim_{x\to\infty} I(x) = 0$, then I(1) is equal to

(1) $\frac{e+1}{e+2} - \log_e(e+1)$ (2) $\frac{e+2}{e+1} + \log_e(e+1)$

(3) $\frac{e+2}{e+1} - \log_e(e+1)$ (4) $\frac{e+1}{e+2} + \log_e(e+1)$

Sol.

$$I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx$$

$1 + xe^x = t$

$(xe^x + e^x)dx = dt$

$(x+1)dx = \frac{1}{e^x}dt$

$$\therefore \int \frac{dt}{xe^x . t^2} \Rightarrow \int \frac{dt}{(t-1)t^2} \Rightarrow \int \frac{dt}{t(t-1)\cdot t} \Rightarrow \int \frac{t-(t-1)}{t(t-1)t}dt$$

$$\Rightarrow \int \frac{dt}{t(t-1)} - \int \frac{dt}{t^2} \Rightarrow \int \frac{t-(t-1)}{t(t-1)}dt + \frac{1}{t} + c$$

$$\Rightarrow \ln|t-1| - \ln|t| + \frac{1}{t} + c$$

$$\Rightarrow \ln|xe^x| - \ln|1+xe^x| + \frac{1}{1+xe^x} + c$$

$$I(x) = \ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + c$$

$$\lim_{x\to\infty} I(x) = c = 0$$

$$\therefore I(1) = \ln\left|\frac{e}{1+e}\right| + \frac{1}{1+e}$$

$$= \ln e - \ln(1+e) + \frac{1}{1+e}$$

$$= \frac{e+2}{e+1} - \ln(1+e)$$

SECTION - B

21. Let A = {0,3 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R ={x,y) $\in$ A × A: x – y is odd positive integer or x – y = 2}. The minimum number of elements that must be aadded to the relation R, so that it is a symmetric relation, is equal to ______.

Sol. **(19)**

A = {0, 3, 4, 6, 7, 8, 9, 10} 3, 7, 9 $\to$ odd

$R=\{x-y=odd+ve\text{ or } x-y=2\}$ 0, 4, 6, 8, 10 $\to$ even

${}^3C_1\cdot{}^5C_1=15+$ (6, 4), (8, 6), (10, 8), (9, 7)

Min^m ordered pairs to be added must be

: 15 + 4 = 19

22. Let (t) denote the greatest integer $\le$ t, If the constant term in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$ is α, then [α] is equal to ______.

Sol. **(1275)**

$$\left(3x^2-\frac{1}{2x^5}\right)^7$$

$$T_{r+1}={}^7C_r\left(3x^2\right)^{7-r}\left(-\frac{1}{2x^5}\right)^r$$

14 – 2r – 5r = 14 – 7r = 0

$\therefore r=2$

$$\therefore T_3={}^7C_2.3^5\left(-\frac{1}{2}\right)^2=\frac{21\times243}{4}=1275.75$$

$$\therefore\left[\alpha\right]=1275$$

23. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2},1,\lambda\right)$ and (–2, 0, 1) are at equal distance from the plane 2x + 3y – 6z + 7 = 0. If $\lambda_1>\lambda_2$, then the distance of the point ($\lambda_1-\lambda_2$, λ_2, λ_1) from the line $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$ is ______.

Sol. **9**

2x + 3y – 6z + 7 = 0 $\left(\frac{5}{2},1,\lambda\right),\left(-2,0,1\right)$

$$d_1=\left|\frac{5+3-6\lambda+7}{7}\right|=d_2=\left|\frac{-4-6+7}{7}\right|$$

$$\Rightarrow\left|15-6\lambda\right|=\left|3\right|$$

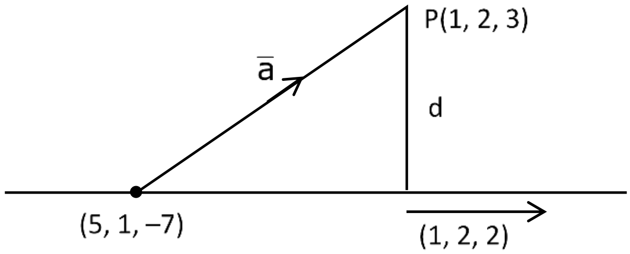
$15-6\lambda=3$ or $15-6\lambda=-3$

$6\lambda=12$ $6\lambda=18$

$\lambda=2$ $\lambda=3$

$\lambda_1=3$, $\lambda_2=2$

$\therefore P\left(1,2,3\right)$ $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$



$$d = \left|\frac{(4,-1,-10)\times(1,2,2)}{3}\right| = \left|\frac{18\hat{i}-18\hat{j}+9\hat{k}}{3}\right| = 9$$

24. If the solution curve of the differential equation $(y\text{-}2\log_e x)dx + (x\log_e x^2)\,dy = 0$, $x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and $\left(e^4, \alpha\right)$, then α is equal to ______.

Sol. **(3)**

$$(y-2\ln x)dx + (2x\ln x)dy = 0$$

$$dy(2x\ln x) = [(2\ln x) - y]dx$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{y}{2x\ln x}$$

$$\frac{dy}{dx} + \frac{y}{2x\ln x} = \frac{1}{x}$$

$$\text{I.F} = e^{\int \frac{1}{2x\ln x}dx}$$

$$= e^{\frac{1}{2}\int\frac{dt}{t}} = e^{\frac{1}{2}\ln(\ln x)}$$

$$\Rightarrow \text{I.F} = (\ln x)^{1/2}$$

$$\therefore y\sqrt{\ln x} = \int \frac{\sqrt{\ln x}}{x}dx \qquad \left(\text{Let}, \ln x = u^2\right)$$

$$= 2\int u^2 du \qquad \frac{1}{x}dx = 2udu$$

$$y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{3/2} + c \leftarrow \left(e, \frac{4}{3}\right)$$

$$\frac{4}{3} = \frac{2}{3} + c \Rightarrow c = \frac{2}{3}$$

$$y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{3/2} + \frac{2}{3} \leftarrow \left(e^4, \alpha\right)$$

$$\alpha \cdot 2 = \frac{2}{3}\times 8 + \frac{2}{3}$$

$$\alpha = 3$$

25. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} - = \vec{a} \times \vec{b}$. If $\vec{a}.\vec{c} = -12, \vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c}.(\hat{i} + \hat{j} + \hat{k})$ is equal to ___.

Sol. **(11)**

$\vec{a} \times \vec{c} = \vec{a} \times 5$

$\Rightarrow \vec{a} \times \left(\vec{c} - \vec{b}\right) = 0$

$\vec{a} \, ||^{rl} \left(\vec{c} - \vec{b}\right)$

$\therefore \vec{a} = \lambda\left(\vec{c} - \vec{b}\right)$

$(6,9,12) = \lambda[x - \alpha, y - 11, z + 2]$

$\frac{x-\alpha}{2} = \frac{y-11}{3} = \frac{z+2}{4}$

4y – 44 = 3z + 6

4y – 3z = 50

6x + 9y + 12z = – 12

2x + 3y + 4z = – 4 $\quad (\because x - 2y + z = 5)$

2x – 4y + 2z = 10

$\quad$ + $\quad$ – $\quad$ –

7y + 2z = – 14 …(2)

8y – 6z = 100

21y + 6z = – 42

29y = 58

y = 2, z = – 14

$\therefore$ x – 4 – 14 = 5

x = 23

$\bar{c} = (23, 2, -14)$

$\bar{c} \cdot (1,1,1) = 23 + 2 - 14 = 11$

26. The largest natural number n such that 3^n divides 66! is ______.

Sol. **(31)**

$\left[\frac{66}{3}\right] + \left[\frac{66}{9}\right] + \left[\frac{66}{27}\right]$

22 + 7 + 2 = 31

27. If a_a is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}$, n = 1,2,3,..., then a is equal to ______.

Sol. **(0.158)**

$f(x) = \frac{x^3}{x^4 + 147}$

$f'(x) = \frac{\left(x^4 + 147\right)3x^2 - x^3\left(4x^3\right)}{\left(x^4 + 147\right)^2}$

$= \frac{3x^6 + 147 \times 3x^2 - 4x^6}{+ve} = x^2\left(44 - x^4\right)$

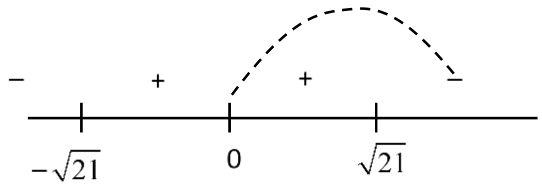
$f'(x) = 0$ at $x^6 = 147 \times 3x^2$

$x^2 = 0,\ x^4 = 147 \times 3$

$x = 0,\ x^2 = \pm\sqrt{147 \times 3}$

$x^2 = \pm 21$

$x = \pm\sqrt{21}$



fmax at f(4) or f(5)

$f(4) = \frac{64}{403} \simeq 0.158 \qquad f(5) = \frac{125}{772} \simeq 0.161$

$\therefore\ a = 5$

28. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If x > y, then 3x -2y is equal to _________.

Sol. **(25)**

$\frac{x+y+52}{8} = 9 \Rightarrow\ x + y = 20$

For variance

$x - 9,\ y - 9,\ 3,\ 3,\ 1,\ -5,\ -1,\ -3$

$\bar{x} = 0$

$\therefore\ \frac{(x-9)^2 + (y-9)^2 + 54}{8} - \bar{0}^2 = 9.25$

$(x-9)^2 + (11-x)^2 = 20$

$x = 7$ or $13\ \therefore\ y = 13, 7$

$3x - 2y = 3 \times 13 - 2 \times 7 = 25$

29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to _______.

Sol. **(2)**

$x^2 + y^2 - 4x - 2y + 5 - \alpha = 0,$

$C_1(2,1)\ \ r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

Image of (2, 1)

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$

$x = 2 - \frac{16}{5} = \frac{-6}{5},\ y = 1 + \frac{8}{5} = \frac{13}{5}$

$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$

$C_2(f, g)$

$$r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$$

$$\alpha = f^2 + g^2 - \frac{36}{5}$$

$$\therefore f = -\frac{6}{5}, g = \frac{13}{5}$$

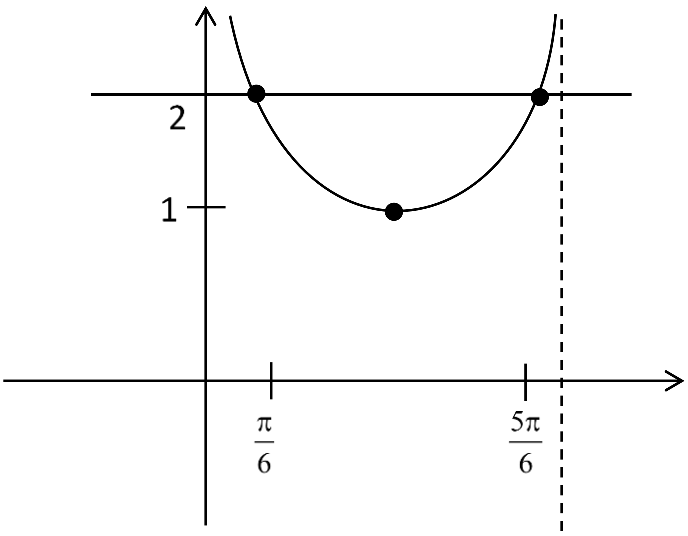
$$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}$$

$$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$$

$$\therefore \alpha + r = 2$$

30. Let [t] denote the greatest integer ≤ t. The $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])\,dx$ is equal to ______.

Sol. **(14)**



$$8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\operatorname{cosec} x]\,dx$$

$$8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dx = \frac{16\pi/3}{16\pi/3}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x]\,dx$$

$$x \to \pi - x$$

$$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [-\cot x]\,dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left([\cot x] + [-\cot x] \right) dx$$

$$I = -\frac{1}{2} \int_{\pi/6}^{5\pi/6} dx \Rightarrow -\frac{1}{2}\left(\frac{4\pi}{6}\right)$$

$$= -\pi/3 .$$

$$\therefore \frac{2}{\pi}\left[\frac{16\pi}{3} + \frac{5\pi}{3}\right] = \frac{2}{\pi}\left(\frac{21\pi}{3}\right)$$

$= 14$

SECTION - A

31. A cylindrical wire of mass (0.4 ± 0.01)g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be:

(1) 4% (2) 1% (3) 3.5% (4) 5%

Sol. (1)

Cylindrical wire $m = (0.4 \pm 0.01)$ g

$\ell = (8 \pm 0.04)$ cm

$r = (6 \pm 0.03)$ mm

Density $\rho = \frac{m}{\pi r^2 \ell} \Rightarrow \rho r^2 \ell m^{-1} = \frac{1}{\pi}$ = const.

Differentiating after taking log on both side

$$\frac{d\rho}{\rho} + \frac{2dr}{r} + \frac{d\ell}{\ell} - \frac{dm}{m} = 0$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta\ell}{\ell} - \frac{2\Delta r}{r}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left[\frac{0.01}{0.4} + \frac{0.04}{8} + 2\left(\frac{0.03}{6}\right)\right]$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = 0.04$$

Percentage error = $0.04 \times 100 = 4\%$

32. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is : (neglect air speed. Speed of sound in air = 330 ms^{-1})

(1) 400 Hz (2) 388 Hz (3) 200 Hz (4) 412 Hz

Sol. (1)

The passenger inside the train is at rest wrt train so frequency heard by passenger inside the train is same as the source frequency i.e., 400 Hz.

33. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) 300 N (2) Zero (3) 100 N (4) 200 N

Sol. (4)

Weight on the earth surface = mg

mg = 400 N (given)

Weight at a depth d $w = m\left(\frac{GM(R-d)}{R^3}\right)$

$$W = mg\left(1 - \frac{d}{R}\right)$$

$$d = \frac{R}{2} \Rightarrow w = mg\left(1 - \frac{1}{2}\right) \Rightarrow w = \frac{mg}{2}$$

w = 200 N

34. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km^2 (2) 1549 km^2 (3) 4868 km^2 (4) 3942 km^2

Sol. (4)

Max. distance covered $d = \sqrt{2Rh_T}$

(R = radius of earth, h_T = height of antenna)

Area $A = \pi d^2$

$A = \pi(2Rh_T)$

$A = 2 \times 3.14 \times 6400 \times 98 \times 10^{-3}$

$A \approx 3942\ km^2$

35. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

(1) To produce large deflecting torque on the coil
(2) To bring the coil to rest quickly
(3) To oscillate the coil in magnetic field for longer period of time
(4) To make the magnetic field radial

Sol. **(2)**

To bring the coil at rest quickly

36. Dimension of $\frac{1}{\mu_0 \in_0}$ should be equal to

(1) T/L (2) T^2 / L^2 (3) L/T (4) L^2/T^2

Sol. **(4)**

Dimension of $\frac{1}{\mu_0\varepsilon_0}$

$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = c^2$

$\left[\frac{1}{\mu_0\varepsilon_0}\right] = [c^2]$

$= \left[\frac{L^2}{T^2}\right]$

37. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ($g = 10\ m/s^2$)

(1) $2 : \sqrt{3}$ (2) $\sqrt{3} : 2$ (3) 4 : 9 (4) 1 : 1

Sol. **(3)**

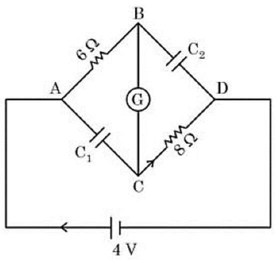
$R = \frac{u^2 \sin 2\theta}{g}$

$\{u_1 = 40\ m/s,\ \theta_1 = 30°,\ u_2 = 60\ m/s,\ \theta_2 = 60°\}$

$\frac{R_1}{R_2} = \left(\frac{u_1}{u_2}\right)^2 \frac{\sin 2\theta_1}{\sin 2\theta_2}$

$\frac{R_1}{R_2} = \left(\frac{40}{60}\right)^2 \times \frac{\sin 60°}{\sin 120°} \Rightarrow \frac{R_1}{R_2} = \frac{4}{9}$

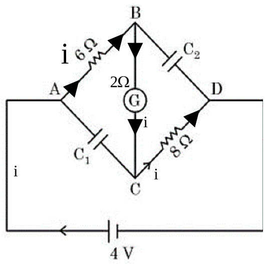
38. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(1) $\frac{5}{4}$ (2) 1 (3) $\frac{4}{5}$ (4) $\frac{3}{4}$

Sol. **(3)**

At steady state current will not be in the capacitor branch.



$$i = \frac{4}{6+2+8}$$

$$i = \frac{1}{4}A$$

$$\Delta V_{C_1} = i(6+2)$$

$$\Delta V_{C_2} = i(2+8)$$

$$\frac{\Delta V_{C_1}}{\Delta V_{C_2}} = \frac{4}{5}$$

39. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(1) Helical path with the axis along magnetic field B

(2) Straight along the direction of magnetic field B

(3) Helical path with the axis perpendicular to the direction of magnetic field B

(4) Circular path

Sol. **(1)**

Path will be helical with axis along uniform $\vec{B}$ -.

40. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$):

(1) 1 : 43 (2) 43 : 1 (3) 1 : 1849 (4) 1 : 1

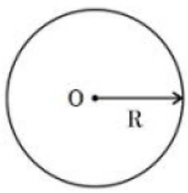
Sol. **(4)**

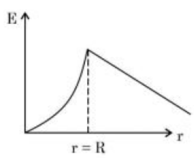
Debroglie wavelength $\lambda = \frac{h}{p}$

$\lambda_p = \lambda_e$

$$\frac{h}{p_p} = \frac{h}{p_e} \Rightarrow \frac{p_p}{p_e} = 1$$

41. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



(1)


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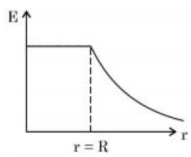

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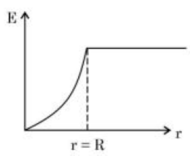

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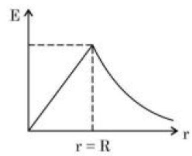
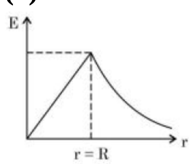


Sol. **(4)**



Electric field due to uniformly charged insulating solid sphere

$$E = \begin{cases} \dfrac{kQr}{R^3} & r \le R \\ \dfrac{kQ}{r^2} & r \ge R \end{cases}$$

42. For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon (b_s) = $-a_1 A^{2/3}$.

B. The Coulomb contribution to the binding energy $b_c = -a_2 \dfrac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the most appropriate answer from the options given below:

(1) B, C only (2) A, B, C, D only (3) B, C, E only (4) C, D only

Sol. **(4)**

$$\mathbf{E_B = a_vA - a_s\,A^{2/3} - a_A}\,\frac{(A-2Z)^2}{A^{1/3}} - a_c\,\frac{Z(Z-1)}{A^{1/3}} + \delta\,(A,Z)$$

Volume term **Surface term** **Asymmetry term** **Coulomb term** **Pairing term**

Most appropriate is option (4)

43. At any instant the velocity of a particle of mass 500 g is $\left(2t\hat{i} + 3t^2\hat{j}\right)$ ms^{-1}. If the force acting on the particle at t = 1s is $\left(\hat{i} + x\hat{j}\right)N$. Then the value of x will be:

(1) 2 (2) 6 (3) 3 (4) 4

Sol. **(3)**

$\vec{V} = (2t\hat{i} + 3t^2\hat{j})\,m/s$, mass m = 500 gm

Force, $\vec{F} = m\vec{a}$ -

$\vec{F} = \frac{1}{2}\left(\frac{d\vec{v}}{dt}\right) \Rightarrow \vec{F} = \frac{1}{2}\left(2\hat{i} + 6t\hat{j}\right)$

$\vec{F} = \left(\hat{i} + 3t\hat{j}\right)$

At t = 1 s $\Rightarrow$ $\vec{F} = \left(\hat{i} + 3\hat{j}\right)$

x = 3

44. Given below are two statements:

Statement I : If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$

Statement II : The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. **(1)**

For satellite K.E. $= \frac{1}{2}\ mv^2 = \frac{1}{2}\ m\left(\sqrt{\frac{GM}{r}}\right)^2$

K.E. $= \frac{GMm}{2r}$

Potential energy $U = -\frac{GMm}{r}$

Total energy = K.E + U

$E = -\frac{GMm}{2r}$

U = 2E   St I – incorrect

K.E. = |E| St II - incorrect

45. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is:

(1) $\frac{5A}{2}$ (2) $\frac{\sqrt{5}A^2}{2}$ (3) $\frac{\sqrt{5}A}{4}$ (4) $\frac{\sqrt{5}A}{2}$

Sol. **(4)**

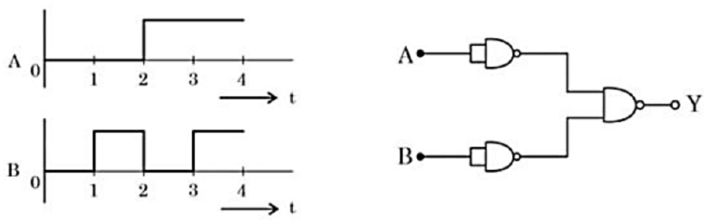
$\left|\vec{F}_1\right| = A\,,\ \left|\vec{F}_2\right| = \frac{A}{2} \qquad \theta = \frac{\pi}{2}$

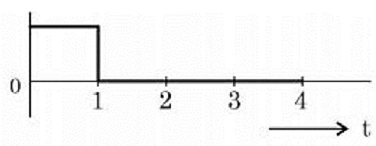
$\left|\vec{F}_{net}\right| = \sqrt{F_1^2 + F_2^2}$

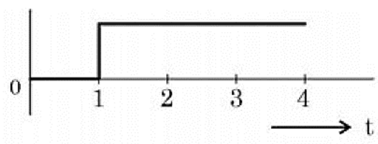
$= \sqrt{A^2 + \left(\frac{A}{2}\right)^2}$

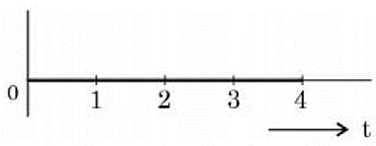
$\left|\vec{F}_{net}\right| = \frac{\sqrt{5}A}{2}$

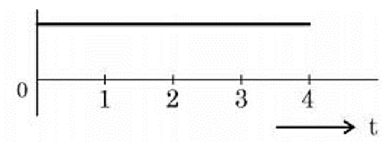
46. For the logic circuit shown, the output waveform at Y is:



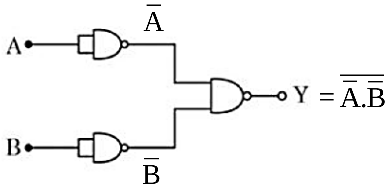
(1)


(2)


(3)


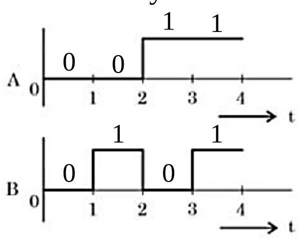
(4)


Sol. **(2)**

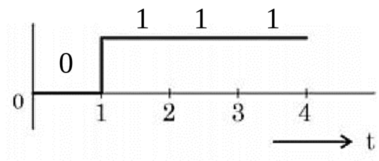


$y = \overline{\overline{A}.\overline{B}} \Rightarrow y = \overline{\overline{A}} + \overline{\overline{B}}$

$y = A + B$



A	B	y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



47. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(1) 5600 (2) 2800 (3) 11200 (4) 8400

Sol. **(1)**

Aluminium rod Young's modulus

$y = 7.0 \times 10^{10} \frac{N}{m^2}$

strain 0.04%

strain = $\frac{0.04}{100}$

Energy per unit volume = $\frac{1}{2}$ stress × strain

$= \frac{1}{2}$ y strain × strain

$= \frac{1}{2}$ y (strain)^2

$= \frac{1}{2} \times 7 \times 10^{10} \times \left(\frac{0.04}{100}\right)^2$

Energy per unit volume = $5600 \frac{J}{m^3}$

48. Given below are two statements:

Statement I : If heat is added to a system, its temperature must increase.

Statement II : If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true (2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false (4) Statement I is false but Statement II is true

Sol. **(4)**

St I False

Ex. in isothermal process temp. is constant but heat can be added.

ST II True

$w = \int PdV$

If volume increases the w = +ve

49. An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be:

(1) 3 cm^3 (2) 4 cm^3 (3) 2 cm^3 (4) 5 cm^3

Sol. **(4)**

Pressure at surface = $P_{atm} = 1 \times 10^5$ Pa

$v_{surface} = ?$

Pressure at h = 40 m depth

$P = P_{atm} + \rho gh$

$P = 10^5 + 10^3 \times 10 \times 40$

$P = 5 \times 10^5$ Pa

$\nu = 1\ cm^3$

Temp. is constant

$P_1V_1 = P_2V_2$

$10^5 \times \nu = 5 \times 10^5 \times 1$

$\nu = 5\ cm^3$

50. In a reflecting telescope, a secondary mirror is used to:

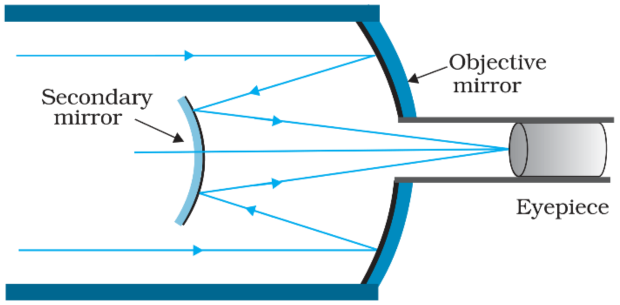
(1) Make chromatic aberration zero

(2) Reduce the problem of mechanical support

(3) Move the eyepiece outside the telescopic tube

(4) Remove spherical aberration

Sol. **(3)**



To move the eye piece outside the telescopic tube

SECTION – B

51. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is __________ %.

Sol. **(125)**

$$K_i = \frac{P_i^2}{2m}$$

$$K_f = \frac{\left(P_i + \frac{P_i}{2}\right)^2}{2m} \Rightarrow K_f = \frac{9}{4}\frac{P_i^2}{2m}$$

Percentage increase in K.E. = $\dfrac{K_f - K_i}{K_i} \times 100$

$$= \frac{\frac{9}{4} - 1}{1} \times 100$$

$$= \frac{5}{4} \times 100 = 125\%$$

52. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is ___________ MeV.

Sol. **(121)**

Gain in binding energy = $B.E_f - BE_i$

$= 2(121 \times 8.1) - 242 \times 7.6$

$= 121$ MeV

53. An electric dipole of dipole moment is 6.0×10^{-6} C m placed in a uniform electric field of 1.5×10^3 NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be ___________ mJ.

Sol. **(18)**

$W_{ext} = U_f - U_i \qquad \left\{U = -\vec{P}.\vec{E}\right\}$

$= -PE\cos\pi - (-PE\cos 0)$

$= 2PE$

$= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^3$

$= 18$ mJ

54. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is ___________ Hz.

Sol. **(900)**

Open organ pipe $\ell = 40$ cm

Speed of sound v = 360 m/s

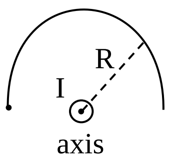
Frequency of second harmonics $f_2 = \dfrac{2v}{2\ell}$

$f_2 = \dfrac{v}{\ell} \Rightarrow f_2 = \dfrac{360}{0.4}$

$f_2 = 900\,\text{Hz}$

55. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\dfrac{1}{x}$ MR^2, where R is the radius and M is the mass of the semicircular ring. The value of x will be ___________.

Sol. **(1)**



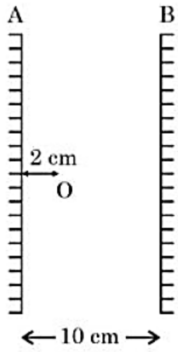
$I = \int dmR^2 \Rightarrow R^2\int dm = MR^2$

$I = MR^2$

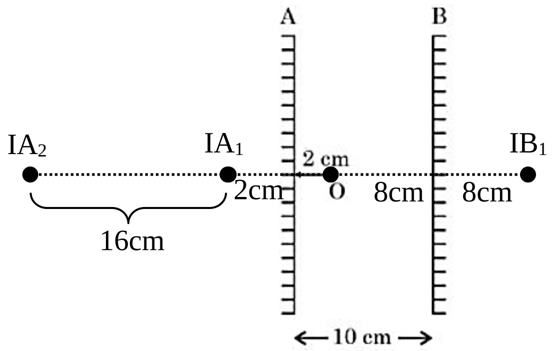
Given $I = \dfrac{1}{x}MR^2$

$x = 1$

56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _________ cm.



Sol. **(18)**



d = 2 + 16

d = 18 cm

57. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^3 Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is ___________ A.

Sol. **(2)**

$H = 1.6 \times 10^3$ A/m, n = 8 per cm = 800 per m

$$H = nI \Rightarrow I = \frac{H}{n}$$

$$I = \frac{1.6 \times 10^3}{8 \times 10^2} \Rightarrow I = 2\text{ A}$$

58. A current of 2 A through a wire of cross-sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is ________ $\times 10^{-6}$ ms^{-1}
(given, charge on electron = 1.6×10^{-19} C).

Sol. **(25)**

$I = neAV_d$

$$V_d = \frac{I}{neA} \Rightarrow V_d = \frac{2}{2 \times 10^{28} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}$$

$V_d = 25$ m/s

59. An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be __________ mA.

Sol. **(9)**

LC oscillation L = 75 mH

C = 1.2 μF

$U_{\max L} = U_{\max C}$

$$\frac{1}{2}\ LI^2_{\max} = \frac{1}{2}\ \frac{q^2_{\max}}{C}$$

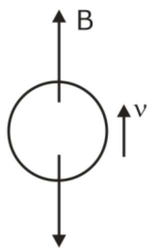
$$I_{\max} = \frac{q_{\max}}{\sqrt{LC}} \Rightarrow I_{\max} = \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}}$$

$I_{\max} = 9 \times 10^{-3}$ A

$I_{\max} = 9$ mA

60. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. TGe co-efficient of viscosity of the solution (neglect density of air) is __________ Pas (given, g = 10 ms^{-2}).

Sol. **(10)**



F_v=6pηrv

For uniform velocity net force = 0

B = 6πηrv

$$\rho\ \frac{4}{3}\pi r^3 g = 6\pi\eta rv$$

$$\eta = \frac{2\rho r^2 g}{9v}$$

$$\eta = \frac{2\times1750\times(3\times10^{-3})^2\times10}{9\times0.35\times10^{-2}}$$

η = 10 Pa-s

SECTION - A

61. The reaction

$$\frac{1}{2}H_2(g)+Ag(Cl)(s) \rightleftharpoons H^+(aq)+Cl^-(aq)+Ag(s)$$

occurs in which of the given galvanic cell.

(1) $Pt|H_2(g)|HCl(sol^n)|AgNO_3(sol^n)|Ag$ (2) $Pt|H_2(g)|HCl(sol^n)|AgCl(s)|Ag$

(3) $Pt|H_2(g)|KCl(sol^n)|AgCl(s)|Ag$ (4) $Ag|AgCl(s)|KCl(sol^n)|AgNO_3|Ag$

Sol. **2**

Anode $\rightarrow H_2 \rightarrow 2H^+ + 2e^-$

Cathode $\rightarrow AgCl + e^- \rightarrow Ag + Cl^-$

62. Sulphur (S) containing amino acids from the following are:

(a) isoleucine (b) cysteine (c) lysine (d) methionine

(e) glutamic acid

(1) b, c, e (2) a, d (3) a, b, c (4) b, d

Sol. **4**

(a) isoleucine : $CH_3-CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(b) cysteine : $HS-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(c) lysine : $H_2N-(CH_2)_4-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(d) methionine : $CH_3-S-CH_2-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(e) glutamic acid : $HOOC-CH_2-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

63. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $K_3[Co(CN)_6]$ (2) $[Ni(NH_3)_6]Cl_2$ (3) $[Co(H_2O)_6]Cl_2$ (4) $Na_3[CoCl_6]$

Sol. **1**

$K_3[Co(CN)_6]$

$+ 3 + x - 6 = 0$

$\boxed{x = +3}$

$\Downarrow$

$Co^{+3} \rightarrow 3d^6$

⇅	↑	↑	↑	↑

$\because CN^-$ is SFL so pairing occur so

⇅	⇅	⇅		

u-e = 0

$\downarrow$

So diamagnetic

64. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu (2) Au (3) Pb (4) Sn

Sol. **4**

Sn due to Amphoteric nature.

65. The correct order of spin only magnetic moments for the following complex ions is

(1) $[CoF_6]^{3-} < [MnBr_4]^{2-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(2) $[Fe(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-} < [Mn(CN)_6]^{3-}$

(3) $[MnBr_4]^{2-} < [CoF_6]^{3-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(4) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

Sol. **4**

$[CoF_6]^{3-}$	$[MnBr_4]^{2-}$	$[Fe(CN)_6]^{3-}$	$[Mn(CN)_6]^{3-}$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
CO^{+3}	Mn^{+2}	$3d^5$	Mn^{+3}
F^- WFL	Br^- WFL	CM^- SFL	CN^- is SFL
So no pairing	So no pairing	Pairing occur	So pairing occur
$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$	$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$	$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ $\downarrow$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{}\boxed{}$	$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}$
u.e. = 4	u.e. = 5	u.e. = 1	u.e. = 2

$[Fe(CN)_6]^{-3} < [Mn(CN)_6]^{-3} < [CoF_6]^{-3} < [MnBr_4]^{-2}$

66. The water gas on reacting with cobalt as a catalyst forms

(1) Methanoic acid (2) Methanal (3) Ethanol (4) Methanol

Sol. **4**

$$(CO + H_2) + H_2 \xrightarrow[ZnO,\ Cr_2O_3]{700K} CH_3OH$$

$\downarrow$

Catalyst

67. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

What is the value of x?

(1) 12 (2) 10 (3) 2 (4) 6

Sol. **2**

n factor of IO_3^- and I^- in the given redox reaction are 5 and 1 respectively. Therefore, IO_3^- will always react in the molar ratio 1 : 5 to get I_2.

$IO_3^- + 6H^+ + 5I^- \rightarrow 3I_2 + 3H_2O$

To get 6 molar I_2, multiple equation by 2

$2IO_3^- + 12H^+ + 10I^- \rightarrow 6I_2 + 6H_2O$

So, x = 10

68. What is the purpose of adding gypsum to cement?

(1) To give a hard mass (2) To speed up the process of setting

(3) To facilitate the hydration of cement (4) To slow down the process of setting

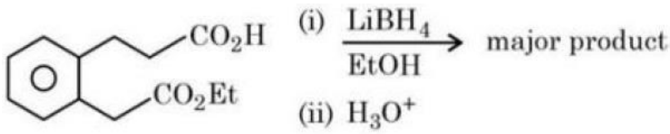
Sol. **4**

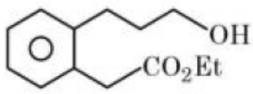
$CaSO_4 \cdot 2H_2O$ Gypsum

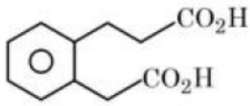
To slow down the process of setting.

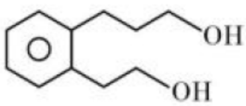
Gypsum is added to control the 'setting of cement'. If not added, the cement will set immediately after miximg of water leaving no time the concrete placing.

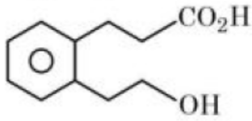
69. The major product formed in the following reaction is:



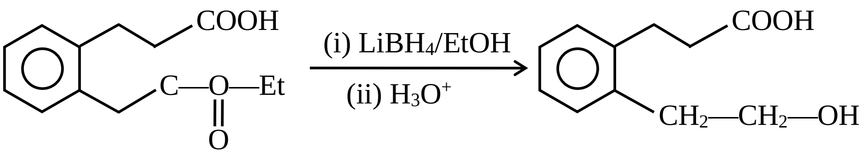
(1) 

(2) 

(3) 

(4) 

Sol. **4**



Note: Lithium borohydride is commonly used for selective reduction of esters and lactones to the corresponding alcohol.

70. Match list I with list II:

List I (species)	List II (Maximum allowed concentration in ppm in drinking water)
A. F^-	I. < 50 ppm
B. SO_4^{2-}	II. < 5 ppm
C. NO_3^-	III. < 2 ppm
D. Zn	IV. < 500 ppm

(1) A-III, B-II, C-I, D-IV
(2) A-II, B-I, C-III, D-IV
(3) A-IV, B-III, C-II, D-I
(4) A-I, B-II, C-III, D-IV

Sol. **Bouns**
Data based

	Maximum allowed (ppm)
F^-	< 2 ppm
SO_4^{2-}	< 5 ppm
NO_3^-	< 50 ppm
Zn	< 500 ppm

71. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)
(1) Fe (III) (2) V (IV) (3) Ti (III) (4) Mn (VII)

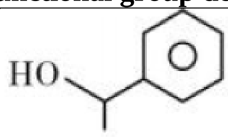
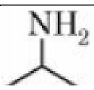
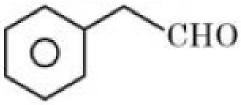
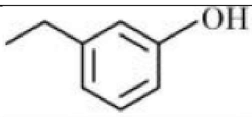
Sol. **4**
$CrO_2Cl_2 \rightarrow$ Chromyl chloride
$\Downarrow$
$Cr^{+6} \rightarrow 4s^0\ 3d^0$
Mn(vii) $\rightarrow Mn^{+7}$ — Same
$\Downarrow$
$4s^0\ 3d^0$

72. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Butan-1-ol has higher boiling point than ethoxyethane.
Reason R : Extensive hydrogen bonding leads to stronger association of molecules.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are true but R is not the correct explanation of A
(2) Both A and R are true and R is the correct explanation of A
(3) A is false but R is true
(4) A is true but R is false

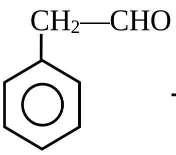
Sol. **2**
At comparable molecular mass, alcohol has higher b.p. than ether due to H-bond, because H-bond leads to stronger associated of molecules.

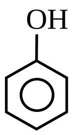
73. Match List I with List II:

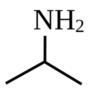
List I (Reagents used)	**List II (Compound with Functional group detected)**
A. Alkaline solution of copper sulphate and sodium cirate	I. 
B. Neutral $FeCl_3$ solution	II. 
C. Alkaline chloroform solution	III. 
D. Potassium iodide and sodium hypochlorite	IV. 

Choose the correct answer from the options given below:
(1) A-III, B-IV, C-II, D-I
(2) A-II, B-IV, C-III, D-I
(3) A-IV, B-I, C-II, D-III
(4) A-III, B-IV, C-I, D-II

Sol. **1**

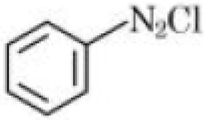
(A)  $\xrightarrow[\text{Sulphate and sodium citrate}]{\text{Alkaline solution of copper}}$ Red PPt

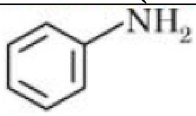
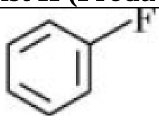
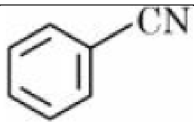
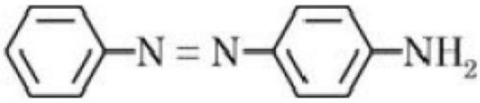
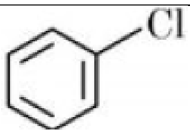
(B) Neutral $FeCl_3$  $\longrightarrow$ Voilet colour

(C) Alkaline chloroform solution  $\longrightarrow$ Carbyl amine reaction

(D) $KI + \overset{+}{Na}OCl \xrightarrow{CH_3-\underset{}{\overset{OH}{\overset{|}{C}}}H-Ph}$ Haloform reaction

74. Match List I with List II:

 is reacted with reagents in List I to form products in List II.

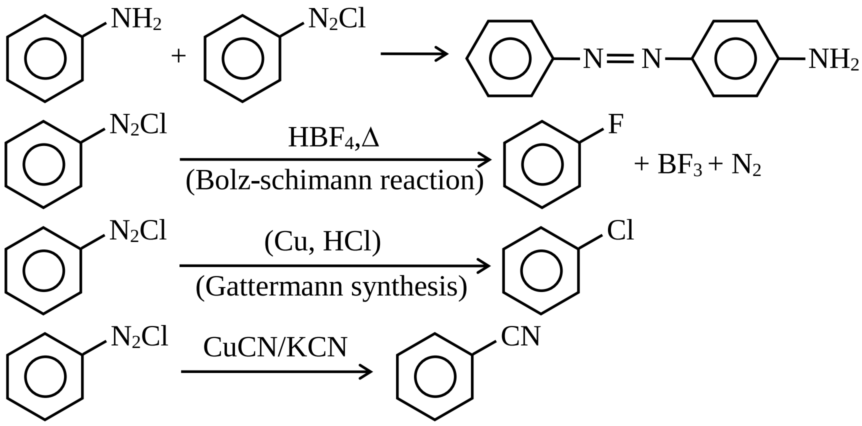
List I (Reagent)	List II (Product)
A. 	I. 
B. HBF_4, Δ	II. 
C. Cu, HCl	III. 
D. CuCN/KCN	IV. 

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II
(2) A-III, B-I, C-II, D-IV
(3) A-III, B-I, C-IV, D-II
(4) A-IV, B-III, C-II, D-I

Sol. **3**

(A) (B) (C) (D)



75. Match List I with List II:

List I	List II
A. Saccharin	I. High potency sweetener
B. Aspartame	II. First artificial sweetening agent
C. Alitame	III. Stable at cooking temperature
D. Sucralose	IV. Unstable at cooking temperature

Choose the **correct** answer from the options given below:

(1) A-II, B-III, C-IV, D-I
(2) A-II, B-IV, C-I, D-III
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-IV, C-III, D-I

Sol. **2**

(A) Saccharin → First artificial sweetening agent
(B) Aspartame → Unstable at cooking temperature used in soft drink and cold drink.
(C) Alitame → High potency sweetener (2000 more sweeter than cane sugar)
(D) Sucralose → Stable at coocking temperature. Also it does not provide calories.

76. The correct order of electronegativity for given elements is:

(1) P > Br > C > At (2) C > P > At > Br (3) Br > P > At > C (4) Br > C > At > P

Sol. **4**

C (2.5)

P (2.1) $\Rightarrow$ Br > C > At > P

Br (2.8)

At (2.2)

77. Given below are two statements :

Statement I : Lithium and Magnesium do not form superoxide

Statement II : The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}

In the light of the above statements, choose the **most appropriate** answer from the options given **below**:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both statement I and Statement II are correct

(4) Both statement I and Statement II are incorrect

Sol. **3 (Fact-based)**

Due to small in size Li and Mg do not from superoxide.

$Li^+ \geq Mg^{+2}$ - radius

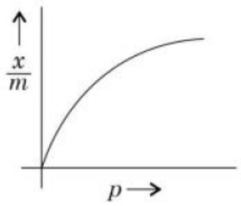
$2e^-$ $10e^-$

$\downarrow$

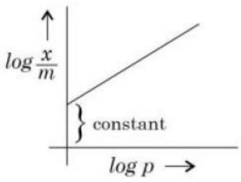
Due to diagonal relationship.

78. Which of the following represent the Freundlich adsorption isotherms?

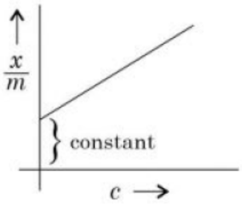
A.



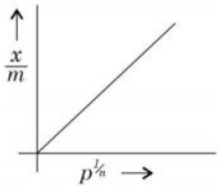
B.



C.



D.

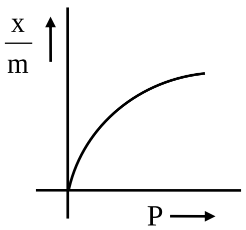


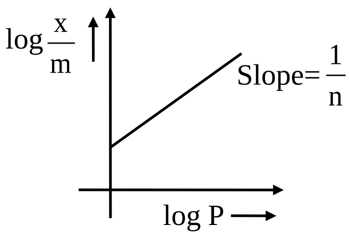
Choose the correct answer from the options given below:

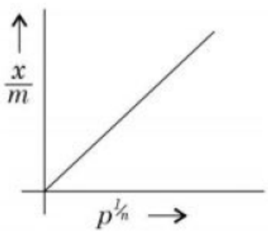
(1) A, C, D only (2) A, B only (3) A, B, D only (4) B, C, D only

Sol. **3**

$\frac{x}{m} = Kp^{1/n}$ $\log\frac{x}{m} = \log K + \frac{1}{n}\log P$







79. Which halogen is known to cause the reaction given below:

$2Cu^{2+} + 4X^{-} \rightarrow Cu_2X_2(s) + X_2$

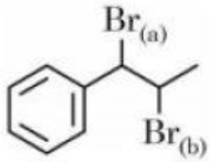
(1) All halogens (2) Only chlorine (3) Only Bromine (4) Only Iodine

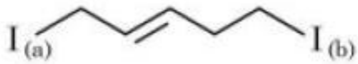
Sol. **4**

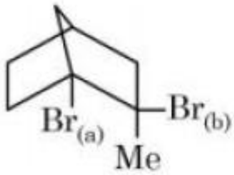
(Only iodine)

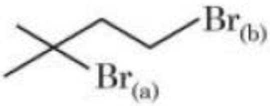
$2Cu^{2+} + 4I^{-} \rightarrow Cu_2I_2 + I_2$

80. Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C, & D)

A. 

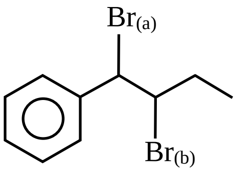
B. 

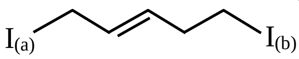
C. 

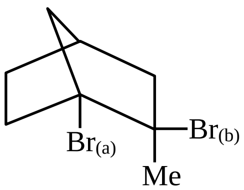
D. 

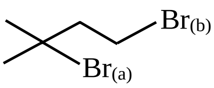
(1) A-$Br_{(a)}$; B-$I_{(a)}$; C-$Br_{(b)}$; D-$Br_{(a)}$
(2) A-$Br_{(b)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$
(3) A-$Br_{(b)}$; B-$I_{(b)}$; C-$Br_{(b)}$; D-$Br_{(b)}$
(4) A-$Br_{(a)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$

Sol. **1**

(A)  $\rightarrow$ Because formed intermediate carbocation formed by $Br_{(a)}$ get stabilised by conjugation with phenyl ring

(B)  $\rightarrow$ Because the intermediate carbocation formed by $I_{(a)}$ become more stable by conjugation

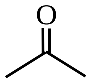
(C)  $\rightarrow$ Becouse, we con't remove $Br_{(a)}$ from bridge head carbon (Bredt's rule)

(D)  $\rightarrow$ Becouse, formed intermediate by $Br_{(a)}$, 3° carbocation is more stable (stability of carbocation 3° > 2° > 1°)

SECTION - B

81. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanol and propanone is ___________g mol^{-1} (Molar mass of C : 12 g mol^{-1}, H : 1 $gmol^{-1}$)

Sol. **70**

Reactant $\xrightarrow{O_3}$  + CH_3CHO

$$CH_3-\overset{\overset{\displaystyle CH_3}{|}}{C}=CH-CH_3$$

(C_5H_{10})

Molecular Mass = 70

82. XeF_4 reacts with SbF_5 to form

$[XeFm]^{n+}$ $[SbF_y]^{z-}$

m+n+y+z =_________

Sol. **11**

$XeF_4 + SbF_5 \rightarrow [XeF_3]^+ (SbF_6)^-$

m + n + x + y = 3 + 1 + 6 + 1 = 11

Xenon fluoride act as F^- donor and F^- acceptor.

83. The number of following statements which is/are incorrect is_________

(1) Line emission spectra are used to study the electronic structure

(2) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet

(3) An absorption spectrum is like the photographic negative of an emission spectrum

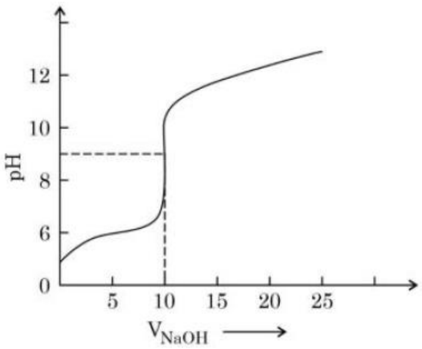
(4) The element helium was discovered in the sun by spectroscopic method

Sol. **1**

Fact

84. The titration curve of weak acid vs. strong base with phenolphthalein as indictor) is shown below. The $K_{phenolphthalein} = 4\times10^{-10}$

Given : log 2 = 0.3



The number of following statements/s which is/are correct about phenolphthalein is__________

(1) It can be used as an indicator for the titration of weak acid with weak base.

(2) It begins to change colour at pH = 8.4

(3) It is a weak organic base

(4) It is colourless in acidic medium

Sol. **2**

(B) $pk_n = -\log(4\times10^{-10}) = 9.4$

Indicator range

$\Rightarrow$ $pk_{In} \pm 1$

i.e. 8.4 to 10.4

(D) In acidic medium, phenolphthalein is in unionized form and is colourless.

85. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5ºC. The heat capacity of the given gas is_______J K^{-1} (Nearest integer)

Sol. **1200**

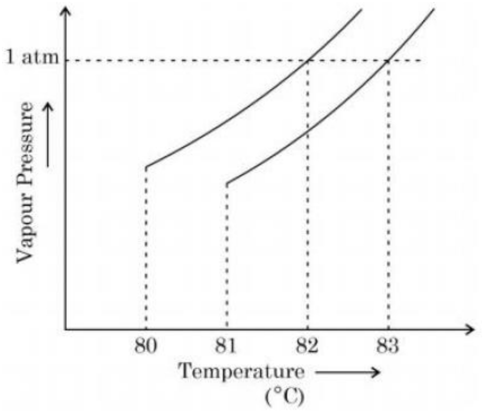
Adiabatic wall {no heat exchange between system and surrounding}

$C_v \times \Delta T = P \times t/sec$

$C_v \times 5 = 60 \times 100$

$C_v = 1200$

86. The vapour pressure vs. temperature curve for a solution solvent system is shown below:



The boiling point of the solvent is______ºC

Sol. **82**

Boiling point of solvent is 82ºC

Boiling point of solvent is 83ºC

87. 0.5 g of an organic compound (X) with 60% carbon will produce______$\times 10^{-1}$ g of CO_2 on complete combustion.

Sol. **11**

Moles of carbon = $\frac{0.5 \times 0.6}{12}$

Moles of CO_2 = $\frac{0.5 \times 0.6}{12}$

Mass of CO_2 = $\frac{0.5 \times 0.6}{12} \times 44 = 11 \times 10^{-1}$ gram

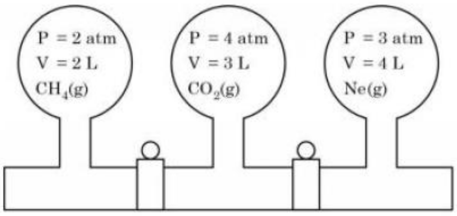
88. The number of following factors which affect the percent covalent character of the ionic bond is_____

(1) Polarising power of cation
(2) Extent of distortion of anion
(3) Polarisability of the anion
(4) Polarising power of anion

Sol. **3**

Percent covalent character of the ionic bond

(1) Polarising power of cation
(2) Extent of distortion of anion
(3) Polarisability of the anion

89. 

Three bulbs are filled with CH_4, CO_2 and Ne as shown the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be________atm. (Nearest integer)

Sol. **3**

$P_f V_f = P_1 V_1 + P_2 V_2 + P_3 V_3$

$P_f \times 9 = 2 \times 2 + 4 \times 3 + 3 \times 4$

$$P_f = \frac{28}{9} = 3.11 \simeq 3$$

90. The number of given statements/s which is/are correct is______

(1) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(2) If a reaction has zero activation energy, its rate is independent of temperature.

(3) The stronger the temperature dependence of the rate constant, the smaller is the activation energy

(4) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Sol. **2**

Clearly, if $E_a = 0$, K is temperature independent

if $E_a > 0$, K increase with increase in temperature

if $E_a < 0$, K decrease with increase in temperature