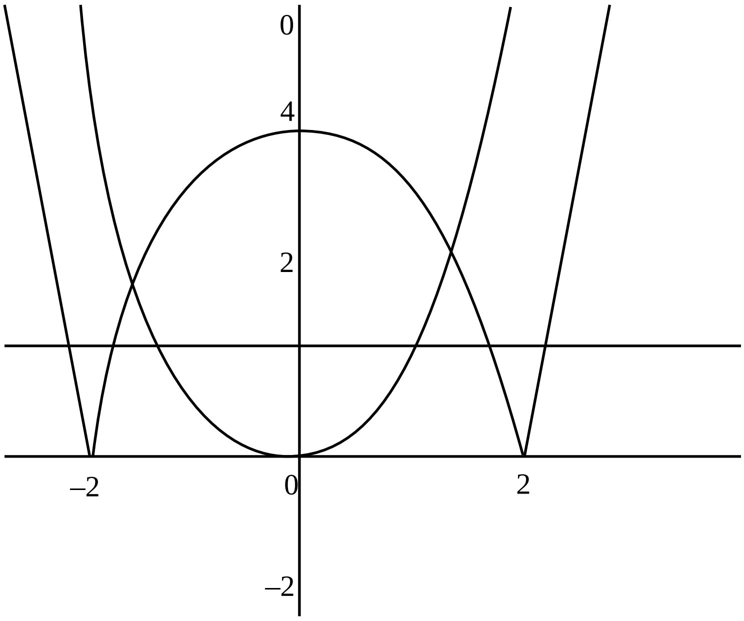
SECTION - A

1. The area of the region $\left\{(x.y): x^2 \le y \le \left|x^2-4\right|, y \ge 1\right\}$ is:

(1) $\frac{3}{4}\left(4\sqrt{2}+1\right)$ (2) $\frac{4}{3}\left(4\sqrt{2}-1\right)$ (3) $\frac{3}{4}\left(4\sqrt{2}-1\right)$ (4) $\frac{4}{3}\left(4\sqrt{2}+1\right)$

Sol. **(2)**



Required area = $2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}\left[4\sqrt{2}-1\right]$

2. If $\lim_{x\to 0} \frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17$, then $5a^2+b^2$ is equal to

(1) 76 (2) 72 (3) 64 (4) 68

Sol. **(4)**

$$\lim_{x\to 0} \frac{e^{ax}-\cos bx-\frac{cxe^{-cx}}{2}}{1-\cos 2x} = 17$$

On expansion

$$\lim_{x\to 0} \frac{\left(1+ax+\frac{(ax)^2}{2!}+...\right)-\left(1-\frac{(bx)^2}{2!}+...\right)-\frac{cx}{2}\left(1-cx+\frac{(cx)^2}{2!}\right)}{\left(\frac{1-\cos 2x}{(2x)^2}\right)\times(2x)^2} = 17$$

$$\lim_{x\to 0} \frac{x\left(a-\frac{c}{2}\right)+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{\frac{1}{2}(4x^2)} = 17$$

For limit to be exist

$$a-\frac{c}{2} = 0 \Rightarrow c = 2a$$

$$\Rightarrow \frac{\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}}{2}=17$$

$$\Rightarrow \frac{a^2}{2}+\frac{b^2}{2}+\frac{4a^2}{2}=34$$

$$\Rightarrow 5a^2+b^2=68$$

3. The line, that is coplanar to the line $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$, is

(1) $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$

(2) $\frac{x+1}{1}=\frac{y-2}{2}=\frac{z-5}{5}$

(3) $\frac{x-1}{-1}=\frac{y-2}{2}=\frac{z-5}{4}$

(4) $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{4}$

Sol. **(1)**

Condition of co-planarity

$$\begin{vmatrix} x_2-x_1 & a_1 & a_2 \\ y_2-y_1 & b_1 & b_2 \\ z_2-z_1 & c_1 & c_2 \end{vmatrix}=0$$

Where a1, b1, c1 are direction cosine of 1st line and a2, b2, c2 are direction cosine of 2nd line.

Now. Solving options

Point (-3, 1, 5) & point (-1, 2, 5)

$$(1)\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

= –3(5) – (10) + 5(– 1 + 4)

= – 15 – 10 + 15 = – 10

(2) point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

= 3(5) – (10) + 5(1 + 4)

– 25 + 5 = 0

(3) point (-1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

–3(4) – (8) + 5(1 + 4)

–12 –8 + 25 = 5

(4) point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

–3(–5) – (–20) + 5(–1 – 8)

15 + 20 – 45=–10

4. The plane, passing through the points (0, -1, 2) and (-1, 2, 1) and parallel to the line passing through (5,1,-7) and (1,-1,-1), also passes through the point

(1) (0, 5, -2) (2) (-2, 5, 0)

(3) (2, 0, 1) (4) (1, -2, 1)

Sol. **(2)**

Plane passing through (0, –1, 0) and (–1, 2, 1)

Then vector in plane $\langle -1, 3, -1\rangle$ vector parallel to plane is $\langle 4, 2, -6\rangle$

Normal vector to plane $(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$

$\vec{n} = \langle 8, 5, 7\rangle$

Equation of plane

8(x – 0) + 5(y+1) + 7(z - 2) = 0

$\Rightarrow$ 8x + 5y + 7z = 9

From given options point (-2, 5, 0) lies on plane.

5. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

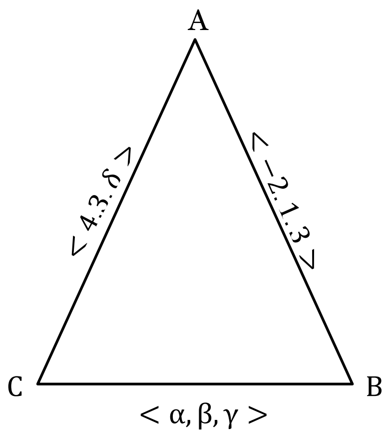
$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to

(1) 108 (2) 60 (3) 54 (4) 120

Sol. **(2)**

5.



$\overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$

$\langle 4, 3, \delta\rangle . + \langle -2, 1, 3\rangle = \overrightarrow{CB}$

$\Rightarrow \overrightarrow{CB} = \langle 2, 4, 3+\delta\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 3\\ -4 & -3 & -\delta\end{vmatrix}$$

$=\hat{\ell}(9-\delta)-\hat{j}(2\delta+12)+k(10)$

$|AB\times AC|^2=(9-\delta)^2+(2\delta+12)^2+(10)^2$

$=5\delta^2+30\delta+325$

Area of $\Delta ABC=5\sqrt{6}$

$\Rightarrow \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=5\sqrt{6}$

$\Rightarrow \left|\overrightarrow{AB}\times\overrightarrow{AC}\right|^2=600$

$\Rightarrow 5\delta^2+30\delta-275=0$

$\Rightarrow S^2+6\delta-55=0$

$\Rightarrow (\delta+11)(\delta-5)=0$

$\delta=5$

$\overrightarrow{CB}=<2,3,8>$

$\overrightarrow{CB}.\overrightarrow{CA}.=<2,4,8>.<4,3,5>$

$=8+12+40=60$

6. Let for A= $\begin{bmatrix}1 & 2 & 3\\ \alpha & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$, $|A|=2$. If $|2\text{ adj }(2\text{ adj }(2A))| = 32^n$, then $3n+\alpha$ is equal to

(1) 10 (2) 9 (3) 12 (4) 11

Sol. **(4)**

$|A| = 2$

$adj(kA) = k^{m-1}\, adjA$ {m = order of matrix}

$adj(2A) = 2^2\, adjA = 4adj(A)$

adj (2adj (2A)) = adj (8adj A)

$= 8^2$ adj adj (A)

$|2\text{ adj }2\text{adj }(2A)| = |2^7\text{ adj adj }(A)|$

$=(2^7)^3\,|A|^{2^2}$

$=2^{21}\,|A|^4$

$=2^{21}.2^4$

$\Rightarrow 2^{25}=(32)^n$

$\Rightarrow 2^{25}=2^{5n}$

$\Rightarrow$ n = 5

$|A| = 2$

$(6-1)-2(2\alpha-1)+3(\alpha-3)=2$

$\Rightarrow 5-4\alpha+2+3\alpha-9$

$\Rightarrow \alpha=-4$

$3n+\alpha=11$

7. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(1) $[0, \pi)$ (2) $[0, \pi]$ (3) $[0, 2\pi)$ (4) $[0, 2\pi]$

Sol. **(3)**

$$f(x) = 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$$

$$0 \le \frac{x^2}{1+x^2} < 1$$

$$\Rightarrow\ 0 \le \sin^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{2}$$

$$\Rightarrow\ 0 \le 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right) < 2\pi$$

Range : $[0, 2\pi)$

8. Let a_1, a_2, a_3, ... be a G. P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(1) 2 (2) 3 (3) $3\sqrt{3}$ (4) $2\sqrt{2}$

Sol. **(2)**

$$a_3 . a_5 = \frac{1}{9}$$

$$\Rightarrow\ ar^2 . ar^4 = \frac{1}{9}$$

$$\Rightarrow\ \left(ar^3\right)^2 = \frac{1}{9}$$

$$\Rightarrow\ ar^3 = \frac{1}{3} \qquad \ldots(i)$$

$$a_6 + a_8 = 2$$

$$\Rightarrow ar^5 + ar^7 = 2$$

$$\Rightarrow ar^3(r^2 + r^4) = 2$$

$$\Rightarrow \frac{1}{3}r^2(1+r^2) = 2$$

$$\Rightarrow r^2(1+r^2) = 2 \times 3$$

$$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$$

$$a = \frac{1}{3} \times \frac{1}{r^3}$$

$$= \frac{1}{3} \times \frac{1}{2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

$$6(a_2 + a_4)(a_4 + a_6)$$

$$\Rightarrow 6(ar + ar^3)(ar^3 + ar^5)$$

$$\Rightarrow 6\left(\frac{ar^3}{r^2} + \frac{1}{3}\right)\left(\frac{1}{3} + \frac{1}{3}r^2\right) = 3$$

9. If the system of equations

$2x+y-z=5$

$2x-5y+\lambda z=\mu$

$x+2y-5z=7$

has infinitely many solutions, then $(\lambda+\mu)^2 + (\lambda-\mu)^2$ is equal to

(1) 904 (2) 916 (3) 912 (4) 920

Sol. **2**

$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$\Rightarrow 2(25-2\lambda)-1(-10-\lambda)-1(4+5)=0$

$\Rightarrow 51-3x=0$

$\Rightarrow \lambda = 17$

$\Delta_x = 0$

$$\begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$\Rightarrow 5(25-34)-1(-5\mu-119)-1(2\mu+35)=0$

$\Rightarrow -45+5\mu+119-2\mu-35=0$

$\Rightarrow 39+3\mu=0 \Rightarrow \mu=-13$

$(\lambda+\mu)^2+(\lambda-\mu)^2 = 4^2+(30)^2$

$=916$

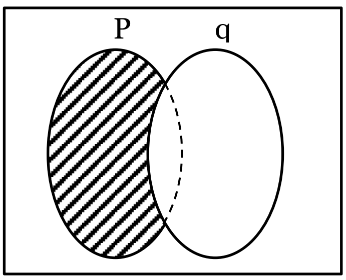
10. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to ______.

(1) $(\sim p) \vee (\sim q)$ (2) $(\sim p) \wedge (\sim q)$ (3) $p \vee (\sim q)$ (4) $p \vee q$

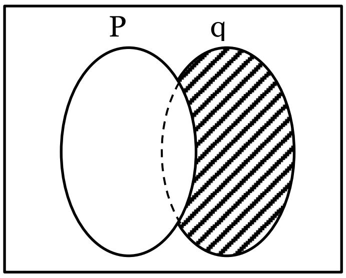
Sol. **(1)**

$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

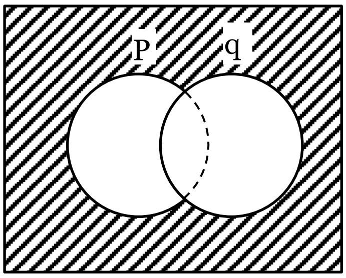
$p \wedge \sim q \Rightarrow$

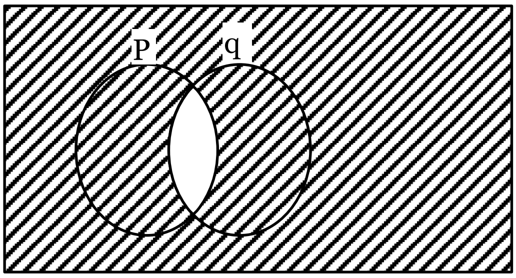


$\sim p \wedge q =$



$\sim p \wedge \sim q =$





$(p \wedge \sim q) \vee (\sim p \wedge q)(\sim p \wedge \sim q)$

(α, β)

$(\sim p) \vee (\sim q)$

Plane passing through $(0,-1,2)$

and $(-1,2,1)$

then vector in plane $< -1,3,-1 >$

vector parallel to plane is $< 4,2,-6 >$

normal vector to plane L_2

$$(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$$

$= i(-16) - \hat{j}(10) + \hat{k}(-14)$

$\vec{n} = < 8,5,7 >$

Equation of plane

$8(x-0)+5(y+1)+7(z-2)=0$

$\Rightarrow 8x+5y+7z=9$

From given options point $(-2,5,0)$ lies on plane.

11. Let $S=\left\{z\in C:\bar{z}=i\left(z^2+\text{Re}\left(\bar{z}\right)\right)\right\}$. Then $\sum_{z\in S}|z|^2$ is equal to

(1) 4 (2) $\frac{7}{2}$ (3) 3 (4) $\frac{5}{2}$

Sol. **(1)**

Let z = x + iy

$$\bar{z}=i(z^2+\text{Re}(\bar{z}))$$

$$\Rightarrow x-iy=i(x^2-y^2+2ixy+x)$$

$$\Rightarrow x-iy=-2xy+i(x^2-y^2+x)$$

$$x+2xy=0 \text{ and } x^2-y^2+x+y=0$$

$$x(1+2y)=0 \text{ and } x^2-y^2+x+y=0$$

If x = 0 then $-y^2+y=0$

$$\Rightarrow y=1,0$$

If $y=\frac{-1}{2}$ then $x^2-\frac{1}{4}+x-\frac{1}{2}=0$

$$\Rightarrow x=-\frac{3}{2},\frac{1}{2}$$

$$=\left\{0+i0,0+i,-\frac{3}{2}-\frac{1}{2}i,\frac{1}{2}-\frac{1}{2}i\right\}$$

$$\sum_{Z\in S}|Z|^2=0+1+\frac{9}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=4$$

12. Let α, β be the roots of the equation $x^2-\sqrt{2}x+2=0$, Then $\alpha^{14}+\beta^{14}$ is equal to

(1) $-128\sqrt{2}$ (2) $-64\sqrt{2}$ (3) –128 (4) –64

Sol. **(3)**

$$x^2-\sqrt{2}x+2=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$$

$$x=\frac{\sqrt{2}\pm\sqrt{-6}}{2}$$

$$=\sqrt{2}\left(\frac{1\pm i\sqrt{3}}{2}\right)$$

$$=-\sqrt{2}\omega,\ -\sqrt{2}\omega^2$$

$$\Rightarrow \alpha-\sqrt{2}\omega,\ \beta=-\sqrt{2}\omega^2$$

$$\alpha^{14}+\beta^{14}=2^7(\omega^{14}+\omega^{28})=2^7(\omega^2+\omega)=-128$$

13. Let $|\vec{a}|=2$, $|\vec{b}|=3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left|\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-3\vec{b}\right)\right|^2$ is equal to

(1) 482 (2) 841 (3) 882 (4) 441

Sol. **(3)**

$$\cos\left(\frac{\pi}{4}\right)=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}}=\frac{\vec{a}.\vec{b}}{(2)(3)} \Rightarrow \vec{a}.\vec{b}=3\sqrt{2}$$

Let $\vec{p}=\vec{a}+2\vec{b}$

$\vec{q}=2\vec{a}-3\vec{b}$

$$|\vec{p}|^2=|\vec{a}|^2+4|\vec{b}|^2+4(\vec{a}.\vec{b})$$

$$=4+36+12\sqrt{2}$$

$$=40+12\sqrt{2}$$

$$|\vec{q}|^2=4|\vec{a}|^2+9|\vec{b}|^2-12(\vec{a}-\vec{b})$$

$$=16+81-36\sqrt{2}$$

$$=97-36\sqrt{2}$$

$$\vec{p}.\vec{q}=2|\vec{a}|^2-6|\vec{b}|^2+\vec{a}.\vec{b}$$

$$=8-54+3\sqrt{2}$$

$$=-46+3\sqrt{2}$$

$$|\vec{p}\times\vec{q}|=(|\vec{p}||\vec{q}|)^2-(\vec{p}.\vec{q})^2$$

$$=\left(40+12\sqrt{2}\left(97-36\sqrt{2}\right)\right)-\left(3\sqrt{2}-46\right)^2$$

$$=\left(3016-276\sqrt{2}\right)-\left(2134-276\sqrt{2}\right)$$

$$=882$$

14. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50} x dx}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49} x+\tan^{51} x\right)dx}$ is

(1) 25 (2) 51 (3) 50 (4) 49

Sol. **(3)**

let $I_1=e^{-\pi/4}+\int_0^{\pi/4} e^{-x}\tan^{50} x dx$

$$I_2=\int_0^{\pi/4} e^{-x}\left(\tan^{49} x+\tan^{51} x\right)dx$$

$$= \int_0^{\pi/4} e^{-x} \tan^{49} x \left(\sec^2 x\right) dx$$

$$= \left| e^{-x} \frac{\tan^{50} x}{50} \right|_0^{\pi/4} + \frac{1}{50} \int_0^{\pi/4} e^{-x} \tan^{50} x dx$$

$$= \frac{e^{-\pi/4}}{50} + \frac{1}{50} \int_0^{\pi/4} e^{-x} \tan^{50} x dx = \frac{I_1}{50}$$

then $\frac{I_1}{I_2} = 50$

15. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

(1) $\frac{80}{9}$ (2) 8 (3) 9 (4) $\frac{26}{3}$

Sol. **(1)**

general term for $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r \left(-\frac{1}{3x^2}\right)^r (2x^3)^{5-r}$$

$$= {}^5C_r (-1)^r 3^{-r} 2^{5-r}.x^{15-5r}$$

$15 - 5r = 5 \Rightarrow r = 2$

Coeff. Of $x^5 = {}^5C_2 (-1)^2 3^{-2} 2^3$

$$= 10 \times \frac{1}{9} \times 8$$

$$= \frac{80}{9}$$

16. The random variable X follows binomial distribution B (n, p), for which the difference of the mean and the variance is 1. If 2P(x = 2) = 3P(x = 1), then $n^2P(X > 1)$ is equal to

(1) 16 (2) 11 (3) 12 (4) 15

Sol. **2**

$$2P(x = 2) = 3P(x = 1)$$

$$2 \times {}^n c_2 P^2 (1 - P)^{n-2} = 3\, {}^n c_1 P^1 (1 - P)^{n-1}$$

$$\Rightarrow 2 \frac{n(n-1)}{2} \cdot P = 3n(1 - P)$$

$$\Rightarrow (n-1)P = 3(1 - P) \cdots (i)$$

$$nP - nPq = 1$$

$$\Rightarrow nP - nP(1 - p) = 1$$

$$\Rightarrow nP^2 = 1 \Rightarrow n = \frac{1}{p^2}$$

$\Rightarrow$ put in equ (i)

$$\left(\frac{1}{p^2}-1\right)P = 3(1-p)$$

$$\frac{1}{p} - P = 3 - 3P$$

$$\Rightarrow 1 - P^2 = 3P - 3p^2$$

$$\Rightarrow 2P^2 - 3P + 1 = 0$$

$$\Rightarrow 2P^2 - 2P - P + 1 = 0$$

$$\Rightarrow 2P(P-1) - 1(P-1) = 0$$

$$\Rightarrow P = \frac{1}{2}, P = 1\{\text{Re jected}\}$$

$$n = \frac{1}{(1/2)^2} = 4$$

$$n^2P(x > 1) = n^2p(1 - P(x=0) - P(x=1))$$

$$\Rightarrow n^2P\left(1-(1-P)^n - nP(1-P)^{n-1}\right)$$

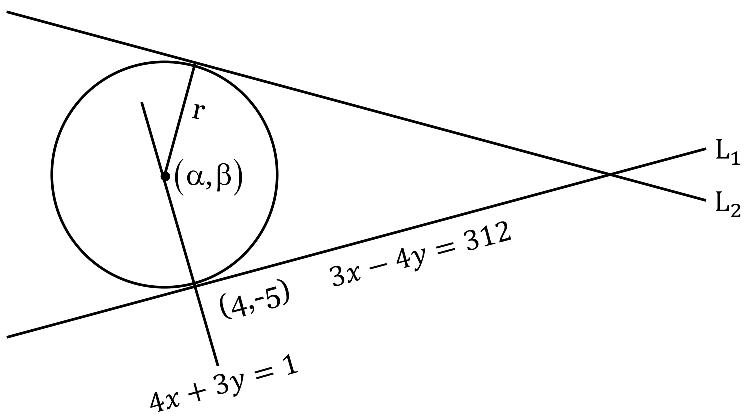
$$\Rightarrow (4)^2\left(1-\left(\frac{1}{2}\right)^4 - 4\left(\frac{2}{2}\right)^4\right)$$

$$\Rightarrow 16 - 1 - 4 = 11$$

17. Let the centre of a circle C be (α,β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to

(1) 5 (2) 6 (3) 7 (4) 9

Sol. **(3)**



Centre lies on normal

$$\Rightarrow 4\alpha + 3\beta = 1$$

Distance of (α,β) from L_1 and L_2 are equal

$$\left|\frac{3\alpha + 4\beta - 24}{5}\right| = \left|\frac{3\alpha - 4\beta - 32}{5}\right|$$

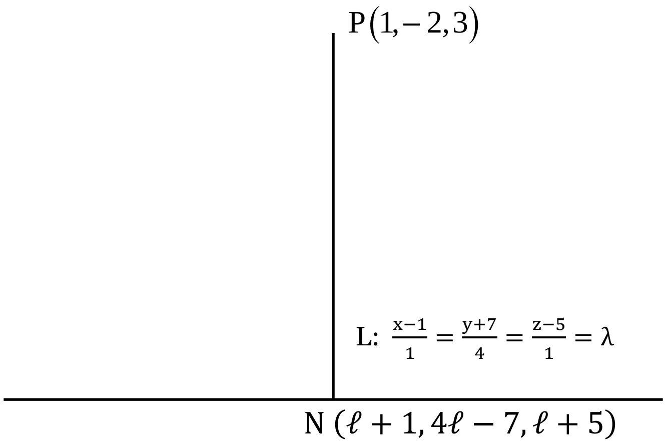
$$3\alpha+4\beta-24=-3\alpha+4\beta+32$$
$$\Rightarrow 6\alpha=56$$
$$\Rightarrow \alpha=\frac{28}{3},\beta=\frac{-109}{3}$$
$$r=\sqrt{\left(\frac{28}{3}-4\right)^2+\left(\frac{-109}{3}+5\right)^2}>8$$
(reject)

$$3\alpha+4\beta-24=-3\alpha-4\beta-32$$
$$8\beta=-8$$
$$\beta=-1,\alpha=1$$
$$\gamma=\sqrt{(4-1)^2+(-5+1)^2}=5$$
$$\alpha-\beta+\gamma=7$$

18. Let N be the foot of perpendicular from the point P (1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7. 5). Then the distance of N from the plane 2x–2y+z+5=0 is

(1) 6 (2) 7 (3) 9 (4) 8

Sol. **(2)**



$$\overrightarrow{PN}=\langle\lambda,4\lambda-5,\lambda+2\rangle$$
$$\overrightarrow{PN}.<1,4,1>=0$$
$$\Rightarrow \lambda+16\lambda-20+\lambda+2=0$$
$$\Rightarrow \lambda=1$$
$$N(2,-3,6)$$

Distance of N from $2x-2y+z+5=0$ is

$$d=\left|\frac{2(2)-2(-3)+6+5}{\sqrt{2^2+(-2)^2+(1)^2}}\right|$$
$$=\left|\frac{21}{3}\right|=7$$

19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

(1) 328 (2) 327 (3) 324 (4) 326

Sol. **(2)**

A					

(5!) $=120$

5!

D					

$= 120$

4!

M	A				

$= 24$

4!

M	D				

$= 24$

4!

M	N				

$= 24$

3!

M	O	A			

$= 6$

3!

M	O	D			

$= 6$

2!

M	O	N	A		

$= 2$

M	O	N	D	A	Y

$= 1$

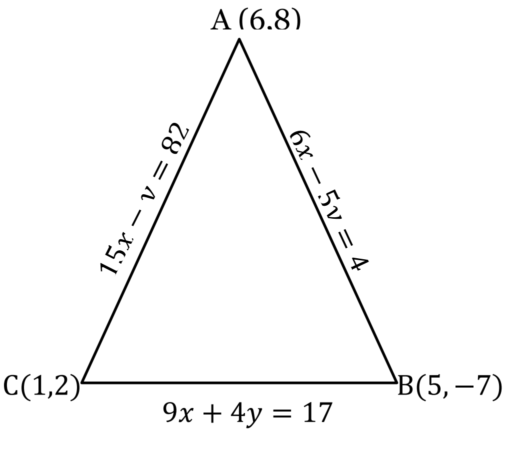
Rank = 120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1

= 327

20. Let (α, β) be the centroid of the triangle formed by the lines 15x–y = 82, 6x–5y= –4 and 9x + 4y =17. Then α + 2β and 2α – β are the roots of the equation

(1) $x^2-13x+42=0$ (2) $x^2-10x+25=0$ (3) $x^2-7x+12=0$ (4) $x^2-14x+48=0$

Sol. **(1)**



Centroid $(\alpha,\beta)=\left(\frac{6+1+5}{3},\frac{8-7+2}{3}\right)=(4,1)$

$\alpha+2\beta=4+2=6$

$2\alpha-\beta=8-1=7$

Quadratic equation

$x^2-(6+7)x+(6\times7)=0$

$\Rightarrow x^2-13x+42=0$

SECTION - B

21. Let A={-4,-3,-2,0,1,3,4} and R = {(a, b) $\in$ A×A: b = |a| or b^2 = a+ 1} be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is __________

Sol. (7)

R = [(–4, 4), (–3, 3), (3, –2), (0, 1), (0, 0), (1, 1),
(4, 4), (3, 3)}

For reflexive, add $\Rightarrow$ (–2, –2), (–4, –4), (–3, –3)

For symmetric, add $\Rightarrow$ (4, –4), (3, –3), (–2, 3), (1, 0)

22. Let $f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right) \cos x\, dx, n \in N.$ Then $f_{21} - f_{20}$ is equal to ________

Sol. (41)

$$f_n = \int_0^{\pi/2} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right)\cos x dx$$

$\sin x = t$

$\cos x dx = dt$

$$f_n \int_0^1 \left(\sum_{k=1}^{n} t^{k-1}\right)\left(\sum_{k=1}^{n} (2k-1)t^{k-1}\right)dt$$

$$= \int_0^1 \left(1+t+t^2+...t^{n-1}\right)\left(1+3t+5t^2+....+(2n-1)t^{n-1}\right)dt$$

$$f_{n+1} = \int_0^1 \left(\sum_{k=1}^{n+1} t^{k-1}\right)\left(\sum_{k=1}^{n+1} (2k-1)t^{k-1}\right)dt$$

$$= \int_0^1 \left(1+t+t^2...t^n\right)\left(1+3t+5t^2+....+(2n+1)t^n\right)dt$$

$$= \int_0^1 \left(1+t+t^2+...t^{n-1}\right)\left(1+3t+5t^2+...+(2n-1)t^{n-1}\right)dt$$

$$+\int_0^1 \left(1+3t+5t^2+...(2n+1)\right)t^n dt$$

$$+\int_0^1 \left(1+t+t^2+...+t^{n-1}\right)(2n+1)t^n dt$$

$$f_{n+1} - f_n = \int_0^1 \left(1+3t+5t^2+...+(2n+1)t^n\right)t^n dt$$

$$+\int_0^1 \left(1+t+t^2+...t^{n+1}\right)\left((2n+1)t^n\right)dt$$

put $n = 20$

$$f_{21} - f_{20} = \int_0^1 \left(1+3t+5t^2...41.t^{20}\right)t^{20} dt + \int_0^1 \left(1+t+t^2...t^{19}\right)\left(41.t^{20}\right)dt$$

$$= \left(\frac{1}{21}+\frac{3}{22}+\frac{5}{23}+\ldots+\frac{39}{40}+\frac{41}{41}\right)+\left(\frac{41}{21}+\frac{41}{22}+\frac{41}{40}\right)$$

$$= \frac{1+41}{21}+\frac{3+41}{22}+...+\frac{39+41}{40}+1 = 40+1 = 41$$

23. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{\left(x^2-1\right)}y=\frac{x+2}{\left(x^2-1\right)^{\frac{5}{2}}}, x>1$ such that $y(2)=\frac{2}{9}\log_e\left(2+\sqrt{3}\right)$ and $y\left(\sqrt{2}\right)=\alpha\log_e\left(\sqrt{\alpha}+\beta\right)+\beta-\sqrt{\gamma}$, $\alpha, \beta, \gamma, \in$ N, then $\alpha\beta\gamma$ is equal to ____.

Sol. **(6)**

given differential equation $\frac{dy}{dx}+\frac{4x}{\left(x^2-1\right)}y=\frac{x+2}{\left(x^2-1\right)^{5/2}}$ is linear D.E.

I.F. $= e^{\int\frac{4x}{x^2-1}dx} = e^{2\ln\left(x^2-1\right)} = e^{\ln\left(x^2-1\right)^2} = \left(x^2-1\right)^2$

$y\left(x^2-1\right)^2=\int\frac{x+2}{\left(x^2-1\right)^{5/2}}\left(x^2-1\right)^2dx$

$=\int\frac{x}{\sqrt{x^2-1}}dx+\int\frac{2dx}{\sqrt{x^2-1}}$

$=\sqrt{x^2-1}+2\ln\left[x+\sqrt{x^2-1}\right]+C$

put $y(2)=\frac{2}{9}\ln\left(2+\sqrt{3}\right)$

$\frac{2}{9}\ln\left(2+\sqrt{3}\right)(9)=\sqrt{3}+2\ln\left[2+\sqrt{3}\right]+C$

$=C=-\sqrt{3}$

put $x=\sqrt{2}$

$y=1+2\ln\left[\sqrt{2}+1\right]-\sqrt{3}$

$\alpha=2,\beta=1=\gamma=3$

$\alpha\beta\gamma=2(1)(3)=6$

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is _____.

Sol. **16**

a	b	2

(a,b) = (1,3), (3,1), (2,2), (2,5), (5,2), (3,4), (4,3), (5,5)
= 8 numbers

a	b	4

(a,b) = (1,1), (1,4), (4,1), (2,3), (3,2)
(4,4), (3,5), (5,3) = 8 numbers
total 8 + 8 = 16

25. The remainder, when 7^{103} is divided by 17, is ______.

Sol. **12**

$7^{103} = 7.7^{102}$

$= 7\left(7^2\right)^{51}$

$= 7(51-2)^{51} \rightarrow \text{remainder} = 7(-2)^{51}$

$-7\left(2^3\right)(16)^{12} = -56(17-1)^{12} \rightarrow \text{remainder} = -56(-1)^{12}$

Remainder = – 56 + 17k

= –56 + 68

= 12

26. Let $f\left(x\right) = \sum_{k=1}^{10} k\,x^k, x \in R$ If 2 f(2) –f ' (2) = $119(2)^n$ + 1 then n is equal to ____

Sol. **10**

$$f(x) = \sum_{k=1}^{10} kx^k$$

$$\Rightarrow f(x) = x + 2x^2 + 3x^3 + \cdots + 9x^9 + 10x^{10} - (i)$$

$$xf(x) = x^2 + 2x^3 + \ldots + 9x^{10} + 10x^{11} \ldots (ii)$$

"(i) - (ii)"

$$f(x)(1-x) = x + x^2 + x^3 + \cdots + x^{10} - 10x^{11}$$

$$f(x)(1-x) = \frac{x\left(1-x^{10}\right)}{1-x} - 10x^{11}$$

$$f(x) = \frac{x\left(1-x^{10}\right)}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(2) = 2 + g(2)^{11}$$

$(1-x)^2 f(x) = x(1-x^{10}) - 10x^{11}(1-x)$

diff. w.r.t. x

$$(1-x)^2 f'(2) + f(2)2(1-x)(-1)$$

$$= x\left(-10x^9\right) + \left(1-x^{10}\right) - 10x^{11}(-1) - (1-x)(110)x^{10}$$

put x = 2

$$f'(2) + f(2)(2) = -10(2)^{10} + 1 - 2^{10} + 10(2)^{11} - 110(2)^{10} + 110(2)^{11}$$

$$= (-121)2^{10} + (120)2^{11} + 1$$

$$= 2^{10}(240-121) + 1$$

$$= 119(2)^{10} + 1$$

$$n = 10$$

27. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1} x = 2\tan^{-1} x$ is equal to _____.

Sol. **2**

$$\sin^{-1} x = 2\tan^{-1} x$$

$$\sin^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow x = \frac{2x}{1+x^2}$$

$$\Rightarrow x\left(\frac{2}{1+x^2} - 1\right) = 0$$

$\Rightarrow x = 0, 1, -1$ but -1 is not included.

Answer 2 solutions

28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively, Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _______.

Sol. **269**

$$\text{Mean } = \frac{\Sigma x_i}{10}$$

$$\Rightarrow 50 = \frac{\Sigma x_i}{10}$$

$$\Rightarrow \Sigma x_i = 500$$

correct $\Sigma x_i = 500 - 45 - 50 + 20 + 25 = 450$

$$\sigma^2 = \frac{\Sigma x_i^2}{10} = \left(\bar{x}\right)^2$$

$$\Rightarrow 144 = \frac{\Sigma x_i^2}{10} - 2500$$

$$\Rightarrow \Sigma x_i^2 = 26440$$

correct $\Sigma x_i^2 = 26440 - (45)^2 - (50)^2 + (20)^2 + (25)^2$

$= 26440 - 2025 - 2500 + 400 + 625$

$= 22940$

$$\sigma^2 = \frac{\text{correct}\Sigma x_i^2}{10} - \left(\frac{\text{correct}\Sigma x_i}{10}\right)^2$$

$$= \frac{22940}{10} - \left(\frac{450}{10}\right)^2 = 2294 - 2025$$

$$= 269$$

29. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x and y - axes are a and b respectively, then $|6a| + |5b|$ is equal to _______.

Sol. **12**

$$2ae = 4$$

$$2a\left(\frac{3}{2}\right) = 4$$

$\Rightarrow a=\frac{4}{3}$

$e^2=1+\frac{b^2}{a^2}$

$\Rightarrow \frac{9}{4}=1+b^2\left(\frac{9}{16}\right)$

$\Rightarrow b^2=\left(\frac{5}{4}\right)\left(\frac{16}{9}\right)=\frac{20}{9}$

slope of tangent $m=\frac{3}{2}$

equation of tangent is

$y=mx\pm\sqrt{a^2m^2-b^2}$

$y=\frac{3}{2}x\pm\sqrt{\frac{16}{9}\left(\frac{9}{4}\right)-\frac{20}{9}}$

$\Rightarrow y=\frac{3x}{2}\pm\frac{4}{3}$

$y=0\Rightarrow a=\pm\frac{8}{9}$

$x=0\Rightarrow b=\pm\frac{4}{3}$

$|6a|+|5b|=\frac{16}{3}+\frac{20}{3}=12$

30. Let [α] denote the greatest integer ≤ α. Then $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+....+\left[\sqrt{120}\right]$ is equal to _____.

Sol. **825**

$S=\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+....+\left[\sqrt{120}\right]$

$\left[\sqrt{1}\right]\to\left[\sqrt{3}\right]=1\times 3$

$\left[\sqrt{4}\right]\to\left[\sqrt{8}\right]=2\times 5$

$\left[\sqrt{9}\right]\to\left[\sqrt{15}\right]=3\times 7$

$\vdots$

$\left[\sqrt{100}\right]\to\left[\sqrt{120}\right]=10\times 21$

$S=1\times3+2\times5+3\times7+...+10\times21$

$=\sum_{r=1}^{10} r(2r+1)$

$=2\sum_{r=1}^{10} r^2+\sum_{r=1}^{10} r$

$=\frac{2\times10\times11\times21}{6}+\frac{10\times11}{2}$

= 770 + 55

= 825

SECTION - A

31. Given below are two statements :

Statements I : An AC circuit undergoes electrical resonance if it contains either a capacitor or an inductor.

Statement II : An AC circuit containing a pure capacitor or a pure inductor consumes high power due to its non-zero power factor.

In the light of above statements, choose the correct answer form the options given below :

(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Sol. **(3)**

Statement-I: An AC circuit for resonance inductor and capacitor both should required.

Statement-II: An AC circuit containing a pure capacitor and pure inductor have no power loss

For resonance, $\phi = 0$

means both capacitor and inductor must be present.

32. A passenger sitting in a train A moving at 90 km/h observes another train B moving in the opposite direction for 8 s. if the velocity of the train B is 54 km/h, then length of train B is :

(1) 120 m (2) 200 m (3) 320 m (4) 80 m

Sol. **(3)**

$$V_A = \frac{90\text{km}}{\text{hr}} = 25\text{ms}^{-1}$$

$$V_B = \frac{54\text{km}}{\text{hr}} = 15\text{ms}^{-1}$$

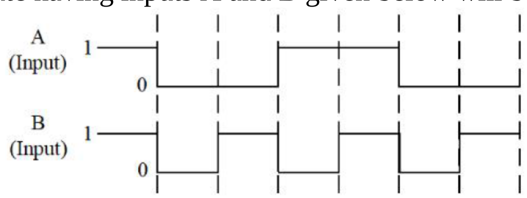
$$\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{VA} = 40\text{ms}^{-1}$$

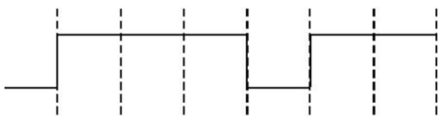
$$\text{Time of crossing } = \frac{\text{Length of train}}{\text{Relative velocity}}$$

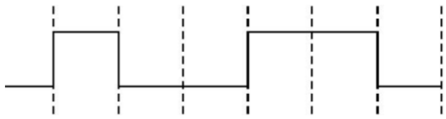
$$8 = \frac{\ell}{40}$$

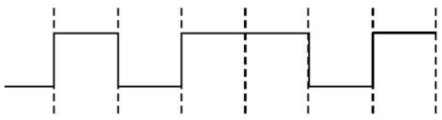
$$\ell = 8 \times 40 = 320\,\text{meter}$$

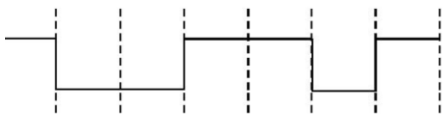
33. The output from a NAND gate having inputs A and B given below will be,



(1)


(2)


(3)


(4)


Sol. **(1)**

Truth table for NAND gate is

A	B	$Y = \overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

On the basis of given input A and B the truth table is

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1

34. The distance travelled by an object in time t is given by $s = (2.5)t^2$. The instantaneous speed of the object at t = 5 s will be :

(1) 25 ms^{-1} (2) 12.5 ms^{-1} (3) 5 ms^{-1} (4) 62.5 ms^{-1}

Sol. **(1)**

$S = 2.5t^2$

Speed $(v) = \frac{ds}{dt} = 5t$

At, $t = 5\,sec.$

$v = 5 \times 5 = 25\,ms^{-1}$

35. In a Young's double slits experiment, the ratio of amplitude of light coming from slits is 2 : 1. The ratio of the maximum to minimum intensity in the interference pattern is :

(1) 9 : 1 (2) 9 : 4 (3) 2 : 1 (4) 25 : 9

Sol. **(1)**

Given that

$\frac{A_1}{A_2} = \frac{2}{1}$

$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{9}{1}$

$= 9:1$

36. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R
Assertion A : The binding energy per nucleon is practically independent of the atomic number for nuclei of mass number in the range 30 to 170.
Reason R : Nuclear force is short ranged.
In the light of the above statements, choose the correct answer from the options given below
(1) Both A and R are true but R is NOT the correct explanation of A
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) A is false but R is true

Sol. **(2)**

37. Two planets A and B of radii R and 1.5 R have densities ρ and $\rho/2$ respectively. The ratio of acceleration due to gravity at the surface of B to A is :
(1) 2 : 3 (2) 2 : 1 (3) 4 : 3 (4) 3 : 4

Sol. **(4)**

$$g=\frac{GM}{R^2}=\frac{G\times\frac{4}{3}\pi R^3\times\rho}{R^2}=\frac{4}{3}G\pi PR$$

$$\frac{g_2}{g_1}=\frac{R_2}{R_1}\times\frac{\rho_2}{\rho_1}=1.5\times\frac{1}{2}=\frac{3}{4}$$

38. The mean free path of molecules of a certain gas at STP is 1500d, where d is the diameter of the gas molecules. While maintaining the standard pressure, the mean free path of the molecules at 373 K is approximately :
(1) 750d (2) 1500d (3) 1098d (4) 2049d

Sol. **(4)**
mean free path λ

$$\lambda=\frac{RT}{\sqrt{2}\Pi d^2N_AP}$$

$$\lambda\propto T$$

$$\frac{1500d}{\lambda}=\frac{273}{373}$$

$$\lambda=2049d$$

39. To radiate EM signal of wavelength λ with high efficiency, the antennas should have a minimum size equal to:
(1) λ (2) $\frac{\lambda}{2}$ (3) 2λ (4) $\frac{\lambda}{4}$

Sol. **(4)**
Minimum length of antenna
Should be $\frac{\lambda}{4}$

40. A particle executes SHM of amplitude A. The distance from the mean position when its's kinetic energy becomes equal to its potential energy is :
(1) $\sqrt{2A}$ (2) $\frac{1}{2}A$ (3) $\frac{1}{\sqrt{2}}A$ (4) 2A

Sol. **(3)**
K.E = P.E

$$\frac{1}{2}M\omega^2(A^2-x^2)=\frac{1}{2}M\omega^2x^2$$

$$(A^2-x^2)=x^2$$

$$2x^2=A^2$$

$$x=\pm\frac{A}{\sqrt{2}}$$

41. In an electromagnetic wave, at an instant and at a particular position, the electric field is along the negative z axis and magnetic field is along the positive x-axis. Then the direction of propagation of electromagnetic wave is :

(1) negative y-axis
(2) at 45° angle from positive y-axis
(3) positive y-axis
(4) positive z-axis

Sol. **(1)**

Direction of propagation of EM wave will be in the direction of $\left(\vec{E} \times \vec{B}\right)$

42. Given below are two statements :

Statement I : Out of microwaves, infrared rays and ultraviolet rays, ultraviolet rays are the most effective for the emission of electrons from a metallic surface.

Statement II : Above the threshold frequency, the maximum kinetic energy of photoelectrons is inversely proportional to the frequency of the incident light.

In the light of above statements, choose the correct answer form the options given below

(1) Statement I is false but statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but statement II is false
(4) Both Statement I and Statement II are false

Sol. **(3)**

UV rays have maximum frequency hence are most effective for emission of electrons from the metallic surface.

$KE_{max} = hf - hf_0$

43. Given below are two statements :

Statement I : For a planet, if the ratio of mass of the planet to its radius increases, the escape velocity from the planet also increases.

Statement II : Escape velocity is independent of the radius of the planet.

In the light of above statements, choose the most appropriate answer form the options given below

(1) Both Statement I and Statement II are correct
(2) Statement I is correct but statement II is incorrect
(3) Statement I is incorrect but statement II is correct
(4) Both Statement I and Statement II are incorrect

Sol. **(2)**

$$V_e = \sqrt{\frac{2GM}{R}}$$

As, $\frac{M}{R}$ increases $\Rightarrow V_e$ increases

$$V_e \propto \frac{1}{\sqrt{R}}$$

As, V_e depends on R

44. A vehicle of mass 200 kg is moving along a levelled curved road of radius 70 m with angular velocity of 0.2 rad/s. The centripetal force acting on the vehicle is:

(1) 2800 N
(2) 560 N
(3) 2240 N
(4) 14 N

Sol. **(2)**

$$F_c = m\omega^2 r = 200 \times (0.2)^2 \times 70$$

$$= 560N$$

45. A 10 μC charge is divided into two parts and placed at 1 cm distance so that the repulsive force between them is maximum. The charges of the two parts are:

(1) 7 μC, 3 μC
(2) 8 μC, 2 μC
(3) 9 μC, 1 μC
(4) 5 μC, 5 μC

Sol. **(4)**

Divide $q = 10\mu c$ into parts (x) and $(q - x)$

$F = \frac{(K)(x)(q-x)}{r^2}$

For F to be maximum

$\frac{dF}{dx} = 0$

$x = \frac{q}{2} = \frac{10\mu C}{2} = 5\ \mu C$

$q - x = 10\mu C - 5\mu C = 5\ \mu C$

46. In the equation $\left[x + \frac{a}{y^2}\right]$[Y – b] = RT, X is pressure, Y is volume, R is universal gas constant ant T is temperature. The physical quantity equivalent to the ratio $\frac{a}{b}$ is :

(1) Coefficient of viscosity
(2) Energy
(3) Impulse
(4) Pressure gradient

Sol. **(2)**

x and $\frac{a}{y^2}$ have same dimensions

y and b have same dimensions

$[a] = [ML^5T^{-2}]$

$[b] = [L^3]$

$\frac{[a]}{[b]} = ML^2T^{-2}$ has dimension of energy

47. An electron is moving along the positive x-axis. If the uniform magnetic field is applied parallel to the negative z-axis, then

A. The electron will experience magnetic force along positive y-axis
B. The electron will experience magnetic force along negative y-axis
C. The electron will not experience any force in magnetic field
D. The electron will continue to move along the positive x-axis
E. The electron will move along circular path in magnetic field

Choose the correct answer from the options given below :

(1) B and E only
(2) A and E only
(3) B and D only
(4) C and D only

Sol. **(1)**

$\vec{F} = q(\vec{V} \times \vec{B})$

$\vec{F} = -e(\vec{V} \times \vec{B})$

Force will be along $-ve$ y axis As magnetic force is $\perp_r$ to velocity, path of electron must be circle.

48. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : A spherical body of radius (5 ± 0.1) mm having a particular density is falling through a liquid of constant density. The percentage error in the calculation of its terminal velocity is 4%

Reason R : The terminal velocity of the spherical body falling through the liquid is inversely proportional to its radius.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are true but R is NOT the correct explanation of A

(2) Both A and R true and R is the correct explanation of A

(3) A is false but R is true

(4) A is true but R is false

Sol. **(4)**

Terminal velocity of a spherical body in liquid

$V_t \propto r^2$

$\frac{\Delta V_t}{V_t} = 2\frac{\Delta r}{r}$

$\frac{\Delta V_t}{V_t} \times 100\% = 2 \times \frac{0.1}{5} \times 100 = 4\%$

Also, $V_t \propto r^2$

Reason (R) is false

49. The initial pressure and volume of an ideal gas are P_o and V_o. The final pressure of the gas when the gas is suddenly compressed to volume $\frac{V_o}{4}$ will be :

(Given γ = ratio of specific heats at constant pressure and at constant volume)

(1) $P_0(4)^{\frac{1}{\gamma}}$ (2) $4P_0$ (3) P_0 (4) $P_0(4)^{\gamma}$

Sol. **(4)**

As, gas in suddenly compressed, the process is adiabatic

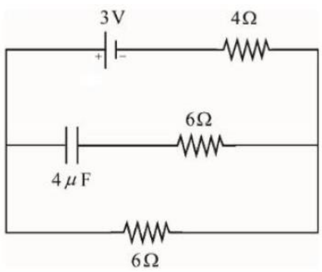
So equation of gas for adiabatic process is:

$PV^{\gamma} = \text{constant}$

$P_0V_0^{\gamma} = P_2\left(\frac{V_0}{4}\right)^{\gamma}$

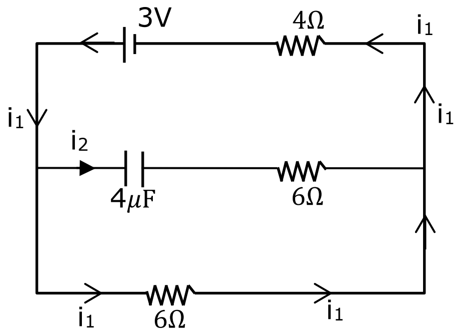
$P_2 = P_0(4)^{\gamma}$

50. In the network shown below, the charge accumulated in the capacitor in steady state will be :



(1) 4.8 μC (2) 12 μC (3) 7.2 μC (4) 10.3 μC

Sol. **(3)**



In steady state, no current will pass through capacitor hence capacitor will act as open circuit.

$i_2 = 0$

$$i_1 = \frac{3}{6+4} = \frac{3}{10}\text{A}$$

Potential difference on 6Ω resistor $= 6 \times \frac{3}{10} = 1.8$ volt capacitor will have same potential so charge

$$= cv = 4 \times 1.8 = 7.2\mu c$$

SECTION - B

51. In an experiment with sonometer when a mass of 180 g is attached to the string, it vibrates with fundamental frequency of 30 Hz. When a mass m is attached, the string vibrates with fundamental frequency of 50 Hz. The value of m is _______ g.

Sol. **(500)**

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

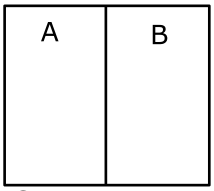
$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\left(\frac{50}{30}\right)^2 = \frac{mg}{180g}$$

$$m = \frac{25}{9} \times 180 = 500\text{gram}$$

52. Two plates A and B have thermal conductivities 84 $Wm^{-1} K^{-1}$ and 126 $Wm^{-1}K^{-1}$ respectively. They have same surface area and same thickness. They are placed in contact along their surfaces. If the temperatures of the outer surfaces of A and B are kept at 100 ºC and 0 ºC respectively, then the temperature of the surface of contact in steady state is ______ ºC.

Sol. **(40)**



$T_A = 100°C$ T $T_B = 0°C$

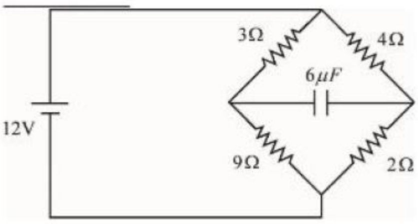
Let the temperature of contact surface is T then,

$$\frac{K_A A(T_A - T)}{L} = \frac{K_B A(T - T_B)}{L}$$

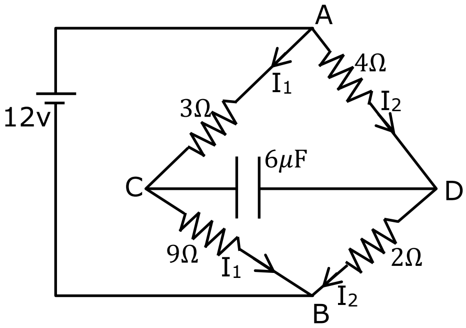
$$84(100 - T) = 126(T - 0)$$

$$T = 40°C$$

53. In the circuit shown, the energy stored in the capacitor is n μ J. The value of n is



Sol. **(75)**



$$I_1 = \frac{12}{3+9} = 1A$$

$$I_2 = \frac{12}{4+2} = 2A$$

$$V_A - V_C = 3I_1 = 3V$$

$$V_A - V_D = 2 \times 4 = 8V$$

So, $V_A - V_D = 5V$

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 5^2 = 75\mu J$$

54. A light rope is wound around a hollow cylinder of mass 5 kg and radius 70 cm. The rope is pulled with a force of 52.5 N. The angular acceleration of the cylinder will be ________ rad s^{-2}.

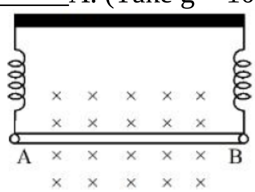
Sol. **(15)**

$$\tau = I\alpha$$

$$FR = mR^2\alpha$$

$$\propto = \frac{F}{mR} = \frac{52.5}{5 \times 0.7} = 15 rads^{-2}$$

55. A straight wire AB of mass 40 g and length 50 cm is suspended by a pair of flexible leads in uniform magnetic field of magnitude 0.40 T as shown in the figure. The magnitude of the current required in the wire to remove the tension in the supporting leads is _______A. (Take g = 10 ms^{-2})



Sol. **(2)**

For equilibrium :

$$Mg = IlB$$

$$I = \frac{Mg}{lB} = \frac{40 \times 10^{-3} \times 10}{50 \times 10^{-2} \times 0.4} = 2A$$

56. An insulated copper wire of 100 turns is wrapped around a wooden cylindrical core of the cross-sectional area 24 cm^2. The two ends of the wire are connected to a resistor. The total resistance in the circuit is 12 Ω. If an externally applied uniform magnetic field in the core along its axis changes from 1.5 T in one direction to 1.5 T in the opposite direction, the charge flowing through a point in the circuit during the change of magnetic field will be ______ mC.

Sol. **(60)**

$$|\Delta Q| = \frac{\Delta\phi}{R} = \frac{2NBA}{R}$$

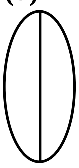
$$= \frac{2\times100\times1.5\times24\times10^{-4}}{12}$$

$$= 6\times10^{-2}\,c$$

$$= 60mc$$

57. A bi convex lens of focal length 10 cm is cut in two identical parts along a plane perpendicular to the principal axis. The power of each lens after cut is _______ D.

Sol. **(5)**

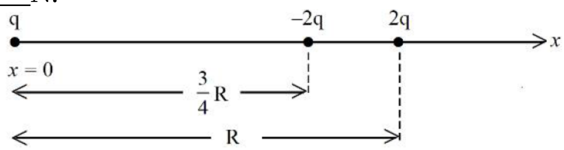


$$P_1 + P_1 = P = \frac{1}{f}$$

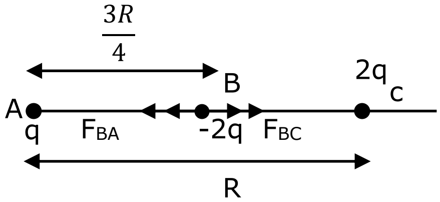
$$2P_1 = \frac{1}{0.1}$$

$$P_1 = 5D$$

58. Three point charges q, -2q and 2q are placed on x-axis at a distance x = 0, $x = \frac{3}{4}R$ and x = R respectively from origin as shown. If q = 2 × 10^{-6} C and R = 2 cm, the magnitude of net force experienced by the charge -2q is ____N.



Sol. **(5440)**



$$F_{BA} = \frac{32kq^2}{9q^2} \qquad F_{BC} = \frac{64kq^2}{R^2}$$

$$F_B = F_{BC} - F_{BA} = \frac{544kq^2}{9R^2}$$

$$= 5440N$$

59. An atom absorbs a photon of wavelength 500 nm and emits another photon of wavelength 600 nm. The net energy absorbed by the atom in this process is n × 10^{-4} eV. The value of n is ______.
[Assume the atom to be stationary during the absorption and emission process]
(Take h = 6.6 × 10^{-34} Js and c = 3 × 10^{8} m/s)

Sol. **(4125)**

$$E = E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$E = 6.6 \times 10^{-20}\,J$$

$$E = 4.125 \times 10^{-1}\,eV$$

$$E = 4125 \times 10^{-4}\,eV$$

60. A car accelerates from rest to u m/s. The energy spent in this process is E J. The energy required to accelerate the car from u m/s to 2 u m/s is nE J. The value of n is ______.

Sol. **(3)**

$$E_1 = \frac{1}{2}mu^2 - 0 = \frac{1}{2}mu^2 = E$$

$$E_2 = \frac{1}{2}m(24)^2 - \frac{1}{2}mu^2$$

$$= \frac{3}{2}mu^2 = 3E$$

SECTION - A

61. Which of the following are the Green house gases ?

(A) Water vapour (B) Ozone (C) I_2 (D) Molecular hydrogen

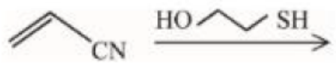
Choose the most appropriate answer from the options given below :

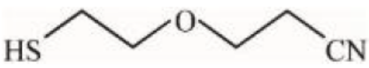
(1) C and D only (2) A and B only (3) B and C only (4) A and D only

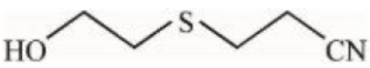
Sol. **2**

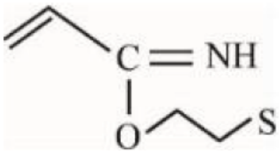
Green house gases are CO_2, CH_4, water vapour, nitrous oxide, CFC_s and ozone.

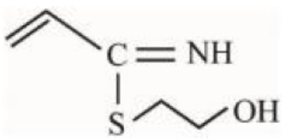
62. The major product for the following reaction is :



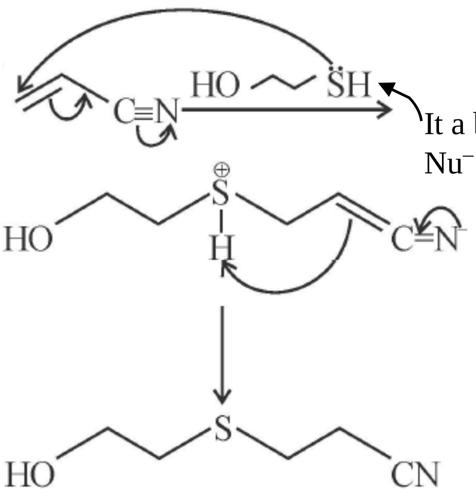
(1) 

(2) 

(3) 

(4) 

Sol. **2**



It a better Nu$^-$ than OH and it soft

Nu$^-$ soft will attack an soft electrophile size

63. In the wet tests for detection of various cations by precipitation, Ba^{2+} cations are detected by obtaining precipitate of :

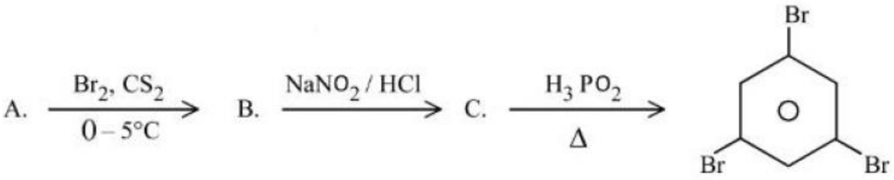
(1) $Ba(OAc)_2$ (2) $BaCO_3$ (3) $BaSO_4$ (4) Ba(ox) : Barium oxalate

Sol. **2**

In wet testing, $(NH_4)_2CO_3$ is used as group reagent for 5th group cations (Ba^{2+}, Ca^{2+}, Sr^{2+})

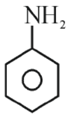
$$Ba^{+2} + (NH_4)_2CO_3 \rightarrow \underset{\text{(white precipitate)}}{BaCO_3 \downarrow} + NH_4^{\oplus}$$

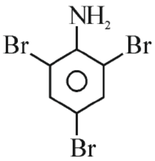
64. Compound A from the following reaction sequence is :

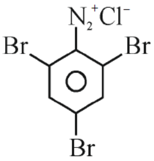


(1) Phenol (2) Benzoic Acid (3) Aniline (4) Salicylic Acid

Sol. **3**

A = 

B = 

C = 

65. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Isotopes of hydrogen have almost same chemical properties, but difference in their rates of reaction.

Reason R : Isotopes of hydrogen have different enthalpy of bond dissociation.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is not correct but R is correct

(2) Both A and R correct but R is NOT the correct explanation of A

(3) Both A and R are correct and R is the correct explanation of A

(4) A is correct but R is not correct

Sol. **3**

Source NCERT

Since the isotopes have the same electronic configuration, they have almost same chemical properties. The only difference is in their rates of reactions, mainly due to their different enthalpy of bond dissociation.

66. Given below are statements related to Ellingham diagram :

Statement I : Ellingham diagram can be constructed for oxides, sulfides and halides of metals.

Statement II : It consists of plosts of $\Delta_f H^0$ vs for formation of oxides of Clements.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but Statement II is incorrect

(2) Statements I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Sol. **1**

Statement I is correct, Ellingham diagram can be constructed for formation of oxides, sulphides and halides of metals. (Ref: NCERT)

Statement II is incorrect because Ellingham diagram consists of Δ_fG^0 vs T for formation of oxides of elements.

67. Better method for preparation of BeF_2, among the following is

(1) $BeH_2 + F_2 \xrightarrow{\Delta} BeF_2$ (2) $Be + F_2 \xrightarrow{\Delta} BeF_2$

(3) $(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2$ (4) $BeO + C + F_2 \xrightarrow{\Delta} BeF_2$

Sol. **3**

As per NCERT (s block), the better method of preparation of BeF_2 is heating $(NH_4)_2BeF_4$

$$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + NH_4F$$

68. Identify the correct order of standard enthalpy of formation of sodium halides.

(1) NaI < NaBr < NaF < NaCl (2) NaF < NaCl < NaBr < Nal

(3) NaCl < NaF < NaBr < Nal (4) Nal < NaBr < NaCl < NaF

Sol. **4**

For a given metal Δ_fH^0 always becomes less negative from fluoride to iodide.

69. Which of the following complexes will exhibit maximum attraction to an applied magnetic field ?

(1) $[Zn(H_2O)_6]^{2+}$ (2) $[Ni(H_2O)_6]^{2+}$ (3) $[Co(en)_3]^{3+}$ (4) $[Co(H_2O)_6]^{2+}$

Sol. **4**

Complex with maximum number of unpaired electron will exhibit maximum attraction to an applied magnetic field

$[Zn(H_2O)_6]^{2+} \rightarrow d^{10}$ system $\rightarrow t_{2g}^6\ eg^4$, 0 unpaired e^-

$[Co(H_2O)_6]^{2+} \rightarrow d^7$ system $\rightarrow t_{2g}^5\ eg^2$, 3 unpaired e^-

$[Co(en)_3]^{3+} \rightarrow d^6$ system $\rightarrow t_{2g}^6\ eg^0$, 0 unpaired e^-

$[Ni(H_2O)_6]^{2+} \rightarrow d^8$ system $\rightarrow t_{2g}^6\ eg^2$, 2 unpaired e^-

70. The correct group of halide ions which can be oxidized by oxygen in acidic medium is

(1) Cl^-, Br^- and I^- only (2) Br^- only (3) Br^- and I^- only (4) I^- only

Sol. **4**

Only I^- among halides can be oxidised to Iodine by oxygen in acidic medium

$4I^-(aq) + 4H^+(aq) + O_2(g) \rightarrow 2I_2(s) + 2H2O(l)$

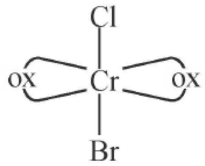
71. The total number of stereoisomers for the complex $[Cr(ox)_2ClBr]^{3-}$ (where ox = oxalate) is :

(1) 3 (2) 1 (3) 4 (4) 2

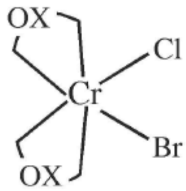
Sol. **1**

$[Cr(Ox)_2ClBr]^{-3}$

- No. of isomers-



- This structure has plane of symmetry, So no optical isomerism will be shown.



- This structure does not contain plane of symmetry, So two forms d as well as 1 will be shown.

72. Match List I with List II

I – Bromopropane is reacted with reagents in List I to give product in List II

LIST I-Reagent	**LIST II – Product**
A KOH (alc)	I. Nitrile
B. KCN (alc)	II. Ester
C. $AgNO_2$	III. Alkene
D. $H_3CCOOAg$	IV. Nitroalkane

Choose the correct answer from the options given below :

(1) A–IV, B-III, C-II, D-I
(2) A–I, B-III, C-IV, D-II
(3) A–I, B-II, C-III, D-IV
(4) A–III, B-I, C-IV, D-II

Sol. **4**

$CH_3 - CH_2 - CH_2 - Br + KOH(Alc) \rightarrow CH_3 - \underset{(Alkene)}{CH} = CH_2$

$CH_3 - CH_2 - CH_2 - Br + KCN(Alc) \rightarrow \underset{(Nitrile)}{CH_3 - CH_2 - CH_2 - CN}$

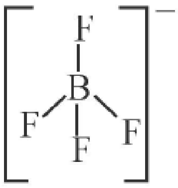
$CH_3 - CH_2 - CH_2 - Br + AgNO_2 + \underset{(Nitroalkane)}{CH_3 - CH_2 - CH_2 - NO_2} + AgBr \downarrow$

$CH_3 - CH_2 - CH_2 - Br + CH_3 - COOAg \rightarrow CH_3 - COO - \underset{(Ester)}{CH_2} - CH_2 - CH_3 + AgBr \downarrow$

73. The covalency and oxidation state respectively of boron in $[BF_4]^-$, are :

(1) 3 and 5
(2) 4 and 3
(3) 4 and 4
(4) 3 and 4

Sol. **2**



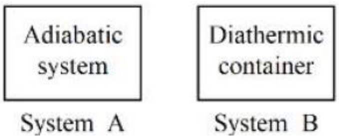
Number of covalent bond formed by Boron is 4

Oxidation number of fluorine is –1,

Oxidation number of B + 4 × (–1) = –1,

Thus, Oxidation number of B = + 3

74. What happens when methance undergoes combustion in systems A and B respectively ?



(1)

System A	System B
Temperature remains same	Temperature rises

(2)

System A	System B
Temperature falls	Temperature rises

(3)

System A	System B
Temperature falls	Temperature remains same

(4)

System A	System B
Temperature rises	Temperature remains same

Sol. **4**

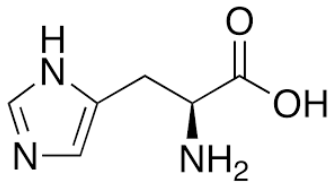
Adiabatic boundary does not allow heat exchange thus heat generated in container can't escape out thereby increasing the temperature. In case of Diathermic container, heat flow can occur to maintain the constant temperature.

75. The naturally occurring amino acid that contains only one basic functional group in its chemical structure is :

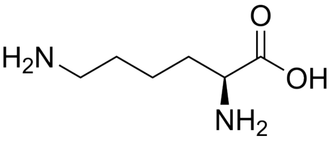
(1) histidine (2) lysine (3) asparagine (4) arginine

Sol. **3**

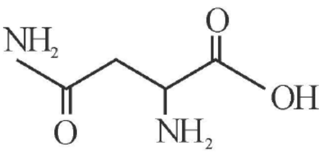
1. histidine



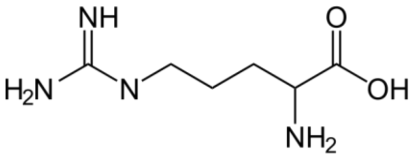
2. Lysine



3.



4. Arginine



76. Given below are two statements :

Statement I : SO_2 and H_2O both possess V-shaped structure.

Statement II : The bond angle of SO_2 less than that of H_2O

In the light of the above statements, choose the most appropriate answer from the options given below :

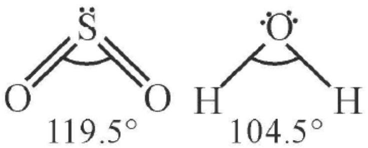
(1) Both Statements I and Statement II are incorrect

(2) Both Statement I and Statements II are correct

(3) Statement I is correct but Statement II is incorrect

(4) Statements I is incorrect but Statement II is correct

Sol. **3**



Both are bent in shape.

Bond angle of SO_2 (sp^2) is greater than that of H_2O (sp^3) due to higher repulsion of multiple bonds.

77. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : The diameter of colloidal particles in solution should not be much smaller than wavelength of light to show Tyndall effect.

Reason R: The light scatters in all direction when the size of particles is large enough.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is true but R is false

(3) Both A and R are correct and R is the correct explanation of A

(4) A is false but R is true

Sol. **3**

Tyndall effect is observed only when the following two conditions are satisfied

(a) The diameter of the dispersed particle is not much smaller than the wave length of light used.

(b) Refractive indices of dispersed phase and dispersion medium differ greatly in magnitude.

78. Match List I with List II

LIST I	**LIST II**
A. Weak intermolecular farces of attraction	I. Hexamethylenendiamine + adipic
B. Hydrogen bonding	II. $AlEt_3 + TiCl_4$
C. Heavily branched polymer	III. 2–chloro –1, 3 – butadiene
D. High density polymer	IV. Phenol + formaldehyde

Choose the correct answer from the options given below :

(1) A–IV, B-I, C-III, D-II

(2) A–III, B-I, C-IV, D-II

(3) A–II, B-IV, C-I, D-III

(4) A–IV, B-II, C-III, D-I

Sol. **2**

- Hexamethylenediamine on reaction with adipic acid forms Nylon 6, 6 which shows H-bonding due to presence of amide group.
- $AlEt_3 + TiCl_4$ is Ziegler-Natta catalyst used to prepare high density polyethylene.
- 2-chloro-1, 3-butadiene (chloroprene) is monomer of neoprene which is a rubber (an elastomer)
- Phenol - formaldehyde forms Bakelite which is heavily branched (cross-linked) polymer

79. Given below are two statements :

Statement I : Tropolone is an aromatic compound and has 8 π electrons.

Statement II : π electrons of > C = O group in tropolone is incolved in aromaticity

In the light of the above statements, choose the correct answer from the options given below :

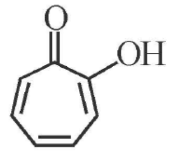
(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

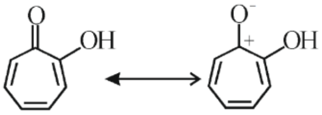
(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. **2**



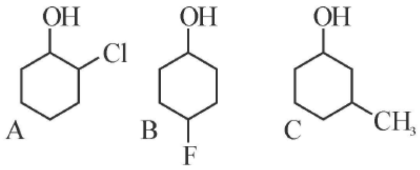
Tropolone is an aromatic compound and has 8π electrons ($6\pi e^-$ are endocyclic and $2\pi e^-$ are exocyclic) and π electrons of $>C = O$ group in tropolone is not involved in aromaticity.



aromatic compound ($6\pi e^-$)

80. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Order of acidic nature of the following compounds is A > B > C.



Reason R : Fluoro is a stronger electron withdrawing group than Chloro group.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are correct and R is the correct explanation of A

(2) A is false but R is true

(3) Both A and R are correct but R is NOT the correct explanation of A

(4) A is true but R is false

Sol. **3**

Acidic strength α –I effect

$$\alpha \ \frac{I}{+I} \text{ effect}$$

F, Cl exerts -1 effect, Methyl exerts +I effect, C is least acidic.

Among A and B; since inductive effect is distance dependent, Extent of –I effect is higher in A followed by B even though F is stronger electron withdrawing group than Cl. Thus, A is more acidic than B.

SECTION - B

81. If the formula of Borax is $Na_2B_4O_x(OH)_y \cdot zH_2O$, then x + y + z = ______.

Sol. **17**

Formula of borax is $Na_2B_4O_5(OH)_4 \cdot 8H_2O$

82. Sea water contains 29.25% NaCl and 19% $MgCl_2$ by weight of solution. The normal boiling point of the sea water is ______°C(Nearest integer)

Assume 100% ionization for both NaCl and $MgCl_2$

Given : $K_b(H_2O) = 0.52$ K kg mol^{-1}

Molar mass of NaCl and $MgCl_2$ is 58.5 and 95 g mol^{-1} respectivley.

Sol. **116**

Amount of solvent = 100 – (29.25 + 19) = 51.75g

$$\Delta T_b = \left[\frac{2\times 29.25\times 1000}{58.5\times 51.75}+\frac{3\times 19\times 1000}{95\times 51.75}\right]\times 0.52$$

$\Delta T_b = 16.075$

$\Delta T_b = (T_b)_{solution} - (T_b)_{solvent}$

$(T_b)_{solution} = 100 + 16.07$

$= 116.07°C$

83. 20 mL of 0.1 M NaOH is added to 50 mL of 0. 1 M acetic acid solution. The pH of the resulting solution is ____ $\times 10^{-2}$ (Nearest integer)

Given : pKa (CH_3 COOH) = 4.76

log 2 = 0.30

log 3 = 0.48

Sol. **458**

	CH_3COOH +	NaOH $\rightarrow$	CH_3COONa +	H_2O
Initially	5mmol	2mmol	0	0
after Rxn	3mmol	0	2 mmole	2 mmole

$$pH = pKa + \log_{10}\frac{[salt]}{[acid]}$$

$$pH = 4.76 + \log_{10}\frac{2}{3}$$

pH = 4.58 = 458 $\times 10^{-2}$

84. At 298 K, the standard reduction potential for Cu^{2+}/Cu electrode is 0.034 V.

Given : K_{sp} $Cu(OH)_2 = 1 \times 10^{-20}$

$$\text{Take}\frac{2.303RT}{F} = 0.059V$$

The reduction potential at pH = 14 for the above couple is (–) x $\times 10^{-2}$ V.

The value of x is _____.

Sol. **25**

$Cu(OH)_2(s) \rightleftharpoons Cu^{2+}(aq) + 2OH^{-}(aq)$

$Ksp = [Cu^{2+}] [OH^{-}]^2$

pH = 14; pOH = 0; $[OH^{-}]$ = 1M

$$\therefore\ [Cu^{2+}] = \frac{Ksp}{[1]^2} = 10^{-20} M$$

$Cu^{2+}(aq) + 2e^{-} \rightarrow Cu(s)$

$$E = E^\circ - \frac{0.059}{2}\log_{10}\frac{1}{[Cu^{2+}]}$$

$$= 0.34 - \frac{0.059}{2}\log_{10}\frac{1}{10^{-20}}$$

$= -0.25 = -25 \times 10^{-2}$

85. Sodium metal crystallizes in a body centred cubic lattice with unit cell edge length of 4 Å. The radius of sodium atom is ____$\times 10^{-1}$ Å (Nearest integer)

Sol. **17**

$\sqrt{3}a = 4r$

$\sqrt{3} \times 4 = 4r$

r = 1.732Å

= 17.32 $\times 10^{-1}$

86. A (g) $\rightarrow$ 2B (g) + C (g) is first order reaction. The initial pressure of the system was found to be 800 mm Hg which increased to 1600 mm Hg after 10 min. The total pressure of the system after 30 min will be ____ mm Hg. (Nearest integer)

Sol. **2200**

$t_{\frac{1}{2}}$ = 10 minutes

$$(P_A)_{30\,min} = (P_A)_0\left(\frac{1}{2}\right)^{30/10}$$

$(P_A)_{30\ min}$ = 100 mm Hg

	A(g) $\longrightarrow$	2B(g) +	C(g)
at t = 0	800 mm	0	0
at t = 30	100 mm	1400 mm	700 mm

Total pressure after 30 minutes = 2200 mm Hg

87. 1g of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01 mol of CO_2. The molar mass of M_2CO_3 is ____ g mol^{-1}. (Nearest integer)

Sol. **100**

$$\underset{1gm}{M_2CO_3} + \underset{Excess}{2HCl} \rightarrow \underset{0.02\,mole}{2MCl} + H_2O + \underset{0.01\,mole}{CO_2}$$

From principle of atomic conservation of carbon atom,

Mole of $M_2CO_3 \times 1$ = Mole of $CO_2 \times 1$

$$\frac{1gm}{\text{molar mass of } M_2CO_3} = 0.01\times 1$$

$\therefore$ Molar mass of M_2CO_3 = 100 gm/mole

88. 0.400 g of an organic compound (X) gave 0.376 g of AgBr in Carius method for estimation of bromine. % of bromine in the compound (X) is ____. (Given : Molar mass AgBr = 188 g mol^{-1}, Br = 80 g mol^{-1})

Sol. **40**

mole of AgBr = $\frac{0.376}{188}$

mole of Br^- = mole of AgBr = $\frac{0.376}{188}$

mass of Br^- = $\frac{0.376}{188}\times 80$

% of Br^- = $\frac{0.376\times 80}{188\times 0.4}\times 100 = 40\%$

89. The orbital angular momentum of an electron in 3s orbital is $\frac{xh}{2\pi}$. The value of x is ____ (nearest integer)

Sol. **0**

Orbital angular momentum = $\sqrt{l(l+1)}\frac{h}{2\pi}$

Value of 1 for s = 0

90. See the following chemical reaction :

$$Cr_2O_7^{2-} + XH^+ + 6Fe^{2+} \rightarrow YCr^{3+} + 6Fe^{3+} + ZH_2O$$

The sum of X, Y and Z is _____

Sol. **23**

$Cr_2O_7^{2-} + 14H^+ + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$

x = 14

y = 2

z = 7

Hence (x + y + z) = 14 + 2 + 7 = 23