SECTION-A

1. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of $y_1, y_2, \ldots y_{100}$ is :

(1) 10051.50 (2) 10100 (3) 10101.50 (4) 10049.50

Sol. (4)

Mean = 200

$$\Rightarrow \frac{\frac{100}{2}(2\times 2+99d)}{100} = 200$$

$$\Rightarrow 4+99d = 400$$

$$\Rightarrow d = 4$$

$$y_i = i(xi - 1)$$

$$= i(2+(i-1)4-i) = 3i^2 - 2i$$

$$\text{Mean } = \frac{\sum y_i}{100}$$

$$= \frac{1}{100}\sum_{i=1}^{100} 3i^2 - 2i$$

$$= \frac{1}{100}\left\{\frac{3\times 100\times 101\times 201}{6} - \frac{2\times 100\times 101}{2}\right\}$$

$$= 101\left\{\frac{201}{2} - 1\right\} = 101\times 99.5$$

= 10049.50

2. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is :

(1) 10 (2) 9 (3) 8 (4) 12

Sol. (2)

$$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$$

$$\Rightarrow 3\cos^2\theta(\cos^2\theta - 1) + 2\sin^2\theta(\sin^4\theta - 1) = 0$$

$$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta(1+\sin^2\theta)\cos^2\theta - 1$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2+2\sin^2\theta - 3) = 0$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta - 1) = 0$$

$$(C1)\sin^2\theta = 0 \rightarrow 3\text{ solution; } \theta = \{0, \pi, 2\pi\}$$

$$(C2)\cos^2\theta = 0 \rightarrow 2\text{ solution; } \theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

$$(C3)\ \sin^2\theta = \frac{1}{2} \rightarrow 4\text{ solution; } \theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

No. of solution = 9

3. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to :

(1) $\log_e\left(\frac{\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$ (2) $\log_e\left(\frac{2\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{\sqrt{2}\left(3-\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$ (4) $\log_e\left(\frac{\sqrt{2}\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

Sol. **(4)**

$$I=\int_{-\ln 2}^{\ln 2} e^x\left(\ln\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$$

Put $e^x=t \Rightarrow e^x dx=dt$

$$I=\int_{1/2}^{2}\ln\left(t+\sqrt{1+t^2}\right)dt$$

Applying integration by parts.

$$=\left[t\ln\left(t+\sqrt{1+t^2}\right)\right]_{\frac{1}{2}}^{2}-\int_{1/2}^{2}\frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$$

$$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\int_{1/2}^{2}\frac{t}{\sqrt{1+t^2}}dt$$

$$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\frac{\sqrt{5}}{2}$$

$$=\ln\left(\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{\frac{1}{2}}}\right)-\frac{\sqrt{5}}{2}$$

4. Let S = {M = [a_{ij}], a_{ij} ∈ {0, 1, 2}, 1 ≤ i, j ≤ 2} be a sample space and A = {M ∈ S : M is invertible} be an event. Then P(A) is equal to :

(1) $\frac{16}{27}$ (2) $\frac{50}{81}$ (3) $\frac{47}{81}$ (4) $\frac{49}{81}$

Sol. **(2)**

M $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d, $\in\{0,1,2\}$

n(s) = 3^4 = 81

we first bound p $(\bar{A})$

$|m|=0 \Rightarrow ad=bc$

$ad=bc=0 \Rightarrow$ no. of (a, b, c, d) = $\left(3^2-2^2\right)^2=25$

ad = bc =1 $\Rightarrow$ no. of (a,b,c,d) = 1^2 = 1

ad = bc = 2 $\Rightarrow$ no. of (a,b,c,d) = 2^2 = 4

ad = bc = 4 $\Rightarrow$ no. of (a,b,c,d) = 1^2 = 1

$:P(\bar{A})=\frac{31}{81} \Rightarrow p(A)=\frac{50}{81}$

5. Let f : [2, 4] → ℝ be a differentiable function such that (x $\log_e$ x) f ' (x) + ($\log_e$ x) f(x) + f(x) ≥ 1, x ∈ [2, 4] with f(2) = $\frac{1}{2}$ and f(4) = $\frac{1}{4}$. Consider the following two statements :

(A) : f(x) ≤ 1, for all x ∈ [2, 4]

(B) : f(x) ≥ $\frac{1}{8}$, for all x ∈ [2, 4]

Then,

(1) Only statement (B) is true
(2) Only statement (A) is true
(3) Neither statement (A) nor statement (B) is true
(4) Both the statements (A) and (B) are true

Sol. **(4)**

$x\ell nxf'(x)+\ln xf(x)+f(x)\geq 1, x\in[2,4]$

And f(2) = $\frac{1}{2}$, f(4) = $\frac{1}{4}$

Now xlnx, $\frac{dy}{dx}+(\ln+1)y\geq 1$

$\frac{d}{dx}(y\cdot x\ln x)\geq 1$

$\frac{d}{dx}(f(x).x\ln x)\geq 1$

$\Rightarrow \frac{d}{dx}(x\ln xf(x)-x)\geq 0, x\in[2,4]$

⇒ The function g(x) = x ln x f(x) – x is increasing in [2,4]

And g(2) = 2 ln 2f(2) – 2 = ln 2– 2

g(2) = 4 ln 4f(4) – 4 = ln 4 – 4

= 2(ln 2 – 2)

Now $g(2)\leq g(x)\leq g(4)$

Ln 2 – 2 ≤ x lnx f(x) – x ≤ 2(ln 2– 2)

$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\leq f(x)\leq \frac{2(\ln 2-2)}{x\ln x}+\frac{1}{\ln x}$

Now for $x\in[2,4]$

$\frac{2(\ell n2-2)}{x\ln x}+\frac{1}{\ell nx}<\frac{2(\ln 2-2)}{2\ln 2}+\frac{1}{\ln 2}=1-\frac{1}{\ln 2}<1$

$\Rightarrow f(x)\leq 1$ for $x\in[2,4]$

$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\geq\frac{\ln 2-2}{4\ln 4}+\frac{1}{\ln 4}=\frac{1}{8}+\frac{1}{2\ln 2}>\frac{1}{8}$

$\Rightarrow f(x)\geq\frac{1}{8}$ for $x\in[2,4]$

Hence both A and B are true.

6. Let A be a 2 × 2 matrix with real entries such that A' = α A + I, where a $\in \mathbb{R} - \{-1, 1\}$. If det $(A^2 - A) = 4$, then the sum of all possible values of α is equal to :

(1) 0 (2) $\frac{5}{2}$ (3) 2 (4) $\frac{3}{2}$

Sol. **(2)**

$A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A(1-\alpha^2) = (\alpha+1)I$

$A = \frac{I}{1-\alpha}$ … (1)

$|A| = \frac{1}{(1-\alpha)^2}$ … (2)

$|A^2 - A| = |A||A-1|$ … (3)

$A - I = \frac{I}{I-\alpha} - I = \frac{\alpha}{1-\alpha} I$

$|A - I| = \left(\frac{\alpha}{1-\alpha}\right)^2$ … (4)

Now $|A^2 - A| = 4$

$|A||A-I| = 4$

$\Rightarrow \frac{1}{(1-\alpha)^2} \frac{\alpha^2}{(1-\alpha^2)} = 4$

$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1)\, 2(1-\alpha)^2 = \alpha$ $(C_2)\, 2(1-\alpha)^3 = -\alpha$

$2\alpha^2 - 5\alpha + 2 = 0 < \alpha_1, \alpha_2$ $2\alpha^2 - 3\alpha + 2 = 0$

$\alpha_1 + \alpha_2 = \frac{5}{2}$ $\alpha \notin R$

7. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

(1) 5 (2) 7 (3) 8 (4) 6

Sol. **(4)**

$\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region: $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$

And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$

And $\frac{x-7}{2x-3} \neq 0$ and $2x-3 \neq 0$

$\Downarrow$ $\Downarrow$

$x \neq 7$ $x \neq \frac{3}{2}$

Taking intersection: $x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$

Now $\log_a b \geq 0$ if a > 1 and b ≥ 1

$a \in (0,1)$ and $b \in (0,1)$

$C - I;\ x + \frac{7}{2} > 1$ and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$x > -\frac{5}{2}\ ;(2x-3)^2 - (x-7)^2 \leq 0$

$(2x-3+x-7)\ (2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in \left[-4, \frac{10}{3}\right]$

Intersection: $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]$

$C - II:\ x + \frac{7}{2} \in (0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in (0,1)$

$0 < x + \frac{7}{2} < 1;\ \left(\frac{x-7}{2x-3}\right)^2 < 1$

$-\frac{7}{2} < x < \frac{-5}{2}\ ;\ (x-7)^2 < (2x-3)^2$

$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x \in \left(\frac{-5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

Integral value of x are {–2, –1, 0, 1, 2, 3}

No. of integral values = 6

8. For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10\,|a_i| < 1$, i = 1, 2, 3, consider the following statements :

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3 \max\{|a_1|, |a_2|, |a_3|\}$

(1) Only (B) is true
(2) Both (A) and (B) are true
(3) Neither (A) nor (B) is true
(4) Only (A) is true

Sol. **(2)**

Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$|\vec{a}|^2 = |\vec{a}_1|^2 + |\vec{a}_2|^2 + |\vec{a}_3|^2 \geq (a_3)^2$

$\Rightarrow |\vec{a}| \geq |a_3|$ = max $\{|a_1|, |a_2|, |a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow |\vec{a}|^2 \leq 3|a_3|^2$

$\Rightarrow |\vec{a}| \leq \sqrt{3}|\vec{a}_3| = \sqrt{3}$ max $\{|a_1|, |a_2|, |a_3|\}$

$\leq 3\max\ \{|a_1|, |a_2|, |a_3|\}$

(2) is true

9. The number of triplets (x,y,z), where x, y, z are distinct non negative integers satisfying x + y + z = 15, is :

(1) 136 (2) 114 (3) 80 (4) 92

Sol. **(2)**

x + y + z = 15

Total no. solution = ${}^{15+3-1}C_3 = 136$ … (1)

Let x = y $\neq$ z

2x+ z = 15 $\Rightarrow$ z = 15 – 2t

$\Rightarrow r \in \{0, 1, 2, \ldots 7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x, y, z are equal … (2)

There is one solution in which x = y = z … (3)

Required answer = 136 – 21 – 1= 144

10. Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is _______.

(1) 36 (2) 40 (3) 32 (4) 38

Sol. **(4)**

$\omega\ A = \{a_1, a_2, a_3, a_4, a_5\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

Given, $\sum_{i=1}^{5} ai = 25, \sum_{i=1}^{5} bi = 40$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^{5} a_i^2 = 185, \quad \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, \ldots C_{10}\}$

$\therefore$ Mean of C, $\bar{C} = \frac{\left(\sum a_i - 15\right) + \left(\sum b_i - 10\right)}{10}$

$\bar{C} = \frac{10 + 50}{10} = 6$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = (\bar{C})^2$$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i - 2)^2 +}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

= 32

$\therefore$ Mean + Variance = $\bar{C} + \sigma^2 = 6 + 32 = 38$

11. Area of the region $\{(x, y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is :

(1) $\pi + \frac{8}{3}$ (2) $2\pi + \frac{16}{3}$ (3) $2\pi - \frac{16}{3}$ (4) $\pi - \frac{8}{3}$

Sol. **(3)**

$x^2 + (y-2)^2 \le 2^2$ and $x^2 \ge 2y$

Solving circle and parabola simultaneously:

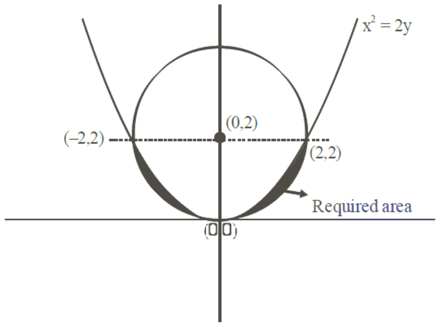
$2y + y^2 - 4y + 4 = 4$

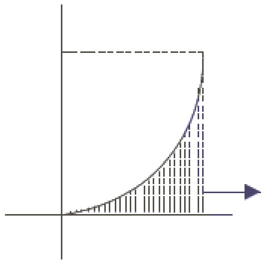
$y^2 - 2y = 0$

y = 0, 2

Put $y = 2$ in $x^2 = 2y \rightarrow x = \pm 2$

$\Rightarrow$ (2, 2) and (–2, 2)





$$= 2 \times 2 - \frac{1}{4} \cdot \pi \cdot 2^2 = 4 - \pi$$

Required area = $2\left[\int_0^2 \frac{x^2}{2}dx - (4 - \pi)\right]$

$$= 2\left[\frac{x^3}{6}\Big|_0^2 - 4 + \pi\right]$$

$= 2\left[\frac{4}{3} + \pi - 4\right]$

$= 2\left[\pi - \frac{8}{3}\right]$

$= 2\pi - \frac{16}{6}$

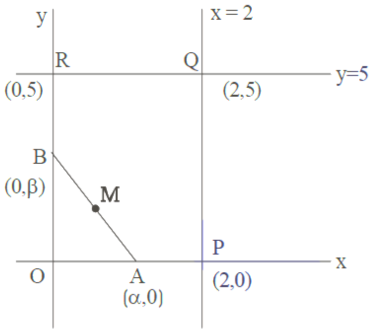
12. Let R be a rectangle given by the line x = 0, x = 2, y = 0 and y = 5. Let A (α, 0) and B (0, β), $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : l. Then, the mid-point of AB lies on a :

(1) straight line (2) parabola (3) circle (4) hyperbola

Sol. **(4)**

$\frac{\text{ar}(\text{OPQR})}{\text{or}(\text{OAB})} = \frac{4}{1}$

Let M be the mid-point of AB.



$M(h, k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$\Rightarrow \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$

$\Rightarrow \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$

$\Rightarrow (2h)(2K) = 4$

$\therefore$ Locus of M is xy = 1

Which is a hyperbola.

13. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two places described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}.\vec{b} = 6$, then ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to :

(1) $\left(\frac{\pi}{3}, 6\right)$ (2) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$ (3) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$ (4) $\left(\frac{\pi}{4}, 6\right)$

Sol. **(4)**

$\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{i} - \hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \lambda|\vec{n}_2 \times \vec{n}_2|$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \lambda\left(-2\hat{j} + 2\hat{k}\right)$$

$$\vec{a} \cdot \vec{b} = \lambda|0 + 4 + 2| = 6$$

$$\Rightarrow \lambda = 1$$

$$\vec{\alpha} = -2\hat{j} + 2\hat{k}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$\cos\theta = \frac{6}{2\sqrt{2} \times 3} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

Now $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |a|^2|b|^2$

$$36 + |\vec{a} \times b^2| = 8 \times 9 = 72$$

$$|\vec{a} \times b|^2 = 36$$

$$|\vec{a} \times \vec{b}| = 6$$

14. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to :

(1) $\pi - \tan^{-1}\frac{8}{9}$ (2) $-\pi + \tan^{-1}\frac{8}{9}$ (3) $\pi - \tan^{-1}\frac{33}{5}$ (4) $-\pi + \tan^{-1}\frac{33}{5}$

Sol. **(1)**

$$W_1 = z_i i = (5 + 4i)i = -4 + 5i \ \dots \text{(i)}$$

$$W_1 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i \ \dots (2)$$

$$W_1 - W_2 = -9 + 8i$$

Principal argument = $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

15. Consider ellipse $E_k : kx^2 + k^2y^2 = 1$, $k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k then the value of $\sum_{k=1}^{20} \frac{1}{r_k^{\ 2}}$ is

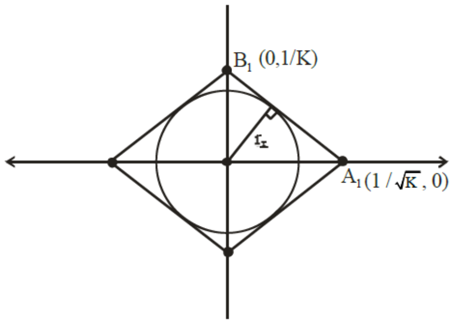
(1) 3320 (2) 3210 (3) 3080 (4) 2870

Sol. **(3)**

$$Kx^2 + K^2y^2 = 1$$

$$\frac{x^2}{1/K}+\frac{y^2}{1/K^2}=1$$

Now



Equation of

$$A_1B_2;\frac{x}{1/\sqrt{K}}+\frac{y}{1/K}=1\Rightarrow\sqrt{K}x+Ky=1$$

$r_K=\perp r$ distance of (0, 0) from line A_1B_1

$$r_K=\left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right|=\frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2}=K+K^2\Rightarrow\sum_{k=1}^{20}\frac{1}{r_K^2}=\sum_{K=1}^{20}\left(K+K^2\right)$$

$$=\sum_{K=1}^{20}K+\sum_{K=1}^{20}K^2$$

$$=\frac{20\times21}{2}+\frac{20.21.41}{6}$$

= 210 + 10 × 7 × 41

= 210 + 2870

= 3080

16. If equation of the plane that contains the point (–2,3,5) and is perpendicular to each of the planes 2x + 4y + 5z = 8 and 3x – 2y + 3z = 5 is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma$ = :

(1) 15 (2) 18 (3) 17 (4) 16

Sol. **(1)**

The equation of plane through (–2,3,5) is

a(x+2) + b(y – 3) + c (z – 5) = 0

it is perpendicular to 2x + 4y + 5z = 8 & 3x – 2y + 3z = 5

$\therefore$ 2a + 4b + 5c =0

3a – 2b + 3c = 0

$$\therefore\frac{a}{\begin{vmatrix}4&5\\-2&3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2&5\\3&3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2&4\\3&-2\end{vmatrix}}$$

$$\Rightarrow\frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$$

$\therefore$ Equation of plane is

22(x + 2) + 9(y – 3) –16(z – 5) = 0

$\Rightarrow$ 22x + 9y – 16z + 97 = 0

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

17. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

(1) 15 (2) 9 (3) 21 (4) 10

Sol. **(3)**

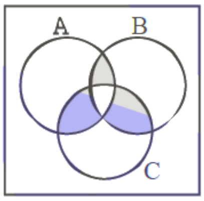
$|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cap B \cap C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$

$= 36$

No. of men who received exactly 2 medals

$\Rightarrow \sum |A \cap B| - 3|A \cap B \cap C|$

$= 36 - 15$

$= 21$

18. Let y = y(x) be a solution curve of the differential equation. $(1 - x^2y^2)dx = ydx + xdy$. If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at y = α, then a value of α is :

(1) $\frac{1+3e^2}{2(3e^2-1)}$ (2) $\frac{1-3e^2}{2(3e^2+1)}$ (3) $\frac{3e^2}{2(3e^2-1)}$ (4) $\frac{3e^2}{2(3e^2+1)}$

Sol. **(1)**

$(1 - x^2y^2)dx = ydx + xdy, y(1) = 2$

$y(2) = \infty$

$dx = \frac{d(xy)}{1-(xy)^2}$

$\int dx = \int \frac{d(xy)}{1-(xy)^2}$

$x = \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| + C$

Put x = 1 and y = 2:

$1 = \frac{1}{2}\ln\left|\frac{1+2}{1-2}\right| + C$

$C = 1 - \frac{1}{2}\ln 3$

Now put x = 2 :

$$2=\frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right|+1-\frac{1}{2}\ln 3$$

$$1+\frac{1}{2}\ln 3=\frac{1}{2}\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$2+\ln 3=\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$\left|\frac{1+2\alpha}{1-2\alpha}\right|=3e^2$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^2,-3e^2$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^2\Rightarrow\alpha=\frac{3e^2-1}{2(3e^2+1)}$$

And $\frac{1+2\alpha}{1-2\alpha}=-3e^2\Rightarrow\alpha=\frac{3e^2+1}{2(3e^2-1)}$

19. Let (α, β, γ) be the image of the point P (2, 3, 5) in the plane 2x + y – 3z = 6. Then α + β + γ is equal to :

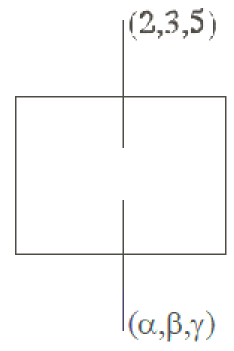
(1) 5 (2) 9 (3) 10 (4) 12

Sol. **(3)**

$$\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-2\left(\frac{2x2+3-3\times5-6}{2^2+1^2+1-3^2}\right)=2$$

$\frac{\alpha-2}{2}=2$ $\quad\beta-3=2$ $\quad\gamma-5=-6$

$\alpha=6$ $\quad\beta=5$ $\quad\gamma=-1$



$$\alpha+\beta+\gamma=10$$

20. Let f(x) = [x^2 – x] + |–x+[x]|, where x ∈ ℝ and [t] denotes the greatest integer less than or equal to t. Then, f is :

(1) not continuous at x = 0 and x = 1

(2) continuous at x = 0 and x = 1

(3) continuous at x = 1, but not continuous at x = 0

(4) continuous at x = 0, but not continuous at x = 1

Sol. **(3)**

Here f(x) = [x(x – 1)] + {x}

$f(0^+)=-1+0=-1$ $\quad f(1^+)=0+0=0$

$f(0)=0$ $\quad f(1)=0$

$f(1^-)=-1+1=0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

SECTION-B

21. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to :

Sol. **(171)**

The number of integral term in the expression of

$\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r\left(3^{\frac{1}{2}}\right)^{680-r}\left(5^{\frac{1}{4}}\right)^{r}$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Values' s of r, where $\frac{r}{4}$ goes to integer

r = 0, 4, 8, 12, …… 680

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

22. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____:

Sol. **(7)**

p	**q**	**r**	**Pvq**	**Pvr**	**(pvq) ∧ (pvr)**	**qvr**	**(pvq) ∧ (pvr) → qvr**
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

23. Let $A=\begin{bmatrix}0 & 1 & 2\\ a & 0 & 3\\ 1 & c & 0\end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval (n – 1, n], where $n \in \mathbb{N}$, then n is equal to ______:

Sol. **(2)**

$$A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$A^3 = A$

$$A^2=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^2=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&1&2+3c\end{bmatrix}$$

$$A^3=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&a&2+3c\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^3=\begin{bmatrix}2ac+3&a+2+3c&2a+4+6c\\a(a+3c)+2a&3+2ac&6+3a+9c\\a+2+3c&ac+c(2+3c)&2ac+3\end{bmatrix}$$

Given $A^3 = A$

$2ac + 3 = 0$ … (1) and $a + 2 + 3c = 1$

$a + 1 + 3c = 0$

$a + 1 - \frac{9}{2a} = 0$

$2a^2 + 2a - 9 = 0$

$f(1) < 0, f(2) > 0$

$a \in (1, 2]$

$n = 2$

24. For m, n > 0, let $\alpha(m, n) = \int_0^2 t^m (1+3t)^n dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p\,(14)^6$, then p is equal to _____:

Sol. **(32)**

$$\alpha(m,n)=\int_0^2 t^m(1+3t)^n\,dt$$

If $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$ then P

$$=11\int_0^2 \underset{II}{t^{10}}\,\underset{I}{(1+3t)^6} + 10\int^2 t^{11}(1+3t)^5\,dt$$

$$=11\left[(1+3t)^6\cdot\frac{t^{11}}{11}-\int 6(1+3t)^5\cdot 3\frac{t^{11}}{11}\right]_0^2+18\int_0^2 t^{11}(1+3t)^5\,dt$$

$$=\left(t^{11}(1+3t)^6\right)_0^2$$

$$=2^{11}(7)^6$$

$$=2^5(14)^6$$

$$=32(14)^6$$

25. Let S = $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ... + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to ______:

Sol. **(2175)**

$$S = 109 + \frac{108}{5} + \frac{107}{5^2} \ldots\ldots + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} \ldots\ldots \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} \ldots\ldots - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{1}{5} \frac{\left(1 - \frac{1}{5^{109}}\right)}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$s = \frac{5}{4}\left(109 - \frac{1}{4} + \frac{1}{4.5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

26. Let H $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1, n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then 21 l is equal to _____:

Sol. **(306)**

$$Hn \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$$

$$e = \sqrt{\frac{2n+4}{n+1}}$$

n = 48 (smallest even value for which $e \in Q$)

$$e = \frac{10}{7}$$

$a^2 = n+1$ $\quad\quad$ $b^2 = n+3$

$=49$ $\quad\quad$ $= 51$

l =length of LR = $\frac{2b^2}{a}$

L = $2 \cdot \frac{51}{7}$

$$1 = \frac{102}{7}$$

$$21\ell = 306$$

27. The mean of the coefficients of x, x^2, …. x^7 in the binomial expansion of $(2 + x)^9$ is ______:

Sol. **2736**

Coefficient of x = ${}^9C_1 2^8$

Coef. $x^2 = {}^9C_2 2^7$

Coef. $x^7 = {}^9C_7 \cdot 2^2$

$$\text{Mean} = \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 \ldots + {}^9C_7 \cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$

28. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ______:

Sol. **(51)**

$x^2 - 7x - 1 = 0 < \begin{matrix} a \\ b \end{matrix}$

By newton's theorem

$S_{n+2} - 7S_{n+1} - S_n = 0$

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

$$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}}$$

$$= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}}$$

$$= 51 \cdot \frac{S_{19}}{S_{19}} = 51$$

29. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _______.

Sol. **(44)**

Derangement of 5 students

$$D_5 = 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$= 120\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$

= 60 – 20 + 5 – 1

= 40+ 4

= 44

30. Let a line *l* pass through the origin and be perpendicular to the lines

$l_1 : \vec{r} = \hat{i} - 11\hat{j} - 7\hat{k} \;\; + \lambda \;\; \hat{i} + 2\hat{j} + 3\hat{k} \;\; , \lambda \in \mathbb{R}$ and

$l_2 : \vec{r} = -\hat{i} + \hat{k} \;\; + \mu \;\; 2\hat{i} + 2\hat{j} + \hat{k} \;\; , \mu \in \mathbb{R}.$

If P is the point of intersection of *l* and l_1, and Q (α, β, γ) is the foot of perpendicular from P on l_2 , then 9 ($\alpha + \beta + \gamma$) is equal to _____:

Sol. **(5)**

Let $\ell = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \gamma\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$

$= \gamma\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma\left(-4\hat{i} + 5\hat{j} - 2\hat{k}\right)$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$

By solving these equation $\gamma = -1, P(4, -5, 2)$

Let Q $(-1 + 2\mu, 2\mu, 1 + \mu)$

$\overrightarrow{PQ} \cdot \left(2\hat{i} + 2\hat{j} + \hat{k}\right) = 0$

$-2 + 4\mu + 4\mu + 1 + \mu = 0$

$9\mu = 1$

$\mu = \dfrac{1}{9}$

$Q\left(\dfrac{-7}{9}, \dfrac{2}{9}, \dfrac{10}{9}\right)$

$9(\alpha + \beta + \gamma) = 9\left(\dfrac{-7}{9} + \dfrac{2}{9} + \dfrac{10}{9}\right)$

= 5

SECTION - A

31. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces ($g_A : g_B$) will be :

(1) 1 : 16 (2) 3 : 16 (3) 3 : 4 (4) 4 : 3

Sol. **(3)**

$$g = \frac{4\pi}{3} GR\delta$$

$$g \propto \delta R$$

$$\frac{g_A}{g_B} = \frac{\delta_A R_A}{\delta_B . R_B} = \frac{\delta . R}{\frac{\delta}{3} . 4R} = \frac{3}{4}$$

32. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table in halved, it will just slip when placed at a distance of ______ from the centre :

(1) 8 cm (2) 4 cm (3) 2 cm (4) 1 cm

Sol. **(2)**

$fr = m\omega^2 r$

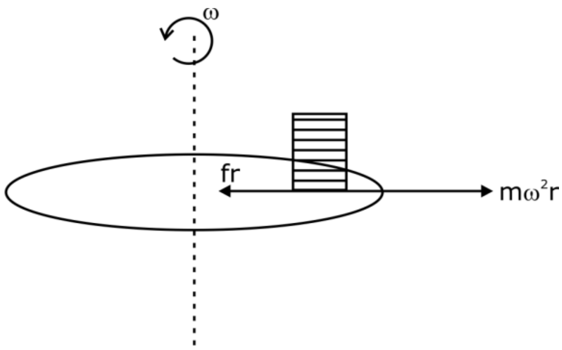
$\mu mg = m\omega^2 r = \text{const.}$

$\omega^2 r = \text{const}$

$\omega_1^2 r_1 = \omega_2^2 r_2$

$\omega^2 (1) = \left(\frac{\omega}{2}\right)^2 r_2$

$r_2 = 4\text{cm}$

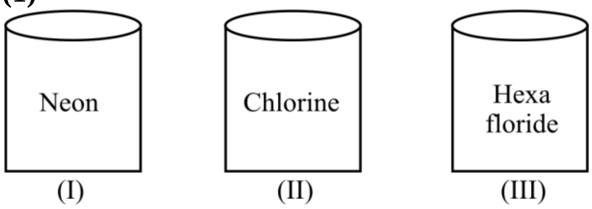


33. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafluoride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below :

(1) V_{rms} (mono) > v_{rms} (dia) > v_{rms} (poly) (2) V_{rms} (dia) < v_{rms} (poly) < v_{rms} (mono)

(3) V_{rms} (mono) < v_{rms} (dia) < v_{rms} (poly) (4) V_{rms} (mono) = v_{rms} (dia) = v_{rms} (poly)

Sol. **(1)**



VRMS = $\sqrt{\frac{\gamma RT}{m}}$ ....(1)

$\gamma = 1 + \frac{2}{f}$ so $r_{monochromic} > r_{diatomic} > r_{poly}$.

$V_{mono} > V_{diatomic} > V_{poly}$. Ans.(1)

34. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.

(1) $\lambda_A = 2\lambda_B$ (2) $\lambda_A = \lambda_B$ (3) $\lambda_A \ln 2 = \lambda_B$ (4) $\lambda_A = \lambda_B \ln 2$

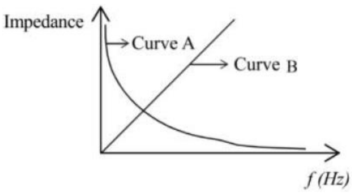
Sol. **(4)**

	A	B	$T \rightarrow$ half life
t = 0	N_0	N_0	$\tau \rightarrow$ ang life

$T_A = \tau_B \quad \rightarrow \quad$ given in question

Now $\dfrac{\ln(2)}{\lambda_A} = \dfrac{1}{\lambda_B} \Rightarrow \boxed{\lambda_A = \lambda_B \cdot \ln(2)}$

35.



As per the given graph, choose the correct representation for curve A and curve B.
{Where X_C = reactance of pure capacitive circuit connected with A.C. source
X_L = reactance of pure inductive circuit connected with A.C. source
R = impedance of pure resistive circuit connected with A.C. source.
Z = impedance of the LCR series circuit}

(1) $A = X_L, B = R$ (2) $A = X_L, B = Z$ (3) $A = X_C, B = R$ (4) $A = X_C, B = X_L$

Sol. **(4)**

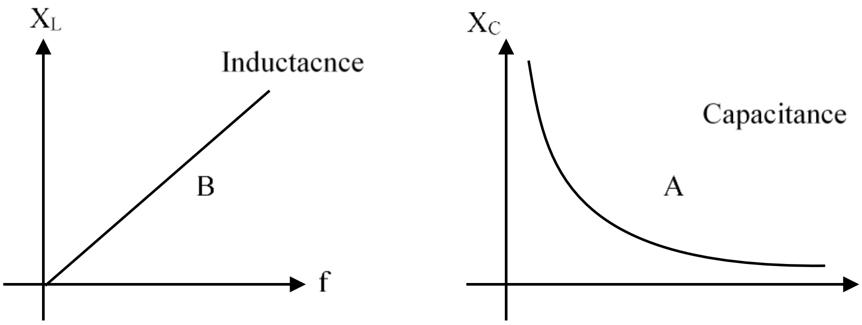
$X_L = W_L = 2\pi fL$

$X_C = \dfrac{1}{Wc} = \dfrac{1}{2\pi fc}$

R = const.

$A \rightarrow X_C$

$B \rightarrow X_L$



36. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is ______.
(Let, radius of earth R = 6400 km)

(1) 125 (2) 12.5 (3) 1250 (4) 1.25

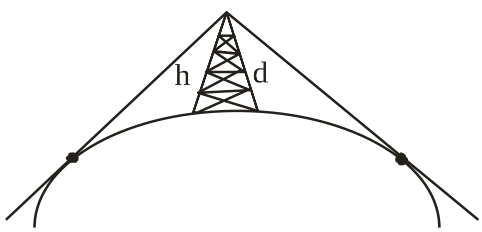
Sol. **(1)**

$d = \sqrt{2R \cdot h}$

$d^2 = 2Rh$

$(4)^2 = 2 \times 6400 \times h$

$\dfrac{16}{2 \times 6400} = h = \dfrac{1}{800}$ km

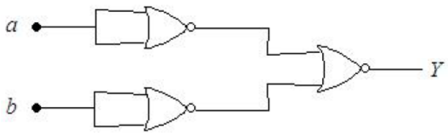


$h = \dfrac{1000}{800} = \dfrac{5}{4}$ m

$x \times 10^{-2} = \dfrac{5}{4}$

$x = \dfrac{500}{4} = 125$ Ans. Option $\rightarrow$ (1)

37. The logic performed by the circuit shown in figure is equivalent to :



(1) NAND (2) NOR (3) AND (4) OR

Sol. **(3)**

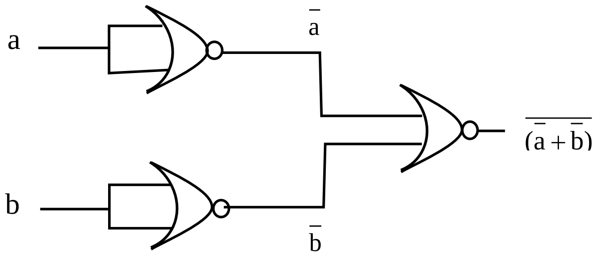
Y = $Y = \overline{(\bar{a} + \bar{b})}$

Y = $Y = (\overline{\overline{a \cdot b}})$

$\boxed{Y = a \cdot b}$

Ans. $\rightarrow$ AND gate

Option $\rightarrow$ 3



38. The electric field in an electromagnetic wave is given as

$\vec{E} = 20 \sin \omega \left(t - \frac{x}{c} \right) \vec{j} \, NC^{-1}$

where ω and c are angular frequency and velocity of electromagnetic wave respectively. the energy contained in a volume of 5 × 10^{-4} m^3 will be (Given $\varepsilon_0 = 8.85 \times 10^{-12} c^2 / Nm^2$)

(1) 88.5 × 10^{-13} J (2) 17.7 × 10^{-13} J (3) 8.85 × 10^{-13} J (4) 28.5 × 10^{-13} J

Sol. **(3)**

$\vec{E}$ = 20 sin w$\left[t - \frac{x}{c} \right]$

E_o 2_o

Energy density = $\frac{1}{2} \varepsilon_o E_o^2$

= $\frac{1}{2}$ × 8.85 × 10^{-12} × 400

= 200 × 8.85 × 10^{-12} × 5 × 10^{-4}

= 8.85 × 10^{-12} × 10^{-4} × 1000

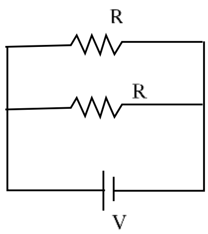
Energy = 8.85× 10^{-13} J option $\rightarrow$ (1)

39. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be :

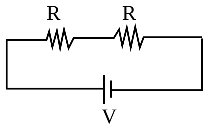
(1) 1 : 2 (2) 4 : 1 (3) 1 : 4 (4) 2 : 1

Sol. **(2)**

$H_1 = \frac{V^2}{(R/2)} t = \frac{2V^2}{R}$ .... (1)



$H_2 = \frac{V^2}{2R}t$ .... (2)



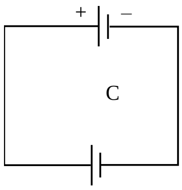
$\frac{H_1}{H_2} = \left(\frac{2V^2t}{R}\right) \times \frac{2R}{V^2t} = \frac{4}{1}$

40. A parallel plate capacitor of capacitance 2 F is charged to a potential V, The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is :

(1) 2 : 3 (2) 1 : 2 (3) 1 : 4 (4) 2 : 1

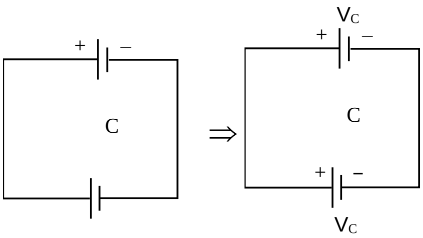
Sol. **(4)**

C = 2F



$E1 = \frac{1}{2}CV^2$ .... (1)

Now



$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$V_C = \frac{CV + O}{2C} = \frac{V}{2}$

$E_2 = CV_C{}^2 = C \cdot \frac{V^2}{4}$ .... (2)

$\frac{E_2}{E_1} = \frac{\left(\frac{CV^2}{4}\right)}{\left(\frac{CV^2}{2}\right)} = \frac{2}{1}$ option $\rightarrow$ (4)

41. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is :

(1) 25 (2) 5 (3) 100 (4) 50

Sol. **(4)**

F = 125N

$F = \frac{dp}{dt}$ n $\rightarrow$ No. of bullets

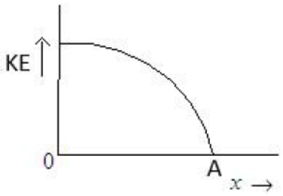
$F = \frac{d(nmv)}{dt} = mv\frac{dn}{dt}$

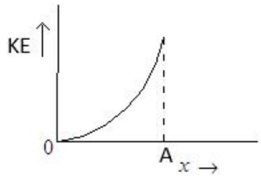
$125 = \frac{10n}{1000} \times 250 \times \frac{dn}{dt}$

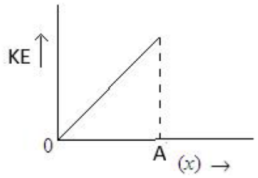
$$\frac{125 \times 1000}{2500} = \frac{dn}{dt}$$

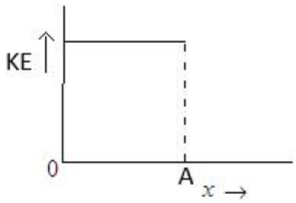
$$\boxed{\frac{dn}{dt} = 50}$$ option $\rightarrow$ (4)

42. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by

(1)


(2)


(3)


(4)


Sol. **(1)**

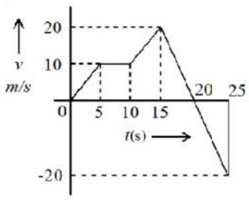
$K \cdot E = T \cdot E - P \cdot E$

$K \cdot E = \frac{1}{2} KA2 - \frac{1}{2} Kx^2$

Graph b/w $K \cdot E$ and x will be parabola

Option $\rightarrow$ (1)

43. From the v – t graph shown, the ratio of distance to displacement in 25 s of motion is :



(1) $\frac{3}{5}$ (2) $\frac{1}{2}$ (3) $\frac{5}{3}$ (4) 1

Sol. **(3)**

Displacement = Area of graph with sign

$$\text{Displacement} = \left(\frac{1}{2} \times 10 \times 5\right) + (10 \times 5) + \left(\frac{1}{2} \times 5 \times 30\right) + \left(\frac{1}{2} \times 5 \times 20\right) - \frac{1}{2}(5)(20)$$

= 25 + 50 + 75 + 50 – 50

= 150 m

Distance $\rightarrow$ Area of graph with positive value

Distance = 25 + 50 + 75 + 50 = 250

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{250}{150} = \frac{5}{3}$$ option $\rightarrow$ (3)

44. On a temperature scale 'X', the boiling point of water is 65º X and the freezing point is-15º X. Assume that the X scale is linear. The equivalent temperature corresponding to -95º X on the Farenheit scale would be :

(1) -63º F (2) -148º F (3) -48º F (4) -112º F

Sol. **(3)**

$$\frac{X_T - X_L}{X_H - X_L} = \frac{T_F - 32}{212 - 32}$$

$$\frac{-95^\circ - (-15^\circ)}{65^\circ - (-15^\circ)} = \frac{T_F - 32}{180}$$

$$\frac{-80^\circ}{80^\circ} = \frac{T_F - 32}{180^\circ}$$

$-180 = T_F - 32$

$T_F = -180 + 32 = -148^\circ$ F Ans. option $\rightarrow$ (2)

45. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be

(1) 0.2% (2) 1% (3) 2% (4) 0.1%

Sol. **(3)**

$X = 2 \times 10^{-2}$

$\mu r = 1 + x = 1 + 0.02 = 1.02$

Bo $\rightarrow$ magnetic field due to magnetic material

$B_m \rightarrow$ magnetic field due to magnetic material

$B_m = \mu_r B_0$

$$\Delta B = \frac{B_m - B_0}{B_0} \times 100 = \frac{\mu_r B_0 - B_0}{B_0} \times 100$$

$$\Delta B\% = \frac{(X+1)-1}{1} \times 100 = X \times 100$$

$\Delta B\% = 2 \times 10^{-2} \times 100 = 2\%$ Ans. Option (3)

46. The critical angle for a denser-rarer interface is 45°. The speed of light in rarer medium is 3×10^8 m/s. The speed of light in the denser medium is :

(1) 2.12×10^8 m/s (2) 5×10^7 m/s (3) 3.12×10^7 m/s (4) $\sqrt{2} \times 10^8$ m/s

Sol. **(1)**

$$\text{Sin ic} = \frac{\mu_r}{\mu_d} \quad \Rightarrow \sin 45^\circ = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow \frac{\mu_r}{\mu_d} = \sqrt{2} \qquad \text{...........(1)}$$

We know

$$V \propto \frac{1}{\mu} \quad \Rightarrow \frac{V_d}{V_r} = \frac{\mu_r}{\mu_d}$$

$$= \frac{V_d}{3 \times 10^8} = \frac{1}{\sqrt{2}}$$

$$V_d = \frac{3}{\sqrt{2}} \times 10^8 = 3 \times 0.7 \times 10^8$$

$\boxed{V_d = 2.12 \times 10^8 \text{ m/sec}}$ Ans. Option (1)

47. Given below are two statements :

Statements I : Astronomical unit (Au), Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II : Au < Parsec (Pc) < ly

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statements I and Statements II are incorrect.

(2) Both Statements I and Statements II are correct.

(3) Statements I is incorrect but Statements II are correct.

(4) Statements I is correct but Statements II are incorrect.

Sol. **(4)**

A.V., Par sec and light year are the unit of distance

Light year $\rightarrow$ distance travelled by light in one year

1 ly = 9.5×10^{15} m

parcec = 3.262 light year

A.V. = 1.58×10^{-5} light year

A.V. < 1y < Parsec.

Statement I correct and statement II incorrect.

48. The current sensitivity of moving coil galvanometer is increased by 25%.This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be :

(1) +25% (2) -25% (3) -50% (4) Zero

Sol. **(1)**

$\tau = mB$ A = area of coil

$K\theta = IANB$ B = magnetic field

$\frac{\theta}{I} = \frac{ANB}{K}$ Currect senstivity

$1.25\left(\frac{\theta}{I}\right)_2 = \left(\frac{\theta}{I}\right)_1$ .... (1)

$1.25\left[\frac{AN_2B}{K}\right] = \left[\frac{AN_1B}{K}\right]$

$1.25 = \frac{N_1}{N_2} = \frac{5}{4}$ ...... (2)

$\Rightarrow R = \frac{\delta\ell}{a} = \text{const.}$

$\Rightarrow \ell = a$

Voltage sensitivity = $\frac{\theta}{V} = \frac{\theta}{IR} = \frac{\text{Current sensitivity}}{R}$

R = constant

Voltage sensitivity $\propto$ current sensitivity Ans. option $\rightarrow$ (A)

49. A metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V_o. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V_0}{4}$. The threshold wavelength for this metallic surface will be

(1) $\frac{3}{2}\lambda$ (2) 4λ (3) 3λ (4) $\frac{\lambda}{4}$

Sol. **(3)**

$E = K.E + \phi_0$

Now

$\frac{hc}{\lambda} = ev_0 + \phi_0$ .... (1)

And $\frac{hc}{2\lambda} = \frac{eV_0}{4} + \phi_0$ .... (2)

(2) × 4 .... (1)

$\frac{2hc}{\lambda} - \frac{hc}{\lambda} = 0 + (4\,\phi_0 - \phi_0)$

$$\frac{hc}{\lambda} = 3\phi_0$$

$$\frac{hc}{\lambda} = 3\frac{hc}{\lambda_0}$$

$$\lambda_0 = 3\lambda$$

50. 1 kg of water at 100ºC is converted into steam at 100ºC by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671 m^3 as steam. The change in internal energy of the system during the process will be
(Given latent heat of vaporisation = 2257 kJ/kg, Atmospheric pressure = 1×10^5 Pa)
(1) + 2476 kJ (2) -2426 kJ (3) -2090 kJ (4) +2090 kJ

Sol. **(4)**

V_1		**V_2**
Water	⟶	Steam
1kg		100°C
100°C		

Change in volume at constant pressure and temp →

$\Delta V = V_2 - V_1 = 1.671 - 0.001$

$\Delta V = 1.67$ m^3 ...... (1)

$\Delta Q = \Delta U + w$

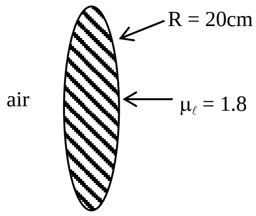
$mL_v = \Delta U + (1.013 \times 10^5)(1.67)$

$\Delta U = (2257 - 170)10^3$

$\Delta U = 2090$ kJ (approx.) Ans. Option → 4

51. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be x : 1. The value of x is _________

Sol. **(4)**



$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_m} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

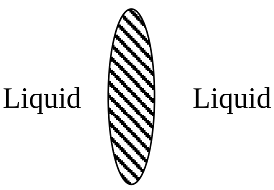
$$P_1 = \frac{2}{R}\left(\frac{1.8}{1} - 1\right]$$

$$P_1 = \frac{2}{R}(0.8) = \frac{1.6}{R} \quad .... (1)$$

Now,

$$P_2 = \frac{2}{R}\left[\frac{1.8}{1.5} - 1\right]$$

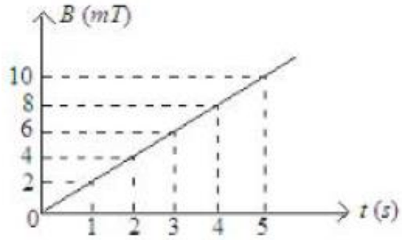
$$P_2 = \frac{2}{R}\left[\frac{0.3}{1.5}\right] = \frac{2}{R} \times \frac{1}{5} = \frac{2}{5R}$$



$$\frac{P_{air}}{P_{liquid}} = \frac{P_1}{P_2} = \frac{\left(\frac{1.6}{R}\right)}{\left(\frac{0.4}{R}\right)} = \frac{4}{1}$$

Ans. → 4

52. The magnetic field B crossing normally a square metallic plate of area 4 m^2 is changing with time as shown in figure. The magnitude of induced emf in the plate during t = 2s to t = 4s, is __________ mV.



Sol. **(8)**

$\text{emf} = \frac{d\phi}{dt}$

$\text{Emf} = \frac{dBA}{dt} = \frac{AdB}{dt}$

Emf = 4 · Slope of B.t curve

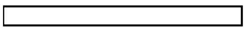
$= 4 \cdot \left[\frac{8-4}{4-2}\right] = 4 \times 2$

$\boxed{\text{Emf} = 8 \text{ Volt}}$

53. The length of a wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If 10 l_2 = 11 l_1, the natural length of wire will be $\frac{1}{x}l_1$. Here the value of x is _______ .

Sol. **(2)**

F = kx



ℓ_0 = natural length

$F = \frac{yA}{\ell_0} \cdot x$.

Sol when F = 100 N

$100 = k(\ell_1 - \ell_0)$ ..... (1)

When F = 120N

$120 = K((\ell_1 - \ell_0)$

Given that $10\ell_2 = 11\ell_1$

$\ell_2 = 1.1\,\ell_1$

So $120 = K(1.1\ell_1 - \ell_0)$ .... (2)

Now (2)\(1)

$\frac{120}{100} = \frac{K(1.1\ell_1 - \ell_0)}{K(\ell_1 - \ell_0)}$

$1.2 = \frac{1.1\ell_1 - \ell_0}{\ell_1 - \ell_0}$

$1.2\ell_1 - 1.2\ell_0 = 1.1\ell_1 - \ell_0$

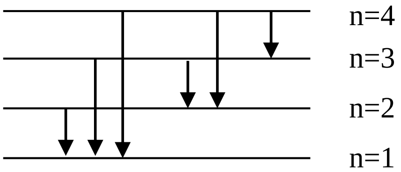
$0.1\ell_1 = 0.2\ell_0$

$\ell_0 = \frac{\ell_1}{2}$ So $\boxed{x = 2}$ Ans.

54. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is _______.

(Given $h = 4.25 \times 10^{-15}$ eVs)

Sol. **(3)**



Total emission lines = 6 (given)

So electron absorbed energy and jump from n = 1 to n = 4

$\Delta E\ 13.6\left[\frac{\ell}{\ell^2}-\frac{1}{4^2}\right]$ ev

$= 13.6\left[1-\frac{1}{16}\right]$ ev

$\Delta E = hf$

$12.75 = 4.5 \times 10^{-15}$,

$f = \frac{12.75}{4.25} \times 10^{15} = 3\times10^{15}$ Hz

$\boxed{x = 3}$ Ans.

55. A force $\vec{F} = (2+3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from x = 0 to x = 4 m, is ______ J.

Sol. **(32)**

$\vec{F} = (2 + 3x)i$

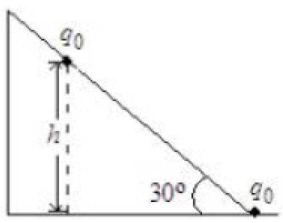
$w = \int_0^4 F.dx = \int_0^4 (2+3x).dx$

$w = \left(2x+\frac{3x^2}{2}\right)_0^4 = (8 + 24)$

$\boxed{w = 32J}$

56. As shown in the figure, a configuration of two equal point charges ($q_0 = +2\,\mu$ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m. The value of x is _________.

(Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,N\,m^2C^{-2}, g = 10ms^{-2}$)



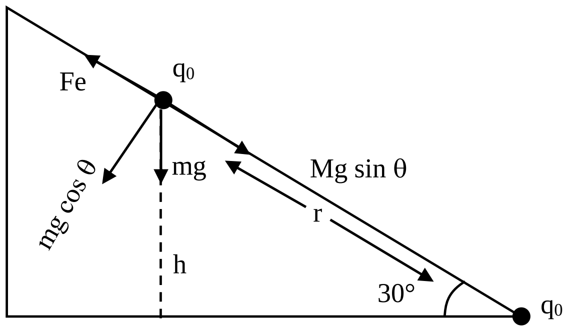
Sol. **(300)**

Point charge on equilibrium is at rest.

So $F_e = mg \sin\theta$

$$\frac{kq_0 \cdot q_0}{r^2} = mg \sin 30°$$

$$\frac{kq_0^2}{\left(\frac{h}{\sin 30°}\right)^2} = \frac{mg}{2}$$



$$\frac{9\times10^9 \times (2\times10^{-6})^2}{4h^2} = \frac{20\times10^{-3}\times10}{2}$$

$$\frac{9\times4\times10^9\times10^{-12}}{4h^2} = 10^{-1}$$

$h^2 = 9\times10^{-2}$

$h = 0.3$ m $= 300 \times 10^{-3}$ m

$\boxed{x = 300}$ Ans.

57. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1}. If the moment of inertia of the sphere about its tangent is $x \times 10^{-2}$ times its angular momentum about the diameter. Then the value of x will be _____________ .

Sol. **(35)**

$$I_1 = \frac{2}{5}mR^2$$

$$I_2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$$

Angular moment about diameter is

$$L_{com} = I_1 w = \frac{2}{5}mR^2 w$$

Now,

$$\frac{I_2}{L_{com}} = \frac{\frac{7}{5}mR^2}{\frac{2}{5}mR^2 w} = \frac{7}{2}w$$

$$\frac{I_2}{L_{com}} = \frac{7}{2\times10} = \frac{7}{20}$$

Now $\frac{7}{20} = x \times 10^{-2}$

$$x = \frac{7}{20} \times 100$$

$\boxed{x = 35}$ Ans.

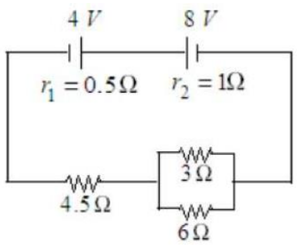
58. The equation of wave is given by $Y = 10^{-2} \sin 2\pi(160t - 0.5x + \pi/4)$ where x and Y are in m and t in s. The speed of the wave is _________ km h^{-1}.

Sol. **(1152)**

$Y = 10^{-2} \sin 2\pi(160t - 0.5x + \pi/4)$

Speed of wave $= \frac{w}{k} = \frac{160}{0.5} = 320$ m/sec $= 320 \times \frac{18}{5} = 1152$ km/h Ans.

59. In the circuit diagram shown in figure given below, the current flowing through resistance 3 Ω is $\frac{x}{3}$ A. The value of x is _________



Sol. **(1)**

Req. = $0.5 + 1 + 4.5 + \left(\frac{3.6}{9}\right)$

Req. = 6 + 2 = 8Ω

$I = \frac{8-4}{8} = \frac{1}{2}$ A = 0.5A

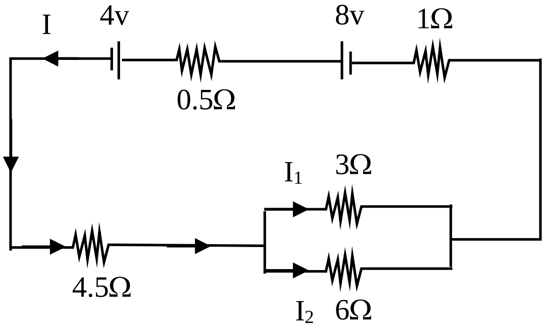
$I_1 : I_2 = \frac{1}{3} : \frac{1}{6}$

$I_1 : I_2 = 2 : 1$

and $I_1 + I_2 = 0.5$ A

$I_1 = \frac{2}{3} \times 0.5 = \frac{1}{3}$ A

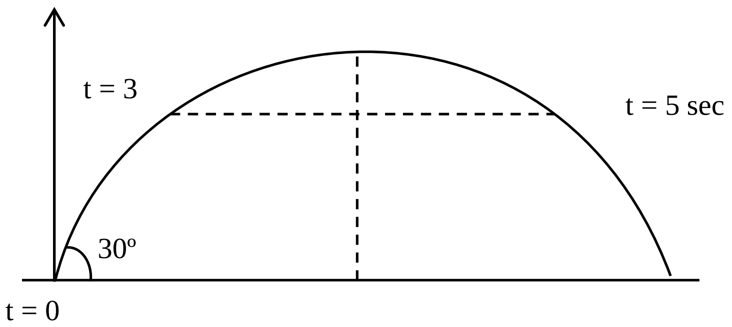
So $\frac{1}{3} = \frac{x}{3} \Rightarrow \boxed{x = 1}$



60. A projectile fired at 30° to the ground is observed to be at same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is ________ ms^{-1}.
(Given g = 10 ms^{-2})

Sol. **(80)**

Time of flight = 5 + 3 = 8 sec.



Now, $T = \frac{2u\sin\theta}{g}$

$8 = \frac{2u \cdot \sin 30°}{10}$

$\Rightarrow \boxed{u = 80 \text{ m/sec}}$ Ans.

SECTION - A

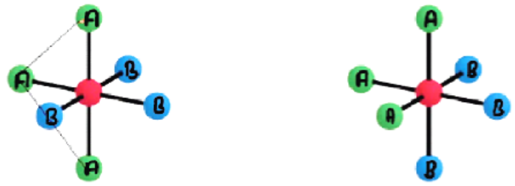
61. Which of the following complex has a possibility to exist as meridional isomer?

(1) $[Co(en)_2Cl_2]$ (2) $[Pt(NH_3)_2Cl_2]$

(3) $[Co(en)_3]$ (4) $[Co(NH_3)_3(NO_2)_3]$

Sol. **4**

$[MA_3B_3]$ type of compound exists as facial and meridional isomer.



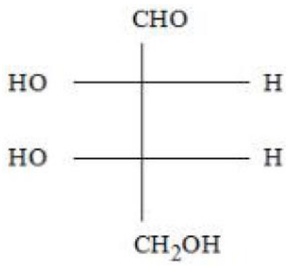
62. L-isomer of tetrose X ($C_4H_8O_4$) gives positive schiff's test and has two chiral carbons. On acetylation. 'X' yields triacetate. 'X' undergoes following reactions

$$'A' \xleftarrow{HNO_3} 'X' \xrightarrow{NaBH_4} 'B'$$

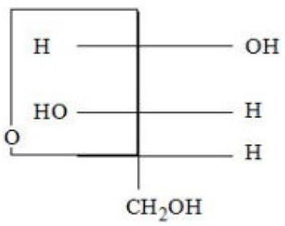
Chiral compound

'X' is

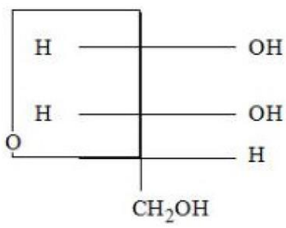
(1)



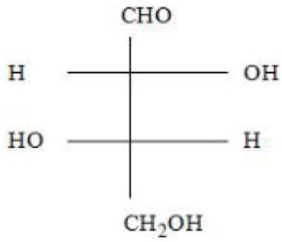
(2)



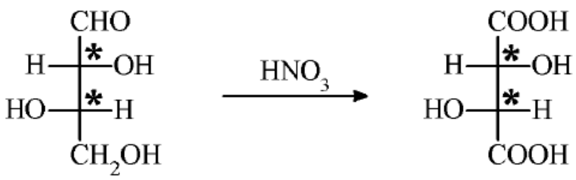
(3)



(4)

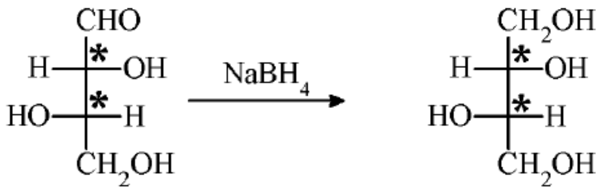


Sol. **4**



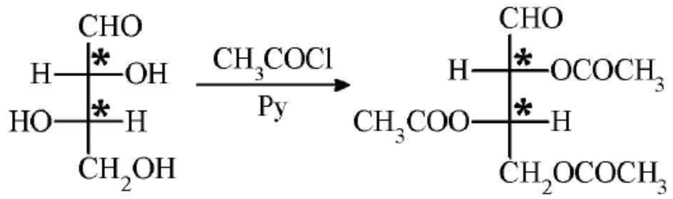
(x) (A)

L-tetrose with two chiral centre



(B)

Optically active



(x) gives positive schiff's test due –CHO group
(x) is L-tetrose.

63. Match list I with list II:

List I	List II
A. K	I. Thermonuclear ractions
B. KCl	II. Fertilizer
C. KOH	III. Sodium potassium pump
D. Li	IV. Absorbent of CO_2

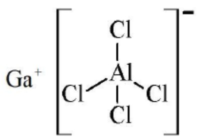
Choose the correct answer from the options given below:
(1) A-III, B-IV, C-II, D-I
(2) A-IV, B-III, C-I, D-II
(3) A-III, B-II, C-IV, D-I
(4) A-IV, B-I, C-III, D-II

Sol. **3**
K^+ –Sodium- Potassium Pump
KCl – Fertiliser
KOH – absorber of CO_2
Li – used in thermonuclear reactions

64. For compound having the formula $GaAlCl_4$, the correct option form the following is
(1) Cl forms bond with both Al and Ga in $GaAlCl_4$
(2) Ga is coordinated with Cl in $GaAlCl_4$
(3) Ga is more electronegative than Al and is present as a cationic part of the salt
(4) Oxidation state of Ga in the salt $GaAlCl_4$ is +3

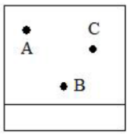
Sol. **3**
Gallous tetrachloro aluminate $Ga^+AlCl_4^-$

$$2Ga + Ga^+Cl_4^- + 2Al_2Cl_6 \xrightarrow{190^\circ} 4Ga^+AlCl_4^-$$

Structure of $Ga^+AlCl_4^-$ 

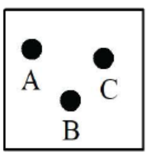
Ga is cationic part of salt $GaAlCl_4$.

65. Thin layer chromatography of a mixture shows the following observation :



The correct order of elution in the silica gel column chromatography is
(1) B, A, C
(2) C, A, B
(3) A, C, B
(4) B, C, A

Sol. **3**



According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.
Hence, the correct order of elution in the silica gel column chromatography is - B < C <A

66. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral $FeCl_3$. it gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains

(1) $C_2O_4^{2-}$ & NO_3^- (2) SO_3^{2-} & $C_2O_4^{2-}$

(3) CH_3COO^- & NO_3^- (4) SO_3^{2-} & CH_3COO^-

Sol. **3**

$CH_3COO^- + FeCl_3 \rightarrow Fe(CH_3COO)_3$ or $[Fe_3(OH)_2(CH_3COO)_6]^+$

Blood red colour

$\downarrow \Delta$

$Fe(OH)_2(CH_3COO)\downarrow$

Red-brown precipitate

$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$

$[Fe(H_2O)_6]^{2+} + NO \rightarrow [Fe(H_2O)_5(NO)]^{2+} + H_2O$

Brown

67. The polymer X-consists of linear molecules and is closely packed. It prepared in the presence of triethylaluminium and titranium tetrachloride under low pressure. The polymer X is-

(1) Polyacrylonitrile (2) Polytetrafluoroethane

(3) High density polythene (4) Low density polythene

Sol. **3**

Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as $AlEt_3$ and $TiCl_4$ (Ziegler – Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6–7 atmosphere.

68. Match list I with list II

List I Species	**List II Geometry/ Shape**
A. H_3O^+	I. Tetrahedral
B. Acetylide anion	II. Linera
C. NH_4^+	III. Pyramidal
D. ClO_2^-	IV. Bent

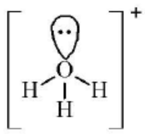
Choose correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II (2) A-III, B-IV, C-II, D-I

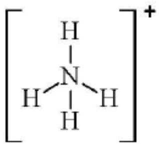
(3) A-III, B-I, C-II, D-IV (4) A-III, B-II, C-I, D-IV

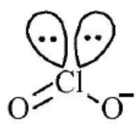
Sol. **4**

Molecule/Ion Hybridisation Shape

H_3O^+ sp^3 Pyramidal 

Acelylide sp linear $\overline{C}\equiv\overline{C}$

NH_4^+ sp^3 tetrahedral 

ClO_2^- sp^3 Bent 

69. Given below are two statement :
Statement I : Methane and steam passed over a heated Ni catalyst produces hydrogen gas
Statement II : Sodium nitrite reacts with NH_4Cl to give H_2O, N_2 and NaCl
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both the statement I and II are incorrect
(2) Statement I is incorrect but statement II is correct
(3) Statement I is correct but statement II is incorrect
(4) Both the statements I and II are correct

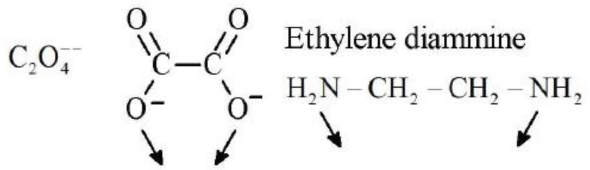
Sol. **4**

$$CH_4(g) + \underset{\text{Steam}}{H_2O(g)} \xrightarrow[1270K]{Ni} CO(g) + 3H_2(g)$$

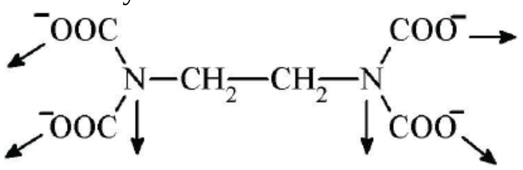
$$NaNO_2(aq) + NH_4Cl(aq) \rightarrow N_2(g) + NaCl(aq) + 2H_2O(\ell)$$

70. The set which does not have ambidentate ligand (s) is
(1) $C_2O_4^{2-}$, NO_2^-, NCS^-
(2) $EDTA^{4-}$, NCS^-, $C_2O_4^{2-}$
(3) NO_2^-, $C_2O_4^{2-}$, $EDTA^{4-}$
(4) $C_2O_4^{2-}$, ethylene diamine, H_2O

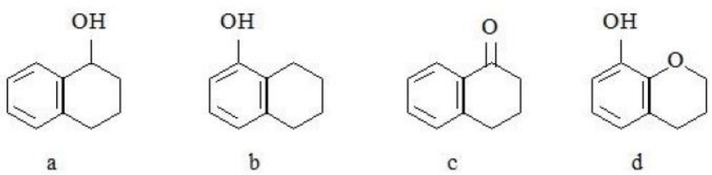
Sol. **4**
NO_2^- NCS^- are ambidentate ligand



EDTA Ethylene diamine tetra acetate

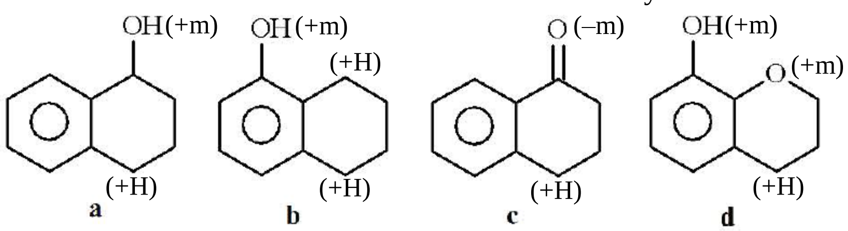


71. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction



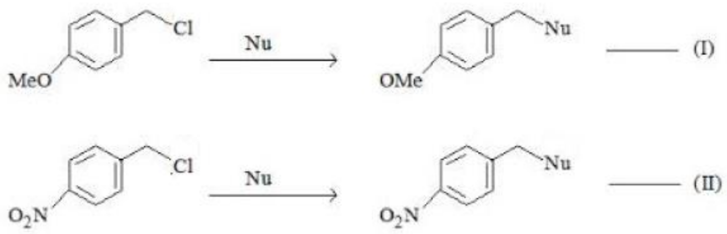
(1) c, a, b, d
(2) d, b, c, a
(3) d, b, a, c
(4) b, c, a, d

Sol. **1**
Benzene becomes more reactive towards EAS when any substituent raises the electron density.



Correct order
c < a < b < d

72.



Where Nu = Nucleophile

Find out the correct statement from the options given below for the above 2 reactions.

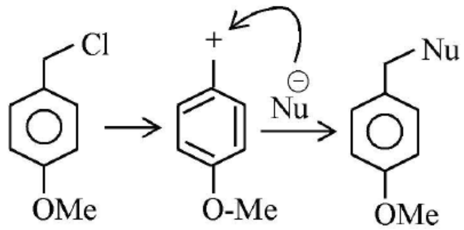
(1) Reaction (I) is of 1^{st} order and reaction (II) is of 2^{nd} order

(2) Reaction (I) and (II) both are 2^{nd} order

(3) Reaction (I) and (II) both are 1^{st} order

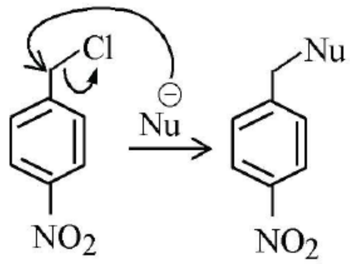
(4) Reaction (I) is of 2^{nd} order and reaction (II) is of 1^{st} order

Sol. **1**



Electron Donating group

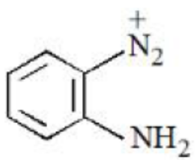
S_N1 Mech. : I^{st} order

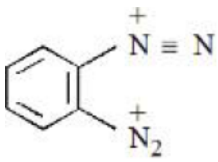


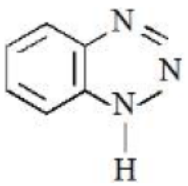
Electron withdrawing group

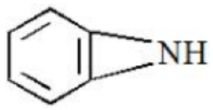
S_N2 Mech: 2^{nd} order

73. o-Phenylenediamine $\xrightarrow{HNO_2}$ 'X' Major Product 'X' is

(1) 

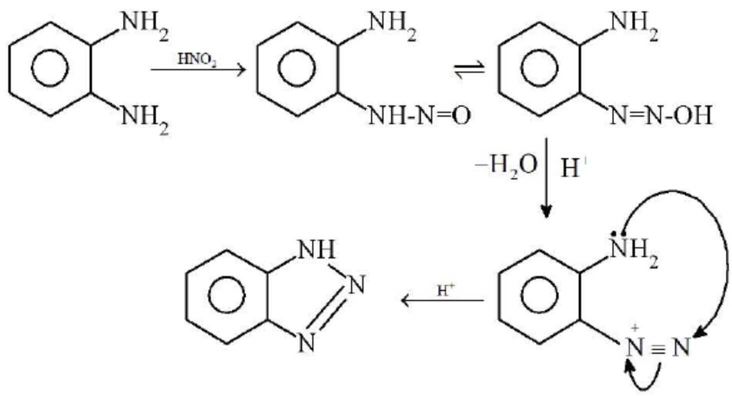
(2) 

(3) 

(4) 

Sol. **3**

o-Phenylenediamine



74. For elements B, C, N, Li, Be, O and F, the correct order of first ionization enthalpy is

(1) B>Li>Be>C>N>O>F (2) Li<Be<B<C<N<O<F

(3) Li<Be<B<C<O<N<F (4) Li<B<Be<C<O<N<F

Sol. **4**

First I.E.

F > N > O > C > Be > B > Li

Li – 520 kJ/mol

Be – 899 kJ/mol

B – 801 kJ/mol

C – 1086 kJ/mol

N – 1402 kJ/mol

O – 1314 kJ/mol

F – 1681 kJ/mol

75. In the extraction process of copper, the product obtained after carrying out the reactions

(i) $2Cu_2S+3O_2 \rightarrow 2Cu_2O+2SO_2$

(ii) $2Cu_2O+Cu_2S \rightarrow 6Cu+SO_2$ is called

(1) Reduced copper (2) Blister copper

(3) Copper matte (4) Copper scrap

Sol. **2**

$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 3SO_2$

$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

Blister copper

Due to evolution of SO_2, the solidified copper formed has a blistered look and is referred to as blister copper.

76. 25 mL of silver nitrate solution (1M) is added dropwise to 25 mL of potassium iodide (1.05 M) solution. The ion(s) present in very small quantity in the solution is/are

(1) NO_3^- only (2) Ag^+ and I^- both (3) K^+ only (4) I^- only

Sol. **2**

On adding $AgNO_3$ into KI, AgI will form and solubility of AgI is very low.

So, $[Ag^+]$ and $[I^-]$ will be present in very small quantity.

77. Given below are two statements:

Statement I : If BOD is 4 ppm and dissolved oxygen is 8 ppm, it is a good quality water.

Statement II : If the concentration of zinc and nitrate salts are 5 ppm each, than it can be good quality water.

In the light of the above statements choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but statement II is correct

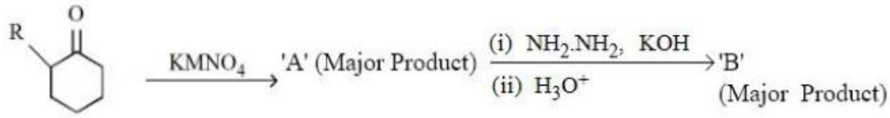
(2) Statement I is correct but statement II is incorrect

(3) Both the statements I and II are incorrect

(4) Both the statement I and II are correct

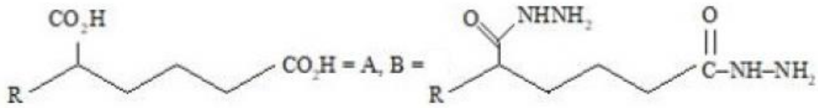
Sol. **4**

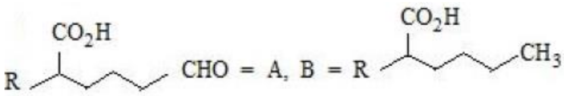
Clean water would have BOD value of less than 5 ppm.
Maximum limit of Zn in clean water = 5.0 ppm or mg dm^{-3}
Maximum limit of NO_3^- in clean water = 50 ppm or mg dm^{-3}

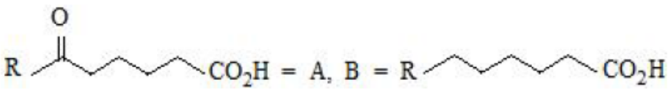
78. 

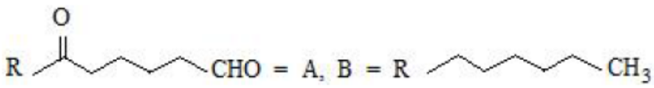
(R = alkyl)

'A' and 'B' in the above reactions are :

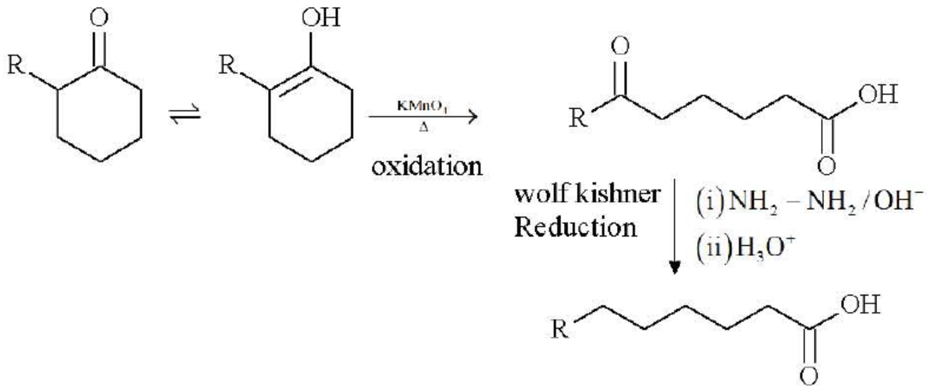
(1) 

(2) 

(3) 

(4) 

Sol. **3**



79. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R:
Assertion A : In the photoelectric effect electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.
Reason R : When the photon of any energy strikes an electron in the atom transfer of energy from the photon to the electron takes place.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) A is correct but R is not correct
(2) A is not correct but R is correct
(3) Both A and R correct and R is the correct explanation of A
(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **1**

Assertion A is correct but Reason is not correct.

80. The complex that dissolves in water is
(1) $[Fe_3(OH)_2(OAc)_6]Cl$
(2) $Fe_4[Fe(CN)_6]_3$
(3) $K_3[Co(NO_2)_6]$
(4) $(NH_4)_3[As(Mo_3O_{10})_4]$

Sol. **1**

$Fe_4[Fe(CN)_6]_3$ Prussian Blue-water insoluble
$K_3[Co(NO_2)_6]$ very poorly water soluble
$(NH_4)_3[As(MO_3O_{10})_4]$ water insoluble
ammonium arseno molybdate
$[Fe_3(OH)_2(OAc)_6]$ Cl is water soluble.

SECTION - B

81. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2) the heat evolved (KJ) per gram of the mixture is _______ (Nearest integer)

Givne $\Delta H_f^\theta (Al_2O_3) = -1700$ KJ mol^{-1}

$\Delta H_f^\theta (Fe_2O_3) = -840$ KJ mol^{-1}

Sol. **4**

$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$

$\Delta H_r = (\Delta H_f)\ Al_2O_3 - \Delta H_f°(Fe_2O_3)$

$= -1700 - (-840)$

$= -860$ kJ

Fe_2O_3 & Al $\rightarrow$ 1 : 2

Fe_2O_3 = 1 mole = (2 × 25 + 48)

= 112 + 48 = 160 gm

Al = 2 mole = 2 × 27 = 54 gm

Total mass = 160 + 54 = 214 gm

Heat evolved per gm = $\frac{-860}{214}$ kJ = –4.01 ≈ 4 kJ

82. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 3H_2O$

The above reaction was studied at 300 K by monitoring the concentration of $FeSO_4$ in which initial concentration was 10 M and after half an hour became 8.8 M. The rate of production of $Fe_2(SO_4)_3$ is______ $\times 10^{-6}$ mol L^{-1} s^{-1}

Sol. **333**

$$\frac{-\Delta FeSO_4}{\Delta t} = \frac{10-8.8}{30\times 60} = \frac{1.2}{1800}$$

From given equation :

$$-\frac{1}{6}\frac{\Delta FeSO_4}{\Delta t} = \frac{1}{3} \times \text{(Rate of production of } Fe_2(SO_4)_3)$$

Rate of production of $Fe_2(SO_4)_3 = \frac{3}{6}\times\frac{1.2}{1800}$

$= \frac{1}{3}\times 10^{-3}$

$= \frac{1000}{3}\times 10^{-6}$

$= 333.33 \times 10^{-6}$

83. 0.004 M K_2SO_4 solution is isotonic with 0.01 M glucose solution. Percentage dissociation of K_2SO_4 is________ (Nearest integer)

Sol. **75**

For isotonic solution

$(ic)_{glucose} = (ic)_{K_2SO_4}$

$0.01 = i\ (0.004)$

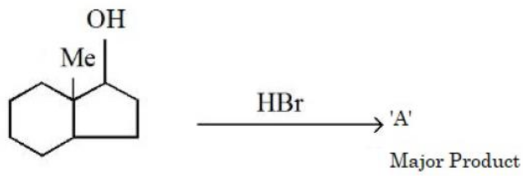
$$i = \frac{0.01}{0.004} = \frac{10}{4} = \frac{5}{2}$$

$$1 + (n - 1)\ \alpha = \frac{5}{2}$$

$$1 + (3 - 1)\ \alpha = \frac{5}{2} \qquad (\because n = 3 \text{ for } K_2SO_4)$$

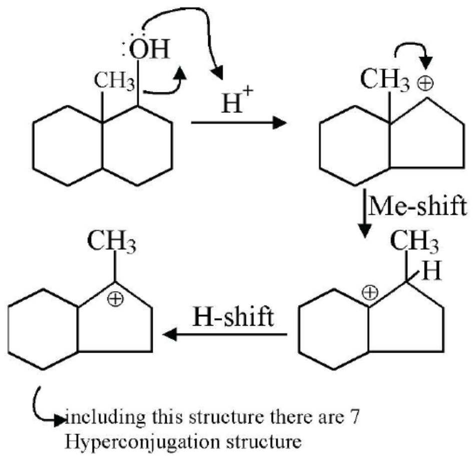
$$2\alpha = \frac{3}{2}$$

$$\alpha = \frac{3}{4} \rightarrow 75\%$$

84. 

The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is______

Sol. **7**



85. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation : $CO(g)+H_2O(g) \rightleftharpoons CO_2(g)+H_2(g)$. The equilibrium constant $K_c \times 10^2$ for the reaction is_________ (Nearest integer)

Sol. **44**

	$CO(g)$ +	$H_2O(g)$ $\rightleftharpoons$	$CO_2(g)$ +	$H_2(g)$
	1mole	1mole		
At equilibrium	1–0.4	1–0.4	0.4	0.4

$$K_c = \frac{0.4 \times 0.4}{0.6 \times 0.6} = \frac{4}{9}$$

$$K_c \times 10^2 = \frac{4}{9} \times 100 = \frac{400}{9} = 44.44 \approx 44$$

86. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300 pm. The no. of atoms present in one unit cell of A is ________ (Nearest integer)

Given the density of A is 3.0 g mL^{-1} and $N_A = 6.02\times10^{23}\ mol^{-1}$

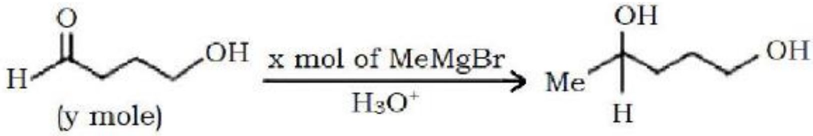
Sol. **4**

$$d = \frac{\frac{Z}{N_A}\times M}{a^3}$$

$$3 = \frac{Z}{6.02\times10^{23}}\times\frac{12}{(300\times10^{-10})^3}$$

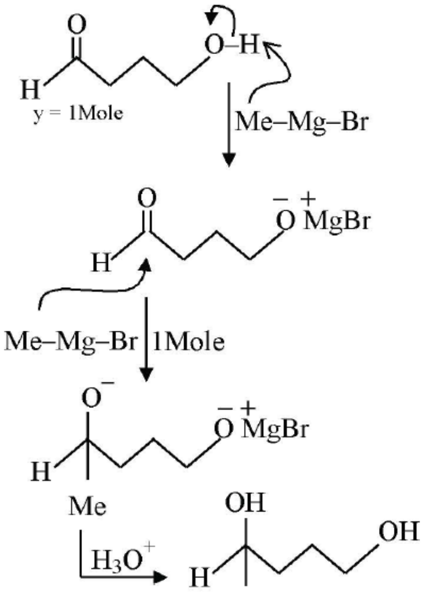
$$Z = \frac{3\times6.02\times27\times10^{6}\times10^{-30}\times10^{23}}{12}$$

$= 40.635 \times 10^{-1} = 4.0635 \approx 4$

87. 

The ratio x/y on completion of the above reaction is_______

Sol. **2**



$\therefore$ x = 2 mole

$$\frac{x}{y} = \frac{2}{1} = 2$$

88. The ratio of spin-only magnetic moment values $\mu_{eff}[Cr(CN)_6]^{3-}/\mu_{eff}[Cr(H_2O)_6]^{3+}$ is_____

Sol. **1**

Spin magnetic moment of $[Cr(CN)_6]^{3-}(t_{2g}^3\ e_g^0)$

$$\mu_1 = \sqrt{3(3+2)} = \sqrt{15}BM$$

Spin magnetic moment of $[Cr(H_2O)_6]^{3+}(t_{2g}^3\ e_g^0)$

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15}BM$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{51}}{\sqrt{51}} = 1$$

89. In an electrochemical reaction of lead, at standard temperature, if $E^o_{(Pb^{2+}/Pb)} = m$ volt and $E^o_{(Pb^{4+}/Pb)} = n$ volt, then the value of $E^o_{(Pb^{2+}/Pb^{4+})}$ is given by m – xn. The value of x is ________ (Nearest integer)

Sol. **2**

$Pb^{2+} + 2e^- \rightarrow Pb$	$E° = m$	$\Delta G_1^o = -2Fm$
$Pb^{4+} + 4e^- \rightarrow Pb$	$E° = n$	$\Delta G_2^o = -4Fn$
$Pb^{2+} \rightarrow Pb^{4+} + 2e^-$	$\Delta G_3^o = \Delta G_1^o - \Delta G_2^o$	

$-2FE° = -2Fm + 4Fn$

$\boxed{E° = m - 2n}$

$\boxed{x = 2}$

90. A solution of sugar is obtained by mixing 200g of its 25% solution and 500g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is________(Nearest integer)

Sol. **36**

Solution (I) $\rightarrow$ Mass of sugar = $200 \times \frac{25}{100}$ = 50 gm

Mass of solution = 200 gm

Solution (II) $\rightarrow$ Mass of solution = 500 gm

Mass of sugar = $\frac{40}{100} \times 500$ = 200 gm

$$\text{Final \% w/w} = \frac{\text{Total mass of sugar}}{\text{Total mass of solution}} \times 100$$

$$= \frac{50 + 200}{200 + 500} \times 100 = \frac{250}{7}$$

= 35.71% ≈ 36