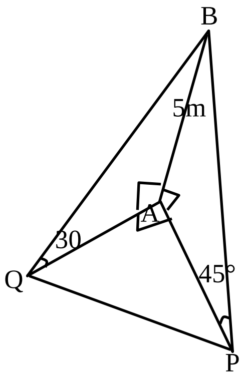
SECTION - A

1. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to

(1) 10 (2) $5\sqrt{5}$ (3) $\frac{5}{2}\sqrt{5}$ (4) 5

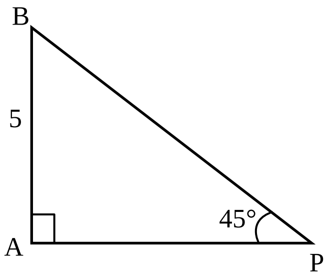
Sol. (1)



Tower AB = 5 m

$\angle APB = 45°$

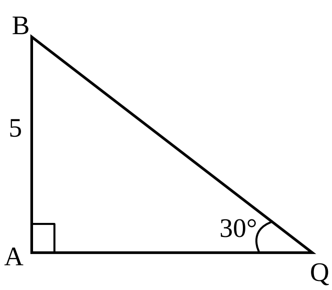
$\angle PAB = 90°$



$\tan 45° = \frac{AB}{AP}$

$1 = \frac{AB}{AP}$

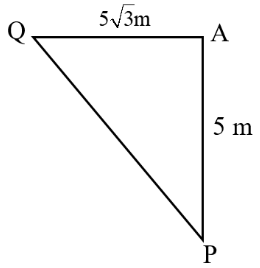
$\boxed{AP = 5m}$



$\tan 30° = \frac{AB}{AQ}$

$\frac{1}{1\sqrt{3}} = \frac{5}{AQ}$

$\boxed{AQ = 5\sqrt{3}}$



$AP^2 + AQ^2 = PQ^2$

$PQ^2 = 5^2 + \left(5\sqrt{3}\right)^2$

$PQ^2 = 25 + 75 = 100$

$\boxed{PQ = 10cm}$

Option (A) 10 cm correct.

2. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5\ b^3c^2\ d$ is 3750β, then the value of β is

(1) 55 (2) 108 (3) 90 (4) 110

Sol. **(3)**

Given a + b + c + d = 11 (a, b, c, d > 0}

$(a^5b^3c^2d)$max. = ?

Let assume Numbers –

$\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{b}{3},\frac{b}{3},\frac{b}{3},\frac{c}{2},\frac{c}{2},$

We know A.M. $\geq$ G.M.

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5.3^3.2^2.1}\right)^{\frac{1}{11}}$$

$$\frac{11}{11} \geq \left(\frac{a^5b^3c^2d}{5^5.3^3.2^2.1}\right)^{\frac{1}{11}}$$

$a^5.b^3.c^2.d \leq 5^5.3^3.2^2,$

$\max(a^5b^3c^2d) = 5^5.3^3.2^2 = 337500$

$= 90 \times 3750 = \beta \times 3750$

$\boxed{\beta = 90}$

Option (C) 90 correct

3. If $f : R \to R$ be a continuous function satisfying $\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\sin x dx + \alpha\int_0^{\frac{\pi}{4}} f\left(\cos 2x\right)\cos x dx = 0$, then the value of α is

(1) $-\sqrt{3}$ (2) $\sqrt{3}$ (3) $-\sqrt{2}$ (4) $\sqrt{2}$

Sol. **(3)**

$F : R \to R$

$$\Rightarrow \int_0^{\frac{\pi}{2}} F(\sin 2x)\sin dx + \alpha\int_0^{\frac{\pi}{4}} F(\cos 2x).\cos x dx = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{4}} F(\sin 2x)\sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} F(\sin 2x).\sin x dx + \alpha\int_0^{\frac{\pi}{4}} F(\cos 2x).\cos x dx = 0$$

$$\int_0^a F(x)\,dx = \int_0^a F(a-x)\,dx$$

Let $x = t + \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} F(\cos 2x)\sin\left(\frac{\pi}{4} - x\right)dx + \int_0^{\frac{\pi}{4}} F(\cos 2t)\sin\left(t + \frac{\pi}{4}\right) + \alpha\int_0^{\frac{\pi}{4}} F(\cos 2x)\cos x dx = 0$$

$$\int_0^{\frac{\pi}{4}} F(\cos 2x)\left\{\sin\left(\frac{\pi}{4} - x\right) + \sin\left(x + \frac{\pi}{4}\right) + \alpha\cos x = 0\right.$$

$$\int_0^{\frac{\pi}{4}} F(\cos 2x)\left\{\left(\sqrt{2} + \alpha\right)\cos x\right\}dx = 0$$

$$\left(\sqrt{2} + \alpha\right)\int_0^{\frac{\pi}{4}} F(\cos 2x)\cos x\,dx = 0$$

$\because$ in interval $\left(0, \frac{\pi}{4}\right) \Rightarrow F(\cos 2x)$ & $\cos x$ is NOT Zero.

$\therefore \sqrt{2} + \alpha = 0$

$\boxed{\alpha = -\sqrt{2}}$

4. Let f and g be two functions defined by $f(x) = \begin{cases} x+1, & x < 0 \\ |x-1,| & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \geq 0 \end{cases}$

Then (gof) (x) is

(1) continuous everywhere but not differentiable at x = 1

(2) continuous everywhere but not differentiable exactly at one point

(3) differentiable everywhere

(4) not continuous at x = –1

Sol. **(2)**

$$f(x) = \begin{cases} x+1, x < 0 \\ 1-x, 0 \leq x < 1 \\ x-1, 1 \leq x \end{cases}$$

$$g(x) = \begin{cases} x+1, x < 0 \\ 1, x \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} x+2, x<-1 \\ 1, x \geq -1 \end{cases}$$

$\therefore$ g(f (x)) is continuous everywhere

g(f (x)) is not differentiable at x = – 1

Differentiable everywhere else

5. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

(1) 69 (2) 72 (3) 115 (4) 92

Sol. **(4)**

C(2,0)

Ellipse $x^2 + 4y^2 = 36$

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

Equation of Normal at P(6cosθ, 3sinθ) is (6secθ)x – (3cosecθ)y = 27

It passes through (2,0)

$$\Rightarrow \sec\theta = \frac{27}{12} = \frac{9}{4}$$

$$\cos\theta \ \frac{4}{9}, \ \sin\theta = \frac{\sqrt{65}}{9}$$

$$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$$

$$\gamma$$

$$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right) \ c(2,0)$$

$$\gamma = \sqrt{\left(\frac{8}{3} - 2\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2} = \frac{\sqrt{69}}{3}$$

$$\text{Value of } 12\gamma^2 = \left(\frac{\sqrt{69}}{3}\right)^2 \times 12$$

$$\Rightarrow \frac{12 \times 69}{9} = 92$$

6. Let the mean of 6 observations 1, 2, 4, 5 x and y 5 and their variance be 10. Then their mean deviation about the mean is equal to

(1) $\frac{7}{3}$ (2) $\frac{10}{3}$ (3) $\frac{8}{3}$ (4) 3

Sol. **(3)**

Mean of 1, 2, 4, 5, x, y is 5

and variance is 10

$$\Rightarrow \text{mean} \Rightarrow \frac{12 + x + y}{6} = 5$$

$12 + x + y = 30$

$x + y = 18$

and by variance $\frac{x^2+y^2+46}{6}-5^2=10$

$x^2 + y^2 = 164$

$x = 8 \quad y = 10$

mean daviation = $\frac{|x-\bar{x}|}{6}$

$\Rightarrow \frac{4+3+1+0+3+5}{6}=\frac{16}{6}=\frac{8}{3}$

7. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A×B such that R ={((a_1, b_1), (a_2,b_2)): $a_1 \le b_2$ and $b_1 \le a_2$ }. Then the number of elements in the set R is

(1) 52 (2) 160 (3) 26 (4) 180

Sol. **(2)**

Let $a_1 = 1 \Rightarrow$ 5 choices of b_2

$a_1 = 3 \Rightarrow$ 4 choices of b_2

$a_1 = 4 \Rightarrow$ 4 choices of b_2

$a_1 = 6 \Rightarrow$ 2 choices of b_2

$a_1 = 9 \Rightarrow$ 1 choices of b_2

For (a_1, b_2) 16 ways .

Similarly, $b_1 = 2 \Rightarrow$ 4 choices of a_2

$b_1 = 4 \Rightarrow$ 3 choices of a_2

$b_1 = 5 \Rightarrow$ 2 choices of a_2

$b_1 = 8 \Rightarrow$ 1 choices of a_2

Required elements in R = 160

8. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha \in N$, if the distances of the points A(3, 4, α) and B(2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is

(1) 5 (2) 6 (3) 4 (4) 3

Sol. **(3)**

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6)+8\hat{j}-24\hat{k}$$

Normal of the plane $=3\hat{i}-4\hat{j}+12\hat{k}$

Plane : $3x - 4y + 12z = 3$

Distance from A(3,4, α)

$$\left|\frac{9-16+12\alpha-3}{13}\right|=2$$

$\alpha = 3$

$\alpha = -8$ (rejected)

Distance from B(2,3,a)

$$\left|\frac{6-12+12a-3}{13}\right|=3$$

$a = 4$

9. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is

(1) 102 (2) 103 (3) 101 (4) 104

Sol. **(2)**

5	2	1	3	4
T	H	A	M	S
4	1	0	0	0
4!	3!	2!	1!	0!

$\Rightarrow 4 \times 4! + 3! \times 1 + 0 + 0 + 0$

$\Rightarrow 96 + 6 = 102$

Ran k THAMS = 102 + 1 = 103

10. If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar, then $\left[\vec{a}\vec{b}\vec{c}\right]$ is equal to

(1) $\left[\vec{d}\ \vec{c}\ \vec{a}\right] + \left[\vec{b}\ \vec{d}\ \vec{a}\right] + \left[\vec{c}\ \vec{d}\ \vec{b}\right]$ (2) $\left[\vec{d}\ \vec{b}\ \vec{a}\right] + \left[\vec{a}\ \vec{c}\ \vec{d}\right] + \left[\vec{d}\ \vec{b}\ \vec{c}\right]$

(3) $\left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{d}\ \vec{c}\ \vec{a}\right] + \left[\vec{d}\ \vec{b}\ \vec{c}\right]$ (4) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{d}\ \vec{a}\ \vec{c}\right] + \left[\vec{d}\ \vec{b}\ \vec{a}\right]$

Sol. **(1)**

$\vec{a}, \vec{b}, \vec{c}, \vec{d} \rightarrow$ coplanar

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = ?$

$\vec{b} - \vec{a}, \vec{c} - \vec{b}, \vec{d} - \vec{c} \rightarrow$ coplanar

$\left[\vec{b} - \vec{a}\ \ \vec{c} - \vec{b},\ \vec{d} - \vec{c}\right] = 0$

$\Rightarrow \left(\vec{b} - \vec{a}\right).\left(\left(\vec{c} - \vec{b}\right) \times \left(\vec{d} - \vec{c}\right)\right) = 0$

$\left(\vec{b} - \vec{a}\right).\left(\vec{c} \times \vec{b} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d}\right) = 0$

$[b\,c\,d] - [b\,c\,a] - [b\,a\,d] - [a\,c\,d] = 0$

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{d}\ \vec{c}\ \vec{a}\right] + \left[\vec{b}\ \vec{d}\ \vec{a}\right] + \left[\vec{c}\ \vec{d}\ \vec{b}\right]$

11. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

(1) 63 (2) 92 (3) 25 (4) 41

Sol. **(1)**

${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} :: 1 : 3 : 5$

$$\frac{(n+2)!}{(r-1)!(n-r+3)!} \times \frac{r!(n+2-r)!}{(n+2)!} = \frac{1}{3}$$

$$\frac{r}{(n-r+3)} = \frac{1}{3} \Rightarrow n-r+3 = 3r$$

$$\boxed{n = 4r-3} - 0$$

$$\frac{(n+1)!}{r!(n+2-r)!} \times \frac{(r+1)!(n-r+1)!}{(n+2)!} = \frac{3}{5}$$

$$\frac{r+1}{n+2-r} = \frac{3}{5}$$

8r–1= 3n ……..(2)

By equation 1 and 2

$$\frac{8r-1}{3} = 4r-3 \qquad n = 4r-3$$

$$\boxed{r = 2} \qquad n = 4 \times 2 - 3$$

$$\boxed{n = 5}$$

Sum : ${}^7C_1 + {}^7C_2 + {}^7C_3 = 7+21+35 = 63$

12. Let y = y (x) be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}$, x > 0. If y(1) = 2, then y(2) is equal to

(1) $\frac{693}{128}$ (2) $\frac{637}{128}$ (3) $\frac{697}{128}$ (4) $\frac{679}{128}$

Sol. **(1)**

I.F = $= e^{\int \frac{5dx}{x(x^5+1)}} = e^{e^{\int \frac{5x^{-6}dx}{(x^{-5}+1)}}}$

Put, $1 + x^{-5} = t \Rightarrow -5x^{-6}\,dx = dt$

$$\Rightarrow e^{\int \frac{-dt}{1}} = \frac{1}{t} = \frac{x^5}{1+x^5}$$

$$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{(1+x^5)} \times \frac{(1+x^5)^2}{x^7} dx$$

$$= \int x^3 dx + \int x^{-2} dx$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Given than: x = 1 $\Rightarrow$ y = 2

$$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$$

$$c = \frac{7}{4}$$

$$y\cdot\frac{x^5}{1+x^5}=\frac{x^4}{4}-\frac{1}{x}+\frac{7}{4}$$

Now put, x = 2

$$y\cdot\left(\frac{32}{33}\right)=\frac{21}{4}$$

$$y=\frac{693}{128}$$

13. The converse of $((\sim p)\wedge q)\Rightarrow r$ is

(1) $(p v(\sim q))\Rightarrow(\sim r)$ (2) $((\sim p) v q)\Rightarrow r$ (3) $(\sim r)\Rightarrow((\sim p)\wedge q)$ (4) $(\sim r)\Rightarrow p\wedge q$

Sol. **(1)**

$((-P)\wedge 2)\Rightarrow r$

Converse ….

$\sim((\sim P)\wedge q)\Rightarrow(\sim r)$

$(P\vee(\sim q))\Rightarrow(\sim r)$

14. If the 1011th term from the end in the binominal expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$ is 1024 times 1011th term from the beginning, the |x| is equal to

(1) 8 (2) 12 (3) 10 (4) 15

Sol. **(3)– Bouns**

T_{1011} from beginning = T_{1010+1}

$$={}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$$

T_{1011} from end

$$={}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$$

Given: $={}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$$=2^{10}\cdot{}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$$

$$\left(\frac{-5}{2x}\right)^{2}=2^{10}\left(\frac{4x}{5}\right)^{2}$$

$$x^4=\frac{5^4}{2^{16}}$$

$$|x|=\frac{5}{16}$$

15. If the system of linear equations

$7x + 11y + \alpha z = 13$

$5x + 4y + 7z = \beta$

$175x + 194y + 57z = 361$

has infinitely many solutions, then $\alpha + B + 2$ is equal to :

(1) 3 (2) 6 (3) 5 (4) 4

Sol. **(4)**

$7x + 11y + \alpha z = 13$

$5x + 4y + 7z = \beta$

$175x + 194y + 57z = 361$

4sc condition of Infinite Many solution

$\boxed{\Delta = 0}$ & $\Delta x, \Delta y, \Delta z = 0$ check.

After solving we get $\boxed{\alpha + 13 + 2 = 4}$

16. Let the line passing through the point P (2, –1, 2) and Q (5, 3, 4) meet the plane x – y +z = 4 at the point T. Then the distance of the point R from the plane x + 2y + 3z +2 = 0 measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

(1) 3 (2) $\sqrt{61}$ (3) $\sqrt{31}$ (4) $\sqrt{189}$

Sol. **(1)**

$\text{Line}: \frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \qquad \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

$\text{Line}: \frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$

Point T = $(2\mu - 1, 2\mu - 5, \mu)$

It lies on plane

$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$

$\mu = 1$

$\therefore T = (1, -3, 1)$

$\therefore RT = 3$

17. Let the function f : [0, 2] $\to$ R be defined as

$$f(x) = \begin{cases} e^{\min\{x^2, x-[x]\}}, & x \in [0,1) \\ e^{[x - \log_e x]}, & x \in [1,2) \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 x f(x)\, dx$ is

(1) $(e-1)\left(e^2 + \frac{1}{2}\right)$ (2) $1 + \frac{3e}{2}$ (3) $2e - \frac{1}{2}$ (4) $2e - 1$

Sol. **(3)**

$F[0,2] \rightarrow R$

$$F(x)=\begin{cases}\min\{x^2,\{x\}\}; x\in[0,1)\\ [x-\log_e x]=1; x\in[1,2)\end{cases}$$

$$F(x)=\begin{cases}e^{x^2} : x\in[0,1)\\ e \quad x\in[1,2)\end{cases}$$

$$\int_0^2 xf(x)dx = \int_0^1 x.e^{x^2}dx + \int_1^2 x.e\ dx$$

$$= \frac{1}{2}(e-1)+\frac{1}{2}(4-1)e$$

$$\Rightarrow 2e-\frac{1}{2}$$

18. For $a\in C$, let A = $\{z\in C: \mathrm{Re}(a+\bar{z})>\mathrm{Im}(\bar{a}+z)\}$ and B = $\{z\in C: \mathrm{Re}(a+\bar{z})<\mathrm{Im}(\bar{a}+z)\}$. The among the two statements:

(S1) : If Re (a), Im (a) >0, then the set A contains all the real numbers

(S2) : If Re (a), Im (a) < 0, then the set B contains all the real numbers,

(1) only (S1) is true (2) both are false (3) only (S2) is true (4) both are true

Sol. **(2)**

Let $a = x_1 + iy_1$ $z = x + iy$

Now $\mathrm{Re}(a + \bar{z}) > \mathrm{Im}(\bar{a} + z)$

$\therefore x_1 + x > -y_1 + y$

$x_1 = 2, y_1 = 10, x = -12, y = 0$

Given inequality is not valid for these values.

S1 is false.

Now $\mathrm{Re}(a + \bar{z}) < \mathrm{Im}(\bar{a} + z)$

$x_1 + x < -y_1 + y$

$x_1 = -2, y_1 = -10, x = 12, y = 0$

Given inequality is not valid for these values.

S2 is false.

19. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation

(1) $4x^2 -24x -27 = 0$ (2) $4x^2 +24x +27 = 0$ (3) $4x^2 -24x +27 = 0$ (4) $4x^2 +24x -27 = 0$

Sol. **(3)**

$$\begin{vmatrix} x+1 & x & x \\ x & x+d & x \\ x & x & x+d^2 \end{vmatrix} = \frac{9}{8}(103x+81)$$

Put $\boxed{x=0}$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8}$$

$$\boxed{\lambda = \frac{9}{2}}$$

$$\frac{\lambda}{3} = \frac{9}{2 \times 3} \Rightarrow \frac{3}{2}$$

$$\boxed{\frac{\lambda}{3} = \frac{3}{2}}$$

Option (C) $4x^2 - 24x + 27 = 0$

has Root $\frac{3}{2}, \frac{9}{2}$

20. The domain of the function f(x) = $\frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where [x] denotes the greatest integer less than or equal to x)

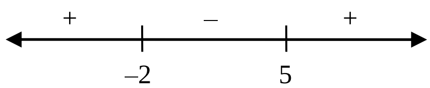
(1) $(-\infty, -3] \cup [6, \infty)$ (2) $(-\infty, -2) \cup (5, \infty)$ (3) $(-\infty, -3] \cup (5, \infty)$ (4) $(-\infty, -2) \cup [6, \infty)$

Sol. **(4)**

$$F(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$$

$$[x]^2 - 3[x] - 10 > 0$$

$$([x] + 2)([x] - 5) > 0$$



$[x] < -2$ or $[x] > 5$

$[x] \le -3$ or $[x] \ge 6$

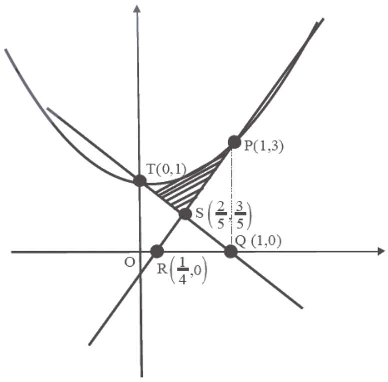
$x < -2$ or $x \ge 6$

$x \in (-\infty, -2) \cup [6, \infty)$

SECTION - B

21. If A is the area in the first quadrant enclosed by the curve C : $2x^2 - y + 1 = 0$, the tangent to C at the point (1,3) and the line x + y = 1, then the value of 60 A is ______.

Sol. 16



$y = 2x^2 + 1$

Tangenet at (1, 3)

$y = 4x - 1$

$A = \int_0^1 \left(2x^2 + 1\right)dx - \text{area of } (\Delta QOT) - \text{area of}$

$(\Delta PQR) + \text{area of } (\Delta QRS)$

$A = \left(\frac{2}{3} + 1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$

22. Let A = {1, 2, 3, 4, 5} and B= {1, 2, 3, 4, 5, 6}. Then the number of functions f:A→B satisfying f(1) + f(2) = f(4)–1 is equal to ______.

Sol. 360

$f(1) + f(2) + 1 = f(4) \le 6$

$f(1) + f(2) \le 5$

Case (i) f(1) = 1 ⇒ f(2) = 1, 2, 3, 4 ⇒ 4 mappings

Case (ii) f(1) = 2 ⇒ f(2) = 1, 2, 3 ⇒ 3 mappings

Case (iii) f(1) = 3 ⇒ f(2) = 1, 2 ⇒ 2 mappings

Case (iv) f(1) 4 ⇒ f(2) = 1 ⇒ 1 mapping

f(5) & f(6) both have 6 mappings each

Number of functions = (4 + 3 + 2 + 1) × 6 × 6 = 360

23. Let the tangent to the parabola $y^2 = 12x$ at the point (3, α) be perpendicular to the line 2x+2y = 3. Then the square of distance of the point (6,–4) from the normal to the hyperbola $\alpha^2 x^2 - 9y^2 = 9\alpha^2$ at its point (α –1, α + 2) is equal to _____.

Sol. **116**

$\because P(3,\alpha)$ lies on $y^2 = 12x$

$\Rightarrow \alpha = \pm 6$

But, $\left.\frac{dy}{dx}\right|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6 \left(\alpha = -6 \text{ reject}\right)$

Now, hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$, normal at $Q(\alpha-1, \alpha+2)$ is $\frac{9x}{5} + \frac{36y}{8} = 45$

$\Rightarrow 2x + 5y - 50 = 0$

Now, distance of (6, –4) from 2x + 5y – 50 = 0 is equal to

$\left|\frac{2(6)-5(4)-50}{\sqrt{2^2+5^2}}\right| = \frac{58}{\sqrt{29}}$

$\Rightarrow$ Square of distance = 116

24. For k∈N, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots$ is 10, then the value of k is___

Sol. **2**

$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots \text{upto} \infty$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots \text{upto} \infty$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \ldots\ldots \text{upto} \infty$

$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} \ldots\ldots \text{upto} \infty$

$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \ldots\ldots \text{upto} \infty$

$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \ldots..\infty$

$9\left(1 - \frac{1}{k}\right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k}\right)}$

$9(k-1)^3 = 4k(k-1) + 1$

k = 2

25. Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in R$ meet the plane P : x + 2y + 3z = 4 at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then α + 2β + 6 γ is equal to _____.

Sol. **11**

$\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$

Dr's of line $\ell(1,2,\lambda)$

Dr's of normal vector of plane P : x + 2y + 3z = 4

are (1, 2, 3)

Now, angle between line ℓ and plane P is given by

$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}.\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{given } \cos\theta = \sqrt{\frac{5}{14}}\right)$

$\Rightarrow \lambda = \frac{2}{3}$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$

line of plane P.

$\Rightarrow t = -1$

$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$

$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

26. The number of points where the curve f(x) = $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in R$ cuts x-axis, is equal to ________

Sol. **2**

Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t_2 + \frac{1}{t_2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1+\sqrt{21}}{2}$

Two real values of t.

27. If the line l_1 : 3y– 2x = 3 is the angular bisector of the line l_2 : x –y + 1 = 0 and l_3 : ax +βy + 17, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to ______.

Sol. **348**

Point of intersection of $\ell_1 : 3y - 2x = 3$

$\ell_2 : x - y + 1 = 0$ is $P \equiv (0,1)$

Which lies on $\ell_3 : \alpha x - \beta y + 17 = 0,$

$\boxed{\Rightarrow \beta = -17}$

Consider a random point $Q \equiv (-1,0)$

on $\ell_2 : x - y + 1 = 0$, image of Q about

$\ell_2 : x - y + 1 = 0$, is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae

$$\frac{x-(-1)}{2} = \frac{y-0}{-3} = 2\left(\frac{-2+3}{13}\right)$$

Now, Q' lies in $\ell_3 : \alpha x + \beta y + 17 = 0$

$\boxed{\Rightarrow \alpha = 7}$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

28. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to ______.

Sol. **27**

$64x^2 + 5Nx + 1 = 0$

$D = 25N^2 - 256 < 0$

$\Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$

$\therefore N = 1, 2, 3$

$\therefore \text{Probability} = \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64}$

$\therefore q - p = 27$

29. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = 11$, $\vec{b}.(\vec{a} \times \vec{c}) = 27$ and $\vec{b}.\vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to ______.

Sol. **285**

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{b}.(\vec{a} \times \vec{c}) = 27, \vec{a}.\vec{b} = 0$

$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}$

Let θ be angle between $\vec{b}, \vec{a} \times \vec{c}$

Then $|\vec{b}|.|\vec{a} \times \vec{c}| \sin\theta = 3\sqrt{14}$

$|\vec{b}|.|\vec{a} \times \vec{c}| \cos\theta = 27$

$\Rightarrow \sin\theta = \frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}| \times |\vec{a} \times \vec{c}| = 3\sqrt{95}$

$\Rightarrow |\vec{a} \times \vec{c}| = \sqrt{3} \times \sqrt{95}$

30. Let $S=\left\{z\in C-\{i,2i\}:\frac{z^2+8iz-15}{z^2-3iz-2}\in R\right\}$. If $\alpha-\frac{13}{11}i\in S,\ a\in R-\{0\}$, then $242\alpha^2$ is equal to ______.

Sol. **1680**

$$\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right)\in R$$

$$\Rightarrow 1+\frac{(1liz-13)}{(z^2-3iz-2)}\in R$$

Put Z = $\alpha-\frac{13}{11}i$

$\Rightarrow (z^2-3iz-2)$ is imaginary

Put z = x + iy

$\Rightarrow (x^2-y^2+2xyi-3ix+3y-2)\in$ Imaginary

$$\Rightarrow \text{Re}\left(x^2-y^2+3y-2+(2xy-3x)i\right)=0$$

$$\Rightarrow x^2-y^2+3y-2=0$$

$$x^2=y^2-3y+2$$

$$x^2=(y-1)(y-2)\therefore z=\alpha-\frac{13}{11}i$$

Put $x=\alpha, y=\frac{-13}{11}$

$$\alpha^2=\left(\frac{-13}{11}-11\right)\left(\frac{-13}{11}-2\right)$$

$$\alpha^2=\frac{(24\times 35)}{121}$$

$$242\alpha^2=48\times 35=1680$$

SECTION - A

31. Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops collapse, the new velocity is :-

(1) 10 cm/s (2) 40 cm/s (3) 16 cm/s (4) 5 cm/s

Sol. **(2)**

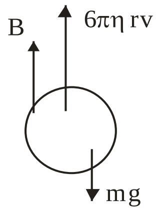
$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 2r$

$V = \frac{2r^2}{9\eta}(\rho_b - \rho_{air})$

$V \propto r^2$

$\frac{V_1}{V_2} = \left(\frac{r}{R}\right)^2$



$\boxed{V_2 = V_1 \times 4 = 10 \times 4 = 40\ \text{km/s}^{-1}}$

32. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non-magnetic bar with same geometry and mass.

Reason R : For the magnetic bar, Eddy currents are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options given below

(1) A is true but R is false

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) A is false but R is true

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **(4)**

Due to Eddy current in the metallic pice which opposes the motion of magnetic bar. So, it takes more time to comes down compared to non-magnetic bar.

33. A space ship of mass 2×10^4 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the space ship in the orbit to overcome the gravitational pull will be (if g = 10 m/s^2 and radius of earth = 6400 km):

(1) $7.9(\sqrt{2}-1)$ km/s (2) $7.4(\sqrt{2}-1)$ km/s (3) $11.2(\sqrt{2}-1)$ km/s (4) $8(\sqrt{2}-1)$ km/s

Sol. **(4)**

$\Delta V = \sqrt{\frac{GM}{R}}(\sqrt{2} - 1)$

$= \sqrt{\frac{GM}{R^2} \times R}(\sqrt{2} - 1)$

$= \sqrt{gR}(\sqrt{2} - 1) = 8000(\sqrt{2} - 1)\ \text{ms}^{-1}$

$= 8(\sqrt{2} - 1)\ \text{kms}^{-1}$

34. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be :

(Given g = 10 m/s^2)

(1) Zero (2) $20\sqrt{3}$ ms^{-1} (3) $40\sqrt{3}$ ms^{-1} (4) 20 ms^{-1}

Sol. **(2)**

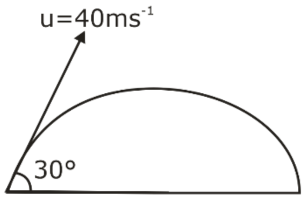
$U_x = 40 \cos 30 = 20\sqrt{3}$

$U_y = 40 \sin 30 = 20$

$Vx = 20\sqrt{3}$

$V_y = u_y - gt = 20 - 10 \times 2 = 0$

$V = \sqrt{{v_x}^2 + {v_y}^2} = 20\sqrt{3} = ms^{-1}$



35. A plane electromagnetic wave of frequency 20 MHz propagates in free space along x-direction. At a particular space and time, $\vec{E} = 6.6\hat{j}\ v/m$. What is $\vec{B}$ at this point?

(1) $-2.2 \times 10^{-8}\ \hat{k}\ T$ (2) $-2.2 \times 10^{-8}\ \hat{i}\ T$ (3) $2.2 \times 10^{-8}\ \hat{k}\ T$ (4) $2.2 \times 10^{-8}\ \hat{i}\ T$

Sol. **(3)**

$|B| = \frac{|E|}{C} = \frac{6.6}{3\times10^8} = 2.2 \times 10^{-8}$

For direction of $\vec{B}$

$= \vec{E} \times \vec{B} = \vec{C}$

$= \hat{j} \times \vec{B} = \hat{i}$

$\vec{B} = (2.2 \times 10^{-8})\hat{k}\,T$

36. A car P travelling at 20 ms^{-1} sounds its horn at a frequency of 400 Hz. Another car Q is travelling behind the first car in the same direction with a velocity 40 ms^{-1}. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound = 360 ms^{-1}]

(1) 471 Hz (2) 514 Hz (3) 421 Hz (4) 485 Hz

Sol. **(3)**

$V_c = 20\ ms^{-1}$

f = 400 Hz

$\boxed{Q} \rightarrow = 40\ ms^{-1}$ $\boxed{P} \rightarrow 20\ ms^{-1}$

0 $\xleftarrow[\oplus]{}$ S

$f_{app} = \left[\frac{Vs - (-V_Q)}{V_S - (-V_p)}\right] f$

$= \left(\frac{360+40}{360+20}\right) \times 400 = \frac{400}{380} \times 400 = 421$ Hz Ans. (3)

37. A body of mass 500 g moves along x-axis such that it's velocity varies with displacement x according to the relation $v = 10\sqrt{x}$ m/s the force acting on the body is:-

(1) 25 N (2) 5 N (3) 166 N (4) 125 N

Sol. **(1)**

$$v = 10\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{10}{2\sqrt{x}} = \frac{5}{\sqrt{x}}$$

$$a = v\frac{dv}{dx}$$

$$a = v \times \frac{5}{\sqrt{x}} = 10\sqrt{x} \times \frac{5}{\sqrt{x}} = 50\ ms^{-2}$$

$$F = ma = \frac{500}{1000} \times 50 = 25\ N$$

38. The ratio of the de-Broglie wavelengths of proton and electron having same Kinetic energy:

(Assume $m_p = m_e \times 1849$)

(1) 1:62 (2) 1:30 (3) 1:43 (4) 2:43

Sol. **(3)**

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

$$\frac{\lambda_P}{\lambda_e} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{m_e}{1840me}} = \frac{1}{\sqrt{1840}}$$

$$\frac{\lambda_P}{\lambda_e} = \frac{1}{43}$$ Ans. (3)

39. If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be:

(1) $FV^{-2}T^2$ (2) FV^4T^{-6} (3) $FV^{-4}T^{-2}$ (4) $F^2V^{-2}T^6$

Sol. **(3)**

$\rho = F^x V^y T^z$

$[ML^{-3}] = [MLT^{-2}]^x [LT^{-1}]^y [T]^z$

$1 = x$ $x = 1$

$-3 = x + y$ $y = -4$

$0 = -2x - y + z$ $z = 2x + 4 = 2 - 4 = -2$

$\rho = F V^{-4} T^{-2}$ Ans. (3)

40. An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

A. The electron will experience magnetic force along the axis of the solenoid.

B. The electron will not experience magnetic force.

C. The electron will continue to move along the axis of the solenoid.

D. The electron will be accelerated along the axis of the solenoid.

E. The electron will follow parabolic path-inside the solenoid.

Choose the correct answer from the options given below:

(1) A and D only (2) B, C and D only (3) B and E only (4) B and C only

Sol. **(4)**

We know that

In the solenoid magnetic field along the axis of solenoid.

When charge particle moving inside solenoid along the axis F = 0

$\overrightarrow{F_m} = q\ (\vec{V} \times \vec{B})$

So, $\overrightarrow{F_m}\ = 0$

And it moves with constant velocity.

41. The Thermodynamic process, in which internal energy of the system remains constant is

(1) Isobaric (2) Isochoric (3) Adiabatic (4) Isothermal

Sol. **(4)**

$\Delta U = nC_V\Delta T$, for all process

For isothermal process, $\Delta T = 0$

So, $\Delta U = 0$

That means internal energy of system remains constant.

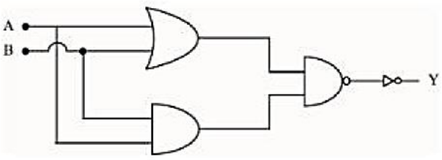
42. In satellite communication, the uplink frequency band used is:

(1) 76 – 88 MHz (2) 420 – 890 MHz (3) 3.7 – 4.2 GHz (4) 5.925 – 6.425 GHz

Sol. **(4)**

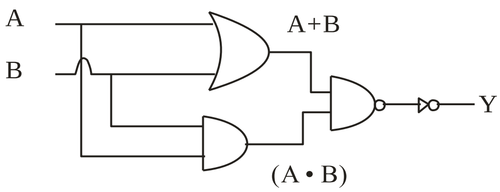
uplink	**Downlink**
5.8-6.2 Ghz	4-4.2 Ghz
I. standard	

43. The logic operations performed by the given digital circuit is equivalent to:



(1) OR (2) NAND (3) NOR (4) AND

Sol. **(4)**



$Y = \overline{\overline{(A+B).(AB)}}$

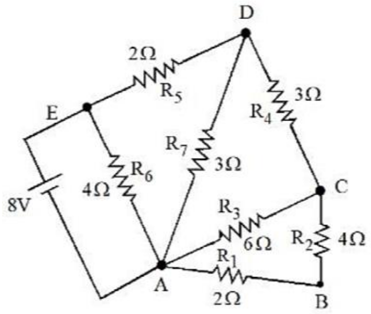
Y = (A + B).(AB)

Y = AB + AB

Y = (A.B)

Y = AND Gate

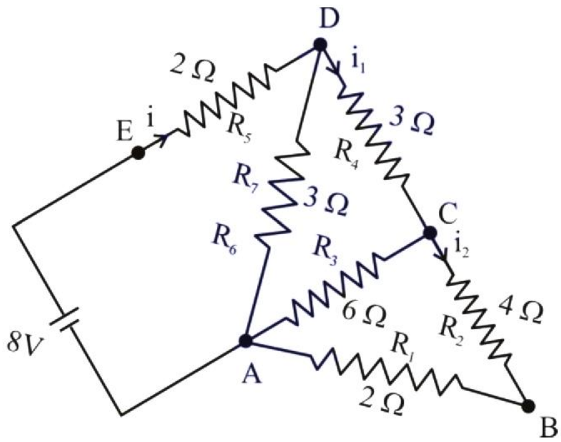
44.



The current flowing through R_2 is:

(1) $\frac{1}{3}$A (2) $\frac{1}{4}$A (3) $\frac{2}{3}$A (4) $\frac{1}{2}$A

Sol. **(1)**



$R_{eq} = 4\Omega$

i = $\frac{8}{4} = 2\text{A}$

i_1 = $\frac{2\times 3}{3+6} = \frac{2}{3}\text{A}$

i_2 = $\frac{2/3}{2} = \frac{1}{3}\text{A}$

45. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\vec{B}$ will be:

(1) 3 (2) $\sqrt{5}$ (3) $\sqrt{13}$ (4) $\sqrt{6}$

Sol. **(Bonus)**

$\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{B} - \vec{A} = 2\hat{j}$

$\vec{B}$ = $2\hat{j} + \vec{A}$ = $2\hat{i} + 5\hat{j} + 2\hat{k}$

$|\vec{B}|$ = $\sqrt{2^2 + 5^2 + 2^2}$ = $\sqrt{4 + 25 + 4}$ = $\sqrt{8 + 25}$ = $\sqrt{33}$

46. If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:-

(1) $\frac{4}{3}V$ (2) V (3) $\frac{3V}{2}$ (4) $\frac{V}{2}$

Sol. **(3)**

$$V = -\frac{GM}{R^3}(1.5R^2 - 0.5r^2)$$

$$V = -\frac{GM}{R} \text{ [At the surface]}$$

$$V_{centre} = -\frac{3GM}{2R} = \frac{3}{2}V$$

47. The root mean square speed of molecules of nitrogen gas at 27°C is approximately : (Given mass of a nitrogen molecule = 4.6×10^{-26} kg and take Boltzmann constant $k_B = 1.4 \times 10^{-23}$ JK^{-1})

(1) 1260 m/s (2) 91 m/s (3) 523 m/s (4) 27.4 m/s

Sol. **(3)**

$$V_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3KT}{m}}$$

$$= \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{4.6 \times 10^{-26}}}$$

$$= \sqrt{\frac{9 \times 1.4 \times 10^{5}}{4.6}}$$

$$= \sqrt{2.73 \times 10^5}$$

$$= \sqrt{27.3 \times 10^4}$$

$= 522.4\ ms^{-1}$

$= 523\ ms^{-1}$

48. The energy of He^+ ion in its first excited state is, (The ground state energy for the Hydrogen atom is –13.6 eV):

(1) –13.6 eV (2) –54.4 eV (3) –27.2 eV (4) –3.4 eV

Sol. **(4)**

$$E = -\left(\frac{13.6z^2}{n^2}\right)eV$$

First excited state

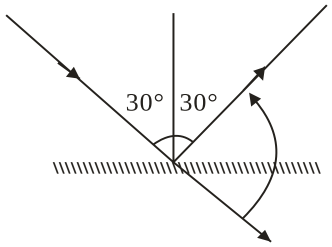
n = 2

$$E = \frac{-13.6 \times z^2}{2^2} = -13.6 \text{ eV}$$

49. When one light ray is reflected from a plane mirror with 30° angle of reflection, the angle of deviation of the ray after reflection is:

(1) 140° (2) 130° (3) 120° (4) 110°

Sol. **(3)**



$\delta = \Pi - 2i \ = \Pi - 2 \times 30$

$= 180 - 60 = 120°$

50. A capacitor of capacitance C is charge to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is :

(1) Zero (2) $\frac{CV}{\varepsilon_0}$ (3) $\frac{2CV}{\varepsilon_0}$ (4) $\frac{CV}{2\varepsilon_0}$

Sol. **(2)**

$\phi = \frac{q}{\varepsilon_0}$

$\phi = \frac{CV}{\varepsilon_0}$

SECTION – B

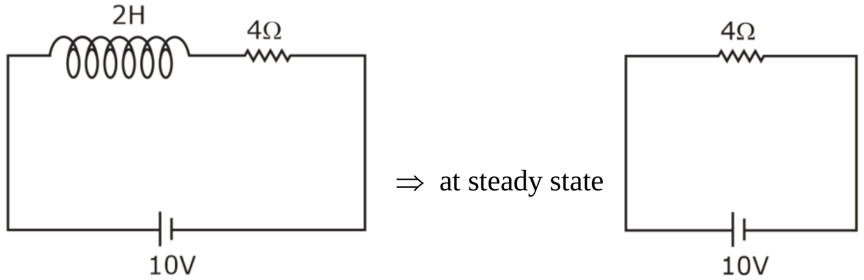
51. A coil has an inductance of 2H and resistance of 4 Ω. A 10 V is applied across the coil. The energy stored in the magnetic field after the current has built up to its equilibrium value will be ___________ $\times 10^{-2}$ J.

Sol. **(625)**

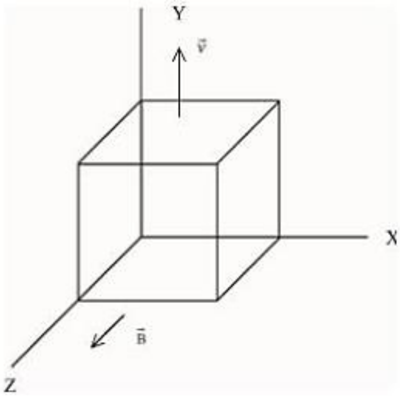
At steady state, inductor will act as a short circuit.

$I = \frac{V}{R} = \frac{10}{4} = \frac{5}{2} A$

$E = \frac{1}{2} LI^2 = \frac{1}{2} \times 2 \times \left(\frac{5}{2}\right)^2 = \frac{25}{4} = 6.25 = 625 \times 10^{-2}$ J



52. A metallic cube of side 15 cm moving along y-axis at a uniform velocity of 2 ms^{-1}. In a region of uniform magnetic field of magnitude 0.5T directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be __________ mV.



Sol. **(150)**

qVB = qE

E = VB

$\Delta V = EL = VBL$

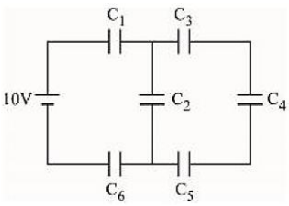
$\Delta V = 2 \times 0.5 \times \frac{15}{100} = \frac{15}{100}$ volt

$= 15 \times 10^{-2}$ volt

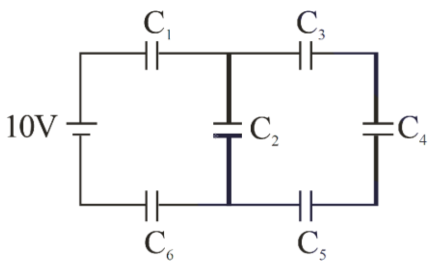
$= 150 \times 10^{-3}$ v

53. In the given circuit,

$C_1 = 2\ \mu F$, $C_2 = 0.2\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$, $C_5 = 2\ \mu F$, $C_6 = 2\ \mu f$, The charge stored on capacitor C_4 is ________ μC.



Sol. **(4)**



$C_{eq} = 0.5\ \mu F$

$Q = 0.5 \times 10 = 5\ \mu C$

$Q' = \frac{5\mu C \times 0.8}{0.8 + 0.2} = 4\mu C$

54. A nucleus disintegrates into two nuclear parts, in such a way that ratio of their nuclear sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of n : 1. The value of n is __________

Sol. **(2)**

From LCM :

$m_1v_1 = m_2v_2$

$$\frac{V_1}{V_2} = \frac{m_2}{m_1} = \frac{r_2^3}{r_1^3} = \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{2^{\frac{1}{3}}}{1}\right)^3 = \frac{2}{1}$$

$\boxed{n = 2}$

55. The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is __________ $\times 10^{-4}$ J. (take $\pi = 22/7$)

Sol. **(264)**

$W = \Delta U$

$W = 25 \times (A_f - A_i)$

$= 2 \times 5 \times 4\pi\ (r_f^2 - r_i 2)$

$$= 2 \times 3.5 \times 10^{-2} \times 4 \times \frac{22}{7} \times 10^{-4}\ (300)$$

$= 264 \times 10^{-4}$ J

56. A circular plate is rotating horizontal plane, about an axis passing through its center perpendicular to the plate, with an angular velocity ω. A person sits at the center having two dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$. The value of x is

Sol. **(3)**

$I_1\omega_1 = I_2\omega_2$

$I\omega = 3I\omega_2$

$$\omega_2 = \frac{\omega}{3}$$

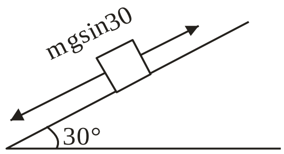
$$E = \frac{1}{2} I\omega^2$$

$$E_f = \frac{1}{2} \times 3I \times \left(\frac{\omega}{3}\right)^2 = \frac{\frac{1}{2} I\omega^2}{3}$$

$x = 3$

57. A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at t = 10 s from the starts is __________ W. [use g = 10 ms^{-2}] (Calculate the nearest integer value)

Sol. F – mg sin 30 = ma

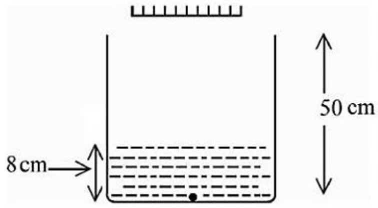


F = 5 × 1 + 25 = 30 N

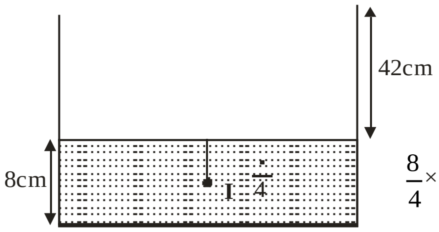
V = u + at = 0 + 1 × 10 = 10

P = FV = 30 × 10 = 300 watt

58. As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is ___________ cm.



Sol. **(98)**



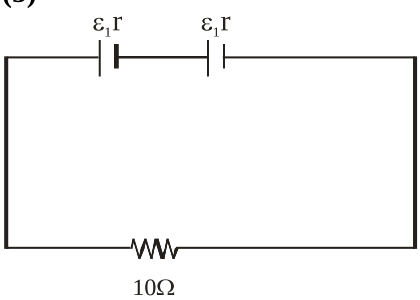
$\frac{8}{4} \times 3 = 6\text{cm}$

Apparent depth of O = $\frac{d}{\mu} = 6$

Distance between O and I_2 = 48 + 50 = 98 cm

59. Two identical cells each of emf 1.5 V are connected in series across a 10 Ω resistance. An ideal voltmeter connected across 10 Ω resistance reads 1.5 V. The internal resistance of each cell is ___________ Ω.

Sol. **(5)**



$I = \frac{2\varepsilon}{10 + 2r} = \frac{1.5}{10}$

$20\varepsilon = 15 + 3r$

$\Rightarrow 20 \times 1.5 = 15 + 3r$

$\Rightarrow 30 = 15 + 3r$

$r = 5\Omega$

60. A wire of density 8×10^3 kg/m^3 is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If Y = 8×10^{10} N/m^2, the fundamental frequency of vibration in the wire will be __________ Hz.

Sol. **(80)**

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$f = \frac{1}{2\ell}\sqrt{\frac{YA\Delta\ell}{\ell\mu}}$$

$$f = \frac{1}{2\times 0.5}\sqrt{\frac{8\times 10^{10}\times 3.2\times 10^{-4}}{8\times 10^{3}\times 0.5}}$$

$$f = \frac{1}{1}\sqrt{6400}$$

f = 80 Hz

SECTION - A

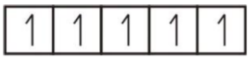
61. The magnetic moment is measured in Bohr Magneton (BM). Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is:

(1) 3.87 B. M. and 1.732 B.M.
(2) 6.92 B.M. in both
(3) 5.92 B.M. and 1.732 B.M.
(4) 4.89 B.M. and 6.92 B.M.

Sol. **3**

$[Fe(H_2O)_6]^{3+}$

$Fe^{3+} \Rightarrow [Ar]\ 3d^5\ 4s^0$

No pairing 

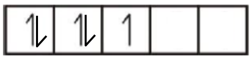
$\therefore$ Unpaired e^- = 5

$\mu = \sqrt{n(n+2)}$

$= \sqrt{5(5+2)}$

$\mu = \sqrt{35} = 5.92 B.M.$

$[Fe(CN)_6]^{-3}$



$\therefore$ Unpaired e^- = 1

$\mu = \sqrt{n(n+2)}$

$= \sqrt{1(1+2)} = \sqrt{3} = 1.732 B.M.$

62. Which one of the following pairs is an example of polar molecular solids?

(1) $SO_2(s), CO_2(s)$
(2) $SO_2(s), NH_2(s)$
(3) $MgO(s), SO_2(s)$
(4) HCl (s), AlN(s)

Sol. **2**

SO_2 and NH_3 are polar molecules. They are constituent particles of polar molecular solids.

63. Match List I with List II

List I Complex		**List II Colour**	
A.	$Mg(NH_4)PO_4$	I.	Brown
B.	$K_3[Co(NO_2)_6]$	II.	White
C.	$MnO(OH)_2$	III.	Yellow
D.	$Fe_4[Fe(CN)_6]_3$	IV.	blue

Choose the correct answer from the options given below:

(1) A–II, B–III, C–IV, D–I
(2) A–II, B–IV, C–I, D–III
(3) A–III, B–IV, C–II, D–I
(4) A–II, B–III, C–I, D–IV

Sol. **4**

$Mg(NH_4)PO_4 \Rightarrow$ White

$K_3[Co(NO_2)_6] \Rightarrow$ Yellow

$MnO(OH)_2 \Rightarrow$ Brown

$Fe_4[Fe(CN)_6]_3 \Rightarrow$ Blue

64. A solution is prepared by adding 2 g of "X" to 1 mole of water. Mass percent of "X" in the solution is

(1) 5% (2) 20 % (3) 2 % (4) 10%

Sol. **4**

Solute (X) = 2 g

Solvent (H_2O) = 1 mole = 18 g

Total mass = 2 + 18 = 20 g

% mass of X = $\frac{2}{20} \times 100 = 10\%$

65. If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]2^-$, which of the following properties are expected to get changed?

A. Geometry

B. Geometrical isomerism

C. Optical isomerism

D. Magnetic properties

(1) A, B and C (2) A and D (3) B and C (4) A, B and D

Sol. **4**

$[NiBr_2Cl_2]^{2-}$ $\rightarrow$ This complex species is tetrahedral as $Br^{\ominus}$ & $Cl^{\ominus}$ are weak field ligands.

$[PtBr_2Cl_2]^{2-}$ $\rightarrow$ As Pt belongs to 5d series. This complex species is square planar.

Both the complex species are optically inactive.

$[NiBr_2Cl_2]^{2-}$, being tetrahedral does not show Geometrical Isomerism.

$[PtBr_2Cl_2]^{2-}$ shows two Geometrical Isomers.

66. Given below are two statements :

Statement I : In the metallurgy process, sulphide ore is converted to oxide before reduction.

Statement II : Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Sol. **1**

$2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2$

Oxides on carbon reduction forms CO_2 while sulphide on carbon reduction gives CS_2.

CO_2 is more volatile compared to CS_2 therefore oxides are easy to reduce.

67. $H_3C-CH_2-\underset{\substack{|\\OH}}{CH}-CH_3 \xrightarrow[\text{(iii)} D_2O]{\text{(i) NaI, } H_3PO_4 \text{ (ii) Mg, Dry ether}} \underset{\text{Product}}{[X]}$

Product [X] formed in the above reaction is:

(1) $H_3C-CH_2-\underset{\substack{|\\D}}{CH}-CH_3$

(2) $H_3C-CH_2-CH=CH_2$

(3) $H_3C-CH=CH-CH_3$

(4) $H_3C-CH_2-\overset{\substack{H\\|}}{\underset{\substack{|\\OH}}{C}}-CH_3$

Sol. **1**

$CH_3-CH_2-\underset{\substack{|\\OH}}{CH}-CH_3 \xrightarrow{\text{(i) NaI, } H_3PO_4} CH_3-CH_2-\underset{\substack{|\\I}}{CH}-CH_3$

$\xrightarrow{Mg} CH_3-CH_2-\underset{\substack{|\\MgI}}{CH}-CH_3 \xrightarrow{D_2O} CH_3-CH_2-\underset{\substack{|\\D}}{CH}-CH_3$

68. Given below are two statements :

Statement I : Ethene at 333 to 343 K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543 K in H_2O through step growth polymerizes to give Nylon 6.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

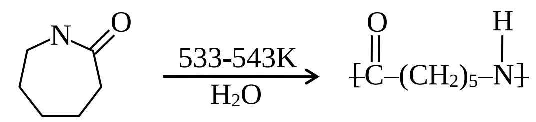
(4) Both Statement I and Statement II are false

Sol. **3**

(Fact-Based)

Statement-I : Ethane at 333 to 343 K and 6-7 atm pressure of $AlEt_3$ and $TiCl_4$ undergo addition polymerization to give HDPE not LDPE

Statement-II :



Caprolactam Nylon-6

69. For a chemical reaction A + B $\rightarrow$ Product, the order is 1 with respect to A and B.

Rate $mol\,L^{-1}\,S^{-1}$	[A] $mol\,L^{-1}$	[B] $mol\,L^{-1}$
0.10	20	0.5
0.40	x	0.5
0.80	40	Y

What is the value of x and y?

(1) 80 and 2 (2) 40 and 4 (3) 80 and 4 (4) 160 and 4

Sol. **1**

$r = K[A]^1[B]^1$

$0.1 = K(20)^1\,(0.5)^1$ …(i)

$0.40 = K(x)^1\,(0.5)^1$ …(ii)

$0.80 = K(40)^1\,(y)^1$ …(iii)

From (i) and (ii)

x = 80

From (i) and (iii)

y = 2

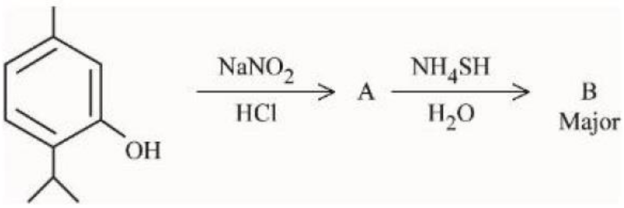
70. Which of the following compounds is an example of Freon?

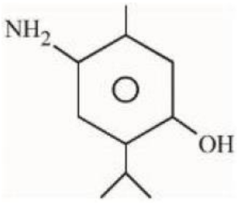
(1) C_2F_4 (2) C_2HF_3 (3) $C_2Cl_2F_2$ (4) $C_2H_2F_2$

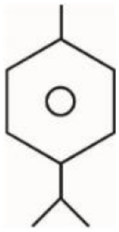
Sol. **3**

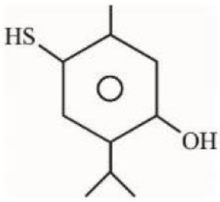
Freons are chlorofluoro carbon.

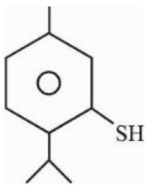
71. Compound ‘B’ is



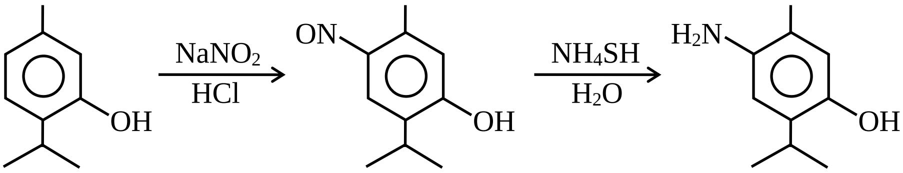
(1) 

(2) 

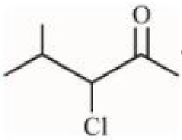
(3) 

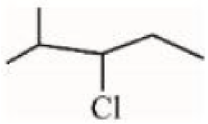
(4) 

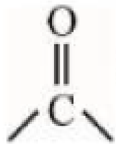
Sol. **1**



72. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: 

can be subjected to Wolff-Kishner reduction to give 

Reason R:Wolff–Kishner reduction is used to convert 

into 

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

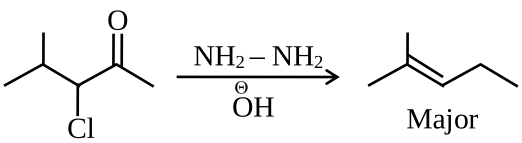
(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

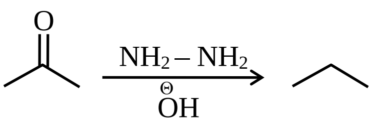
(4) A is false but R is true

Sol. **4**

Assertion (A)



Reason (R)



73. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ absorbs at lower wavelength of light with respect to $[CoCl(NH_3)_5(H_2O)]^{3+}$

Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true and R is the correct explanation of A

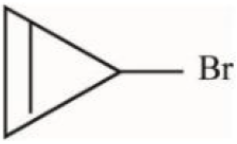
(4) A is false but R is true

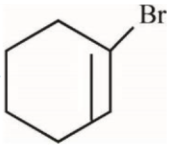
Sol. **4**

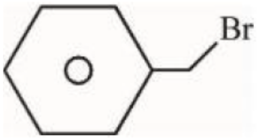
Since H_2O is strong field ligand compared to chloride and Co^{3+} ion is present.

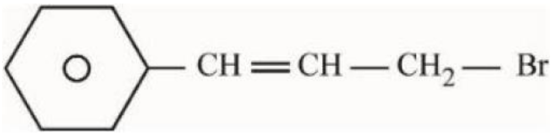
$\therefore$ CFSE is higher for $[Co(NH_3)5H_2O]^{+3}$, hence it will absorb at lower wavelength.

74. Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is :

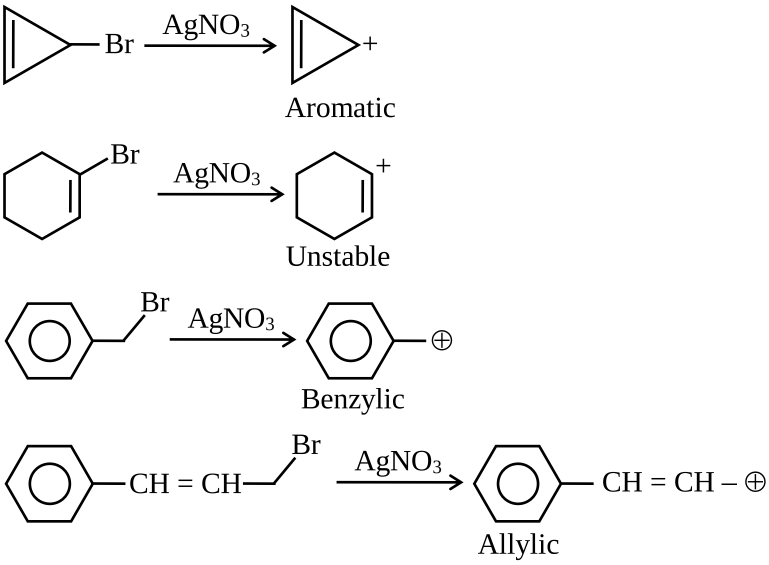
(1) 

(2) 

(3) 

(4) 

Sol. **2**



75. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Sol. **3**

$$H_2N - CH_2 - COOH \xrightarrow[\text{Red P}]{Cl_2} H_2N - \overset{*}{C}H(Cl) - COOH$$

76. Alkali metal from the following with least melting point is:

(1) K (2) Cs (3) Rb (4) Na

Sol. **2**

On moving down the group in alkali metals melting point decreases.

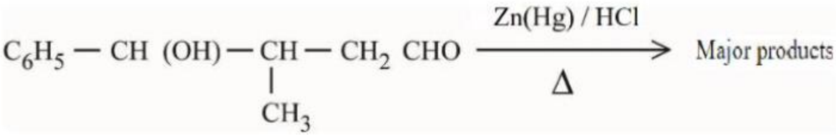
77. Which hydride among the following is less stable?

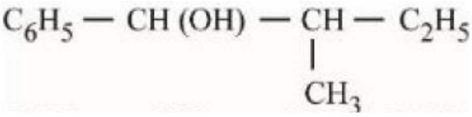
(1) HF (2) NH_3 (3) BeH_2 (4) LiH

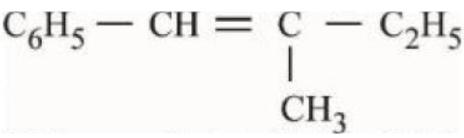
Sol. **3**

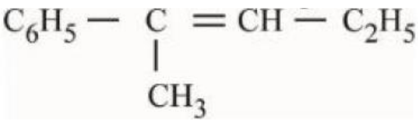
BeH_2 is hypovalent

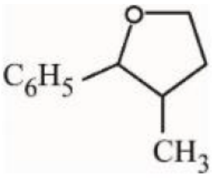
78. The major product formed in the following reaction is



A. 

B. 

C. 

D. 

Choose the correct answer from the options given below:

(1) C only (2) A only (3) B only (4) D only

Sol. **3**

$Ph - CH(OH) - CH(CH_3) - CH_2 - CHO \xrightarrow[\Delta]{Zn(Hg)/HCl} Ph - CH = C(CH_3) - CH_2 - CH_3$

79. One mole of P_4 reacts with 8 moles $SOCl_2$ to give 4 moles of A, x mole of SO_2 and 2 moles of B. A, B and x respectively are

(1) $POCl_3, S_2Cl_2$ and 4 (2) $POCl_3, S_2Cl_2$ and 2 (3) PCl_3, S_2Cl_2 and 4 (4) PCl_3, S_2Cl_2 and 2

Sol. **3**

$P_4 + 8\ SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$

80. What weight of glucose must be dissolved in 100 g of water to lower the vapour pressure by 0.20 mmHg? (Assume dilute solution is being formed)

Given : Vapour pressure of pure water is 54.2 mmHg at room temperature. Molar mass of glucose is $180\,g\,mol^{-1}$

(1) 2.59 g (2) 3.59 g (3) 3.69 g (4) 4.69 g

Sol. **3**

$$\frac{P^0 - P_s}{P^0} = \frac{n}{N} \text{ (for dilute solution)}$$

$$\frac{0.2}{54.2} = \frac{n \times 18}{100}$$

$$n = \frac{100}{271 \times 18}$$

$$w = \frac{100 \times 180}{271 \times 18}; \ w = 3.69 \text{ g}$$

SECTION - B

81. The total number of intensive properties from the following is__________ new line volume, Molar heat capacity, Molarity, E^{θ} cell, Gibbs free energy change, Molar mass, Mole

Sol. **4**

Extensive $\Rightarrow$ Mole, Volume, Gibbs free energy.

Intensive $\Rightarrow$ Molar mass, Molar heat capacity, Molarity, E^{θ} cell.

82. The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is __________ $\times 10^{-2}$ L .

Given: Molar volume of gas is 22.4 L at STP. Molar mass of magnesium is 24 g mol^{-1}

Sol. **224**

$Mg + 2HCl \rightarrow MgCl_2 + H_2 \uparrow$

w = 2.4 g

$N = \frac{2.4}{24} = 0.1 \text{mole}$

1 mole of gas at STP $\Rightarrow$ 22.4 lit.

$\therefore$ 0.1 mole of gas = 0.1 × 22.4

= 2.24 lit. = 224 × 10^{-2} litre

83. The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is

A. The catalyst is diffused over the surface of reactants.

B. Reactants are adsorbed on the surface of the catalyst.

C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.

D. It is a combination of intermediate compound formation theory and the old adsorption theory.

E. It explains the action of the catalyst as well as those of catalytic promoters and poisons.

Sol. **3**

B, C and D are correct.

(NCERT – Surface Chemistry)

84. The number of correct statements from the following is

A. For 1 s orbital, the probability density is maximum at the nucleus

B. For 2 s orbital, the probability density first increases to maximum and then decreases sharply to zero.

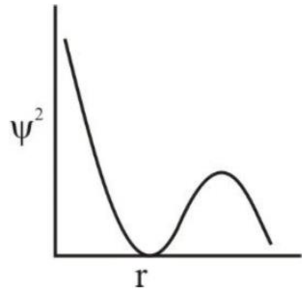
C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively

E. probability density of p-orbital is zero at the nucleus

Sol. **3**

A, D and E statements are correct.



For 2s orbital, the probability density first decreases and then increases.

At any distance from nucleus the probability density of finding electron is never zero and it always have some finite value.

85. The number of correct statements from the following is_____________

A. E_{cell} is an intensive parameter

B. A negative $E^{\ominus}$ means that the redox couple is a stronger reducing agent than the H^+/H_2 couple.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte.

Sol. **4**

Given statements A, B, C and D are correct.

86. $Mg(NO_3)_2 XH_2O$ and $Ba(NO_3)_2 YH_2O$, represent formula of the crystalline forms of nitrate salts. Sum of X and Y is_______________

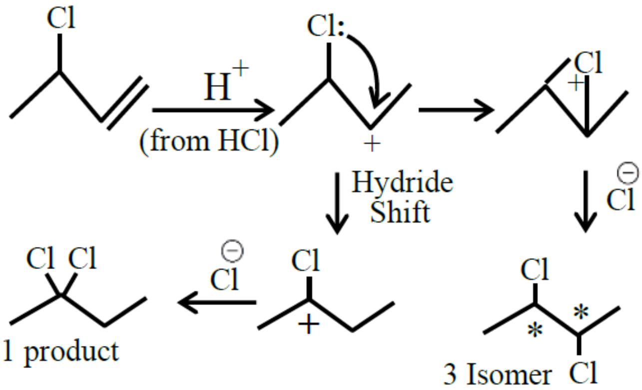
Sol. **6**

$Mg(NO_3)_2 \cdot 6H_2O$ is a hydrated salt whereas $Ba(NO_3)_2$ is a anhydrous salt.

$\therefore$ x + y = 6

87. The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formation is_____________

Sol. **4**



Total Possible Isomeric product = 1 + 3 = 4

88. 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium constant for $H_2(g)+I_2(g) \rightleftharpoons 2HI(g)$ is______________

Sol. **1**

	$H_{2(g)}$	$+ I_{2(g)}$	$\rightleftharpoons$ $2HI_{(g)}$
t = 0	4.5	4.5	–
t_{eq}	3	3	3

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{(3)^2}{3\times3} = \frac{9}{9} = 1$$

89. Number of compounds from the following which will not produce orange red precipitate with Benedict solution is _______________

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose

Sol. **2**

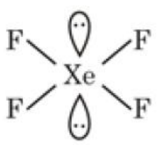
Benedict test :

Glucose	–	✓
maltose	–	✓
sucrose	–	×
ribose	–	✓
2-deoxyribose	–	✓
amylose	–	×
lactose	–	✓

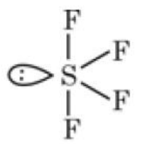
90. The maximum number of lone pairs of electrons on the central atom from the following species is__________

ClO_3^-, XeF_4, SF_4 and I_3^-

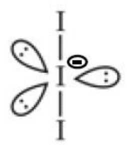
Sol. **3**



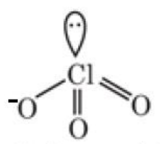
[2 lone pair]



[1 lone pair]



[3 lone pair]



[1 lone pair]