Mathematics Model Paper 2024

PART-I

I. Choose the correct answer. Answer all the questions. [*Answers are in bold*] [20 × 1 = 20]

1. The number of students who take both the subjects Mathematics and Chemistry is 70. This represents 10% of the enrollment in Mathematics and 14% of the enrollment in Chemistry. The number of students take at least one of these two subjects, is ....
   (*a*) 1120 **(*b*) 1130** (*c*) 1100 (*d*) insufficient data

2. If 8 and 2 are the roots of $x^2 + ax + c = 0$ and 3, 3 are the roots of $x^2 + dx + b = 0$, then the roots of the equation $x^2 + ax + b = 0$ are ....
   (*a*) 1, 2 (*b*) −1, 1 **(*c*) 9, 1** (*d*) −1, 2

3. If $\tan 40° = \lambda$ then $\dfrac{\tan 140° - \tan 130°}{1 + \tan 140° \tan 130°} =$ ....
   (*a*) $\dfrac{1-\lambda^2}{\lambda}$ (*b*) $\dfrac{1+\lambda^2}{\lambda}$ (*c*) $\dfrac{1+\lambda^2}{2\lambda}$ **(*d*) $\dfrac{1-\lambda^2}{2\lambda}$**

4. The value of $2 \sin A \cos^3 A - 2 \cos A \sin^3 A$ is ....
   (*a*) sin 4A (*b*) cos 4A **(*c*) $\frac{1}{2}\sin 4A$** (*d*) $\frac{1}{2}\cos 4A$

5. In a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$ then the triangle is .... triangle.
   (*a*) equilateral (*b*) isosceles **(*c*) right** (*d*) scalene

6. The number of ways in which a host lady invite 8 people for a party of 8 out of 12 people of whom two do not want to attend the party together is ....
   (*a*) $2 \times 11C_7 + 10C_8$ (*b*) $11C_7 + 10C_8$ **(*c*) $12C_8 - 10C_6$** (*d*) $10C_6 + 2!$

7. If a is the arithmetic mean and g is the geometric mean of two numbers then ....
   (*a*) $a \le g$ **(*b*) $a \ge g$** (*c*) $a = g$ (*d*) $a > g$

8. The number of rectangles that a chessboard has ....
   (*a*) 81 (*b*) 9^9 **(*c*) 1296** (*d*) 6561

9. The intercepts of the perpendicular bisector of the line segment joining (1, 2) and (3, 4) with coordinate axes are ....
   (*a*) 5, −5 **(*b*) 5, 5** (*c*) 5, 3 (*d*) 5, −4

10. The coordinates of the four vertices of a quadrilateral are (−2,4), (−1,2), (1,2) and (2,4) taken in order. The equation of the line passing through the vertex (−1,2) and dividing the quadrilateral in the equal areas is ....
    (*a*) $x + 1 = 0$ (*b*) $x + y = 1$ **(*c*) $x + y + 3 = 0$** (*d*) $x - y + 3 = 0$

11. The vectors $\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}$ are .... vectors.

(*a*) parallel (*b*) unit

(*c*) mutually perpendicular **(*d*) coplanar**

12. If $\left|\vec{a}+\vec{b}\right|=60$, $\left|\vec{a}-\vec{b}\right|=40$ and $\left|\vec{b}\right|=46$ then $\left|\vec{a}\right|$ is ....

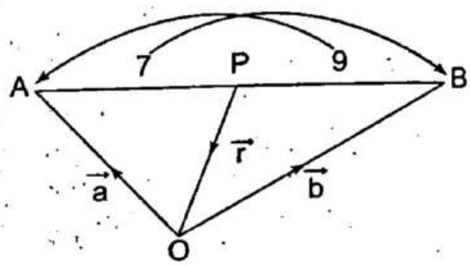
(*a*) 42 (*b*) 12 **(*c*) 22** (*d*) 32

13. Given $\vec{a}=2\vec{i}+\vec{j}-8\vec{k}$ and $\vec{b}=\vec{i}+3\vec{j}-4\vec{k}$ then $\left|\vec{a}+\vec{b}\right|=$ ....

(*a*) 13 (*b*) $\frac{13}{3}$ (*c*) $\frac{4}{13}$ (*d*) $\frac{3}{13}$

14. If $\vec{r}=\frac{9\vec{a}+7\vec{b}}{16}$, then the point P whose position vector $\vec{r}$ divides the line joining the points with position vectors $\vec{a}$ and $\vec{b}$ in the ratio ....

(*a*) 7 : 9 internally (*b*) 9 : 7 internally (*c*) 9 : 7 externally (*d*) 7 : 9 externally



15. If $f(x)=x+2$ then $f'(f(x))$ at $x=4$ is ....

(*a*) 8 **(*b*) 1** (*c*) 4 (*d*) 5

16. The derivative of $\left(x+\frac{1}{x}\right)^2$ w.r.to. x is ....

(*a*) $2x-\frac{2}{x^3}$ (*b*) $2x+\frac{2}{x^3}$ (*c*) $2\left(x+\frac{1}{x}\right)$ (*d*) 0

17. If $y=\frac{1}{a-z}$ then $\frac{dz}{dy}$ is ....

(*a*) $(a-z)^2$ (*b*) $-(z-a)^2$ (*c*) $(z+a)^2$ (*d*) $-(z+a)^2$

18. $\int \sin 7x \cos 5x dx=$ ....

(*a*) $\frac{1}{2}\left[\frac{\cos 12x}{2}+\frac{\cos 2x}{2}\right]+c$

(*b*) $-\frac{1}{2}\left[\frac{\cos 12x}{2}+\frac{\cos 2x}{2}\right]+c$

(*c*) $-\frac{1}{2}\left[\frac{\cos 6x}{6}+\cos x\right]+c$

(*d*) $-\frac{1}{2}\left[\frac{\sin 12x}{2}+\frac{\sin 2x}{2}\right]+c$

19. $\int \frac{1}{e^x}\, dx=$ ....

(*a*) $\log e^x + c$ (*b*) $x+c$ (*c*) $\frac{1}{e^x}+c$ **(*d*) $\frac{-1}{e^x}+c$**

20. A letter is taken at random from the letters of the word 'ASSISTANT' and another letter is taken at random from the letters of the word 'STATISTICS'. The probability that the selected letters are the same is ....

(*a*) $\frac{7}{45}$ (*b*) $\frac{17}{90}$ (*c*) $\frac{29}{90}$ **(*d*) $\frac{19}{90}$**

PART-II

II. Answer any seven questions. Question No. 30 is compulsory. [7 × 2 = 14]

21. For a set A, A × A contains 16 elements and two of its elements are (1, 3) and (0, 2). Find the elements of A.

Ans. $A \times A = 16$ elements $= 4 \times 4$

$\Rightarrow$ A has 4 elements

$\therefore A = \{0, 1, 2, 3\}$

22. Find the area of the triangle whose sides are 13 cm, 14 cm and 15 cm.

Ans. We know that $s = \frac{a+b+c}{2} \Rightarrow s = \frac{13+14+15}{2} = 21\,\text{cm}$

Area of a triangle is $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)} = 84$ sq. cm

23. If $\frac{1}{7!}+\frac{1}{9!}=\frac{x}{10!}$, find x.

Ans. Here $\frac{1}{7!}+\frac{1}{9!}=\frac{x}{10!}$

$$\Rightarrow \frac{1}{7!}+\frac{1}{9\times8\times7!} = \frac{x}{10\times9\times8\times7!}$$

$$\Rightarrow \frac{1}{7!}\left[1+\frac{1}{72}\right] = \frac{1}{7!}\left[\frac{x}{10\times9\times8}\right]$$

$$\Rightarrow \frac{73}{72} = \frac{x}{10\times9\times8}$$

$$\Rightarrow x = \frac{73}{72}\times10\times9\times8 = 730$$

24. Find $\sqrt[3]{1001}$ approximately (two decimal places)

Ans. $$\sqrt[3]{1001} = (1001)^{1/3} = (1000+1)^{1/3} = \left\{1000\left(1+\frac{1}{1000}\right)\right\}^{1/3} = (1000)^{1/3}\left[1+\frac{1}{10^3}\right]^{1/3}$$

$$= 10\left\{1+\frac{1}{3}\left(\frac{1}{10^3}\right)+\frac{\frac{1}{3}\left(-2/3\right)}{2}\left(\frac{1}{10^3}\right)^2 \ldots\right\}$$

$$= 10\left\{1+\frac{1}{3000}-\frac{2}{18000000}\ldots\right\} = 10\left[1+\frac{0.333}{1000}\ldots\right] = 10[1+0.000333..] = 10+0.0033$$

$$= 10.0033$$

33. Solve the following equation for which solutions lies in the interval $0° \leq \theta \leq 360°$.
$\sin^4 x = \sin^2 x$

Ans.

$$\sin^2 x - \sin^4 x = 0$$

$$\sin^2 x\,(1 - \sin^2 x) = 0$$

$$\sin^2 x\,(\cos^2 x) = 0$$

$$\left[\frac{1}{2}(2\sin x\cos x)\right]^2 = 0$$

$\Rightarrow$ $$(\sin 2x)^2 = 0$$

$\Rightarrow$ $$\sin 2x = 0 = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{\pi}{2}, 2\pi$$

34. Find n if $n-1\ P_3 : nP_4 = 1 : 9$

Ans. Here $^{n-1}P_3 : {}^nP_4 = 1 : 9$

$$\therefore \frac{(n-1)!}{(n-4)!} : \frac{n!}{(n-4)!} = 1 : 9$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9} \Rightarrow n = 9$$

35. A family is using Liquefied petroleum gas (LPG) of weight 14.2 kg for consumption. (Full weight 29.5 kg includes the empty cylinders tare weight of 15.3 kg.). If it is used with constant rate then it lasts for 24 days. Then the new cylinder is replaced (*i*) Find the equation relating the quantity of gas in the cylinder to the days. (*ii*) Draw the graph for first 96 days.

Ans. Since the usage is in constant rate and it is the slope $m = \frac{14.2}{24}$

$\therefore y = 14.2 - mx$, (*i.e*) $y = 14.2 - \frac{14.2}{24}x$

(*i.e*) $y = 14.2 - \frac{142}{240}x \qquad 0 \leq x \leq 24$;

$y = 14.2 - \frac{71}{120}x$ which is the equation relating the quantity.

$y - f(x)$ is a periodic function with period 24. (*i.e.*) $f(x) = f(x + 24)$

36. If $\cos 2\theta = 0$, determine $\begin{bmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^2$

Ans. Given $\cos 2\theta = 0$

$\Rightarrow$ $$2\theta = \pi/2 \Rightarrow \theta = \pi/4$$

$$\therefore \quad \cos\theta = \cos \pi/4 = 1/\sqrt{2}$$

and $$\sin\theta = \sin \pi/4 = 1/\sqrt{2}$$

$$\text{Let } \Delta = \begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{vmatrix}$$

$$= 0\,() - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} - 0\right) + \frac{1}{\sqrt{2}}\left(0 - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)$$

$$= -\frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{-1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{-2}{2\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}}$$

$$\therefore \quad \Delta^2 = \left(\frac{-1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

37. Show that the points (4, –3, 1), (2, –4, 5) and (1, –1, 0) form a right angled triangle.

Ans. Trivially they form a triangle. It is enough to prove one angle is $\frac{\pi}{2}$. So find the sides of the triangle.

Let O be the point of reference and A, B, C be (4, –3, 1), (2, –4, 5) and (1, –1, 0) respectively.

$\overrightarrow{OA} = 4\hat{i} - 3\hat{j} + \hat{k}$, $\overrightarrow{OB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, $\overrightarrow{OC} = \hat{i} - \hat{j}$

Now, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -2\hat{i} - \hat{j} + 4\hat{k}$

Similarly, $\overrightarrow{BC} = -\hat{i} + 3\hat{j} - 5\hat{k}$; $\overrightarrow{CA} = 3\hat{i} - 2\hat{j} + \hat{k}$

Clearly $\overrightarrow{AB} \cdot \overrightarrow{CA} = 0$

Thus one angle is $\frac{\pi}{2}$. Hence they form a right angled triangle.

38. Compute $\lim_{x \to 0}\left[\frac{x^2+x}{x} + 4x^3 + 3\right]$

Ans. $$\lim_{x\to 0}\left[\frac{x^2+x}{x} + 4x^3 + 3\right] = \lim_{x\to 0}\left(\frac{x^2+x}{x}\right) + \lim_{x\to 0}(4x^3+3)$$

$$= \lim_{x\to 0}(x+1) + \lim_{x\to 0}(4x^3+3)$$

$$= (0+1) + (0+3)$$

$$= 4$$

39. If for two events A and B, $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$ and $A \cup B = S$ (Sample space) find the conditional probability P(A/B)

Ans. Given $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = 1$

$\Rightarrow$ Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$$\Rightarrow \quad 1 = \frac{3}{4} + \frac{2}{5} - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{4} + \frac{2}{5} - 1 = \frac{15 + 8 - 20}{20}$$

$$P(A \cap B) = 3/20$$

So $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/20}{2/5} = \frac{3}{20} \times \frac{5}{2} = \frac{3}{8}$

40. Evaluate $\int \frac{(x-1)^2}{x^3 + x} dx$

Ans. $\int \frac{(x-1)^2}{x^3 + x} dx = \int \frac{x^2 + 1 - 2x}{x(x^2 + 1)} dx$

$$= \int \left(\frac{(x^2 + 1)}{x(x^2 + 1)} - \frac{2x}{x(x^2 + 1)} \right) dx$$

$$= \int \frac{1}{x} dx - 2 \int \frac{1}{1 + x^2} dx$$

$$= \log |x| - 2 \tan^{-1} x + c.$$

PART-IV

IV. Answer all the questions. **[7 × 5 = 35]**

41. (a) Find the range of the function $\frac{1}{2\cos x - 1}$.

Ans. The range of $\cos x$ is -1 to 1

$-1 < \cos x < 1$

(× by 2) $-2 < 2\cos x < 2$

adding -1 throughout

$-2 - 1 < 2\cos x - 1 < 2 - 1$

(*i.e.*,) $-3 < 2\cos x - 1 < 1$

so $1 < \frac{1}{2\cos x - 1} < \frac{-1}{3}$

The range is outside $\frac{-1}{3}$ and 1

i.e., range is $(-\infty, \frac{-1}{3}] \cup [1, \infty)$

[OR]

(*b*) **In any triangle ABC prove that $a^2 = (b+c)^2 \sin^2 \frac{A}{2} + (b-c)^2 \cos^2 \frac{A}{2}$**

Ans. $$\begin{aligned} \text{RHS} &= (b+c)^2 \sin^2 \frac{A}{2} + (b-c)^2 \cos^2 \frac{A}{2} \\ &= (b^2+c^2+2bc) \sin^2 \frac{A}{2} + (b^2+c^2-2bc)\cos^2 \frac{A}{2} \\ &= (b^2+c^2)\left[\sin^2 \frac{A}{2} + \cos^2 \frac{A}{2}\right] + 2bc\left[\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2}\right] \\ &= b^2 + c^2 - 2bc \cos A = a^2 = \text{LHS} \end{aligned}$$

42. (*a*) **Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$**

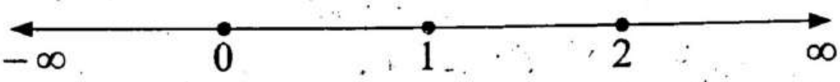
Ans. $\frac{x^3(x-1)}{(x-2)} > 0$

Now we have to find the signs of

x^3, $x-1$ and $x-2$ as follows

$x^3 = 0 \Rightarrow x = 0$; $x - 1 = 0 \Rightarrow x = 1$; $x - 2 = 0 \Rightarrow x = 2$.

Plotting the points in a number line and finding intervals



Intervals	x^3	$x-1$	$x^3(x-1)$	$x-2$	$\frac{x^3(x-1)}{x-2}$
$(-\infty, 0)$	−	−	+	−	−ve
$(0, 1)$	+	−	−	−	+ve
$(1, 2)$	+	+	+	−	−ve
$(2, \infty)$	+	+	+	+	+ve

So the solution set $= (0, 1) \cup (2, \infty)$

[OR]

(*b*) **Resolve $\frac{x}{(x^2+1)(x-1)(x+2)}$ into partial fractions**

Ans. Let $\frac{x}{(x^2+1)(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$

(*i.e.,*) $\frac{x}{(x^2+1)(x-1)(x+2)} = \frac{A(x+2)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+2)}{(x-1)(x+2)(x^2+1)}$

Equating numerator on both sides

$x = A(x+2)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+2)$

This equations is true for any value of x to find A, B, C and D.

put $x = 1$

$1 = A(3)(2) + B(0) + (0)$

$6A = 1 \Rightarrow A = 1/6$

put $x = -2$

$-2 = +0 + B(-3)(5) + 0$

$\Rightarrow \quad -15B = -2 \Rightarrow B = 2/15$

put $x = 0$

$\Rightarrow \quad 2A - B - 2D = 0$

(*i.e.,*) $\frac{2}{6} - \frac{2}{15} - 2D = 0$

$\Rightarrow \quad 2D = \frac{2}{6} - \frac{2}{15} = \frac{10-4}{30} = \frac{6}{30} = \frac{1}{5}$

$\Rightarrow \quad D = \frac{1}{5 \times 2} = \frac{1}{10}$

$D = \frac{1}{10}$

Equating co-efficient of x^3

$A + B + C = 0$

$\frac{1}{6} + \frac{2}{15} + C = 0 \Rightarrow C = \frac{-1}{6} - \frac{2}{15} = \frac{-5-4}{30}$

$C = \frac{-9}{30} = \frac{-3}{10}$

$$\therefore \frac{x}{(x^2+1)(x-1)(x+2)} = \frac{1}{6(x-1)} + \frac{2}{15(x+2)} + \frac{\frac{-3}{10}x + \frac{1}{10}}{x^2+1}$$

$$= \frac{1}{6(x-1)} + \frac{2}{15(x+2)} + \frac{1-3x}{10(x^2+1)}$$

43. (*a*) 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife has also 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of mens relative and 3 of the wifes relatives?

Ans.

Husband		Wife	
L	G	L	G
(4)	(3)	(3)	(4)

We need 3 ladies and 3 gentlemen for the party which consist of 3 Husbands relative and 3 wifes relative.

This can be done as follows

Husband			Wife	
(4) L	G (3)		(3) L	G (4)
3	0	→	0	3
2	1	→	1	2
1	2	→	2	1
0	3	→	3	0

The possible ways are

$$\binom{4}{3}\binom{3}{0}\binom{3}{0}\binom{4}{3}+\binom{4}{2}\binom{3}{1}\binom{3}{1}\binom{4}{2}+\binom{4}{1}\binom{3}{2}\binom{3}{2}\binom{4}{1}+\binom{4}{0}\binom{3}{3}\binom{3}{3}\binom{4}{0}$$

$$\left[\binom{n}{r}={}^{n}C_r\right]$$

${}^4C_0={}^4C_4=1$; ${}^3C_0={}^3C_3=1$

${}^4C_1={}^4C_3=4$; ${}^3C_1={}^3C_2=3$

${}^4C_2=\dfrac{4\times 3}{2\times 1}=6$

$(4)(1)(1)(4)+(6)(3)(3)(6)+(4)(3)(3)(4)+(1)(1)(1)(1)$

$=16+324+144+1=485$ ways

[OR]

(*b*) Show that the points (1, 3), (2, 1) and $\left(\frac{1}{2},4\right)$ are collinear, by using (*i*) Concept of slope (*ii*) Using a straight line and (*iii*) Any other method.

Ans. Let the given points be A (1, 3), B (2, 1), and C $\left(\frac{1}{2},4\right)$

(*i*) Slope of AB $=\dfrac{1-3}{2-1}=\dfrac{-2}{1}=-2=m_1$

Slope of BC $=\dfrac{4-1}{1/2-2}=\dfrac{3}{-3/2}=-2=m_2$

Slope of AB = Slope of BC ⇒ AB parallel to BC but B is a common point

⇒ The points A, B, C are collinear.

(*ii*) Equation of the line passing through A and B is $\dfrac{y-1}{3-1}=\dfrac{x-2}{1-2}\Rightarrow\dfrac{y-1}{2}=\dfrac{x-2}{-1}$

$1-y=2x-4$

$2x+y=5$ .... (1)

Substituting $C\left(\frac{1}{2},4\right)$ in (1),

we get LHS $=2\left(\frac{1}{2}\right)+4=1+4=5=$ RHS

C is a point on AB

⇒ The points A, B, C lie on a line.

⇒ The points A, B, C are collinear.

(*iii*) Area of $\Delta ABC=\frac{1}{2}\left(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right)$

$=\frac{1}{2}\left\{1(1-4)+2(4-3)+\frac{1}{2}(3-1)\right\}=\frac{1}{2}(-3+2+1)=0$

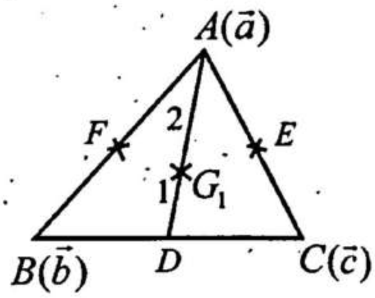
⇒ The points A, B, C are collinear.

44. (a) Prove by vector Method's that the Medians of a triangle are concurrent.

Theorem: The medians of a triangle are concurrent.

Proof: Let ABC be a triangle and let D, E, F be the mid points of its sides BC, CA and AB respectively. We have to prove that the medians AD, BE, CF are concurrent.

Let O be the origin and $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B, and C respectively.



The position vectors of D, E and F are respectively,

$$\frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{a}}{2}, \frac{\vec{a}+\vec{b}}{2}.$$

Let G_1 be the point on AD dividing it internally in the ratio 2 : 1.

Therefore, position vector of $G_1 = \frac{1\overline{OA} + 2\overline{OD}}{1+2}$

$$\overrightarrow{OG_1} = \frac{1\vec{a} + 2\left(\frac{\vec{b}+\vec{c}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}, \quad (1)$$

Let G_2 be the point on BE dividing it internally in the ratio 2 : 1

Therefore $\overrightarrow{OG_2} = \frac{1\overline{OB}+2\overline{OE}}{1+2}$

$$\overrightarrow{OG_2} = \frac{1\vec{b} + 2\left(\frac{\vec{c}+\vec{a}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad (2)$$

Similarly if G_3 divides CF in the ratio 2 : 1 then

$$\overrightarrow{OG_3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad (3)$$

From (1), (2) and (3) we find that the position vectors of the three points G_1, G_2, G_3 are one and the same. Hence they are not different points. Let the common point be G.

Therefore the three medians are concurrent and the point of concurrence is G.. **[OR]**

(b) If $y = Ae^{6x} + Be^{-x}$ prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$

Ans. $y = Ae^{6x} + Be^{-x}$ .... (1)

$y_1 = \frac{dy}{dx} = Ae^{6x}(6) + Be^{-x}(-1)$

$= 6Ae^{6x} - Be^{-x}$ .... (2)

$y_2 = \frac{d^2y}{dx^2} = 6Ae^{6x}(6) - Be^{-x}(-1)$

$= 36Ae^{6x} + Be^{-x}$ .... (3)

eliminating A and B from (1), (2) and (3) we get

$$\begin{vmatrix} y & A & B \\ y_1 & 6A & -B \\ y_2 & 36A & B \end{vmatrix} = 0$$

(i.e.,) $\begin{vmatrix} y & 1 & 1 \\ y_1 & 6 & -1 \\ y_2 & 36 & 1 \end{vmatrix} = 0$

$y(6+36) - y_1(1-36) + y_2(-1-6) = 0$

$42y + 35y_1 - 7y_2 = 0$

$(\div \text{ by } -7)\ y_2 - 5y_1 - 6y = 0$

(i.e.,) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$

45. (*a*) **If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer. [Hint: write $a^n = (a - b + b)^n$ and expand]**

Ans. $a = a - b + b$

So, $a^n = [a - b + b]^n = [(a - b) + b]^n$

$= {}^nC_0 (a-b)^n + {}^nC_1 (a-b)^{n-1} b^1 + {}^nC_2 (a-b)^{n-2} b^2 + \ldots\ldots + {}^nC_{n-1} (a-b)\, b^{n-1} + {}^nC_n (b^n)$

$\Rightarrow a^n - b^n = (a-b)^n + {}^nC_1 (a-b)^{n-1} b + {}^nC_2 (a-b)^{n-2} b^2 + \ldots\ldots + {}^nC_{n-1} (a-b)\, b^{n-1}$

$= (a-b)\,[(a-b)^{n-1} + {}^nC_1 (a-b)^{n-2} b + {}^nC_2 (a-b)^{n-3} b^2 + \ldots\ldots + {}^nC_{n-1}\, b^{n-1}]$

$= (a-b)$ [an integer]

$\Rightarrow a^n - b^n$ is divisible by $(a - b)$ **[OR]**

(*b*) **Verify the property A (B + C) = AB + AC when the matrices A, B and C are given by**

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}$$

Ans. Given $A = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix};\ B = \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{pmatrix};\ C = \begin{pmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}$

Now $B + C = \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 8 \\ 1 & 1 \\ 5 & 1 \end{pmatrix}$

$$\text{LHS} = A(B + C) = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 7 & 8 \\ 1 & 1 \\ 5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 + 0 - 15 & 16 + 0 - 3 \\ 7 + 4 + 25 & 8 + 4 + 5 \end{pmatrix} = \begin{pmatrix} -1 & 13 \\ 36 & 17 \end{pmatrix} \quad (1)$$

$$AB = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} -6 & -4 \\ 19 & 11 \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 17 \\ 17 & 6 \end{pmatrix}$$

$$\text{RHS} = AB + AC = \begin{pmatrix} -6 & -4 \\ 19 & 11 \end{pmatrix} + \begin{pmatrix} 5 & 17 \\ 17 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 13 \\ 36 & 17 \end{pmatrix} \quad (2)$$

$(1) = (2) \Rightarrow A(B + C) = AB + AC$

46. (*a*) Evaluate: $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1}$.

Ans. Here the expression assumes the form ∞ to − ∞ as $x \to \infty$. So, we first reduce it to the rational form $\frac{f(x)}{g(x)}$

$$\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1} = \lim_{x\to\infty} \frac{\left\{\sqrt{x^2+x+1} - \sqrt{x^2+1}\right\}}{\left\{\sqrt{x^2+x+1} + \sqrt{x^2+1}\right\}}\left\{\sqrt{x^2+x+1} + \sqrt{x^2+1}\right\}$$

$$= \lim_{x\to\infty} \frac{x^2+x+1-x^2-1}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x+1}+\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{1}{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1}{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

[OR]

(*b*) Evaluate $\lim_{x\to\infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{1/x}\right]$

Ans. Let $y = \frac{1}{x}$ as $x \to \infty, y \to 0$

$$\lim_{x\to\infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{1/x}\right] = \lim_{y\to 0}\frac{1}{y}(3^y + 1 - \cos y - e^y)$$

$$= \lim_{y\to 0} \frac{3^y-1}{y} + \frac{1-\cos y}{y} - \frac{e^y+1}{y}$$

$$= \lim_{y\to 0} \frac{3^y-1}{y} + \frac{2\sin^2 y/2}{y} - \frac{(e^y-1)}{y}$$

$$= \lim_{y\to 0}\left(\frac{3^y-1}{y}\right) + \frac{\sin^2\frac{y}{2}}{y/2} - \frac{(e^y-1)}{y}$$

$$= \lim_{y\to 0}\left(\frac{3^y-1}{y}\right) + \frac{\sin\frac{y}{2}}{y/2} \times \sin\left(y/2\right) - \left(\frac{e^y-1}{y}\right)$$

$$= \lim_{y\to 0}\left(\frac{3^y-1}{y}\right) + \lim_{y\to 0}\left(\frac{\sin y/2}{y/2}\right)\lim_{y\to 0}\left(\sin y/2\right) - \lim_{y\to 0}\frac{e^y-1}{y}$$

$$= \log 3 + (1)(0) - (1) = \log 3 - 1 = -1 + \log 3 = \log 3 - 1$$

47. (*a*) Evaluate : $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$.

Ans. Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

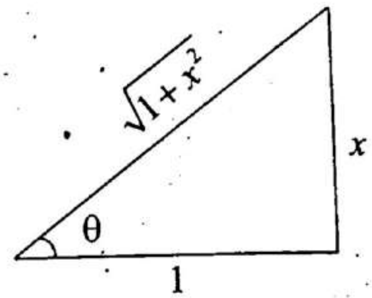
Putting $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

Therefore, $I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\, d\theta$

$= \int 2\theta \sec^2\theta\, d\theta$

$= 2\int (\theta)(\sec^2\theta\, d\theta)$



Applying integration by parts

$I = 2\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$\tan\theta = x$

$= 2(\theta\tan\theta - \log|\sec\theta|) + c$

$\sec\theta = \sqrt{1+x^2}$

$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = 2x\tan^{-1}x - 2\log\left|\sqrt{1+x^2}\right| + c$

[OR]

(*b*) Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

Ans. Given $P(X) = 3/4, P(X') = 1 - 3/4 = 1/4$

$P(Y) = 4/5, P(Y') = 1 - 4/5 = 1/5$

$P(Z) = \frac{2}{3}, P(Z') = 1 - \frac{2}{3} = \frac{1}{3}$

$P(X \cap Y \cap Z') + P(X \cap Y' \cap Z) + P(X' \cap Y \cap Z)$

$= \frac{3}{4}\times\frac{4}{5}\times\frac{1}{3} + \frac{3}{4}\times\frac{1}{5}\times\frac{2}{3} + \frac{1}{4}\times\frac{4}{5}\times\frac{2}{3}$

$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}$

SAMPLE PAPER – 2

(SOLVED)

Time: 2.30 Hours MATHEMATICS **Maximum Marks: 90**

PART-I

I. Choose the correct answer. Answer all the questions. [*Answers are in bold*] [20 × 1 = 20]

1. The range of the function $\frac{1}{1-2\sin x}$ is ....

   (*a*) $(-\infty,-1)\cup(\frac{1}{3},\infty)$ (*b*) $(-1, -]$ (*c*) $[-1, \frac{1}{3}]$ **(*d*) $(-\infty,-1]\cup[\frac{1}{3},\infty)$**

2. The value of $\log_{\sqrt{2}} 512$ is ....

   (*a*) 16 **(*b*) 18** (*c*) 9 (*d*) 12

3. If a and b are the roots of the equation $x^2 - kx + 16 = 0$ and satisfy $a^2 + b^2 = 32$ then the value of k is ....

   (*a*) 10 (*b*) –8 **(*c*) –8, 8** (*d*) 6

4. The value of $\log_9 27$ is ....

   (*a*) $\frac{2}{3}$ **(*b*) $\frac{3}{2}$** (*c*) $\frac{3}{4}$ (*d*) $\frac{4}{3}$

5. The value of $\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta}$ = ...

   (*a*) $\tan 3\theta$ **(*b*) $\tan 6\theta$** (*c*) $\cot 3\theta$ (*d*) $\cot 6\theta$

6. In 3 fingers the number of ways 4 rings can be worn in .... ways.

   (*a*) $4^3 - 1$ (*b*) 3^4 (*c*) 68 **(*d*) 64**

7. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is ....

   (*a*) 11 **(*b*) 12** (*c*) 10 (*d*) 6

8. The H.M of two positive number whose AM and G.M. are 16, 8 respectively is ....

   (*a*) 10 (*b*) 6 (*c*) 5 **(*d*) 4**

9. The co-efficient of the term independent of x in the expansion of $\left(2x+\frac{1}{3x}\right)^6$ is ....

   (*a*) $\frac{160}{27}$ (*b*) $\frac{160}{37}$ (*c*) $\frac{80}{3}$ (*d*) $\frac{80}{9}$

10. The value of $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}^2$ is ....

    (*a*) abc (*b*) $-abc$ (*c*) 0 **(*d*) $a^2b^2c^2$**

11. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is ....

(*a*) Δ (*b*) $k\Delta$ (*c*) $3k\Delta$ **(*d*) $k^3\Delta$**

12. A vector makes equal angle with the positive direction of the co-ordinate axes then each angle is equal to ....

(*a*) $\cos^{-1}\left(\frac{1}{3}\right)$ (*b*) $\cos^{-1}\left(\frac{2}{3}\right)$ **(*c*) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$** (*d*) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

13. If the centroids of ΔABC and A'B'C' are respectively G and G' then $\overline{AA'}+\overline{BB'}+\overline{CC'}$ ....

(*a*) $\overline{GG'}$ **(*b*) $3\overline{GG'}$** (*c*) $2\overline{GG'}$ (*d*) 0

14. If $f(x) = \begin{cases} kx^2 & \text{for } x \le 2 \\ 3 & \text{for } x > 2 \end{cases}$ is continuous at $x = 2$ then the value of k is ....

(*a*) $\frac{3}{4}$ (*b*) 0 **(*c*) 1** (*d*) $\frac{4}{3}$

15. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$ then $\frac{dy}{dx} =$ ....

(*a*) $\frac{y}{x}$ (*b*) $\frac{-y}{x}$ **(*c*) $-\frac{x}{y}$** (*d*) $\frac{x}{y}$

16. $\lim_{x \to \infty} \left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$ is ....

(*a*) e^4 (*b*) e^2 (*c*) e^3 (*d*) 1

17. $\int \frac{e^x\left(x^2 \tan^{-1} x + \tan^{-1} x + 1\right)}{x^2+1} dx$ is ....

(*a*) $e^x \tan^{-1}(x+1) + c$ (*b*) $\tan^{-1}(e^x) + c$

(*c*) $e^x \frac{\left(\tan^{-1} x\right)^2}{2} + c$ **(*d*) $e^x \tan^{-1} x + c$**

18. $\int \frac{\sec x}{\sqrt{\cos 2x}} dx =$ ....

(*a*) $\tan^{-1}(\sin x) + c$ (*b*) $2\sin^{-1}(\tan x) + c$

(*c*) $\tan^{-1}(\cos x) + c$ **(*d*) $\sin^{-1}(\tan x) + c$**

19. $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}} dx =$ ....

(*a*) $x + c$ **(*b*) $\frac{x^3}{3} + c$** (*c*) $\frac{3}{x^3} + c$ (*d*) $\frac{1}{x^2} + c$

20. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$ then $P(B) =$ ....

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{2}$

PART-II

II. Answer any seven questions. Question No. 30 is compulsory. **[7 × 2 = 14]**

21. Find x such that $-\pi \le x \le \pi$ and $\cos 2x = \sin x$

Ans. We have $\cos 2x = \sin x$ which gives

$$2\sin^2 x + \sin x - 1 = 0$$

The roots of the equation are $\sin x = \frac{-1 \pm 3}{4} = -1$ (or) $\frac{1}{2}$

Now, $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

Also $\sin x = -1 \Rightarrow x = -\frac{\pi}{2}$

Thus, $x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

22. If $^{(n-1)}P_3 : {^nP_4} = 1 : 10$, find n.

Ans. Given $\frac{(n-1)P_3}{nP_4} = \frac{1}{10}$

$$\Rightarrow \frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{10} \;;\; (i.e)\ \frac{1}{n} = \frac{1}{10} \Rightarrow n = 10$$

23. Find the 18th and 25th terms of the sequence defined by

$$a_n = \begin{cases} n(n+2), \text{if } n \text{ is even natural number} \\ \frac{4n}{n^2+1}, \text{if } n \text{ is odd natural number} \end{cases}$$

Ans. When $n = 18$ (even)

$a_n = n(n+2) = 18(18+2) = 18(20) = 360$

When $n = 25$ (odd)

$$a_n = \frac{4n}{n^2+1} = \frac{4(25)}{(25)^2+1} = \frac{100}{625+1} = \frac{100}{626} = \frac{50}{313}$$

24. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

Ans. Slope of I line $= m_1 = -\left(\frac{3}{2}\right) = \frac{-3}{2}$

Slope of II line $= m_2 = -\left(\frac{12}{8}\right) = \frac{-3}{2}$

Here $m_1 = m_2 \Rightarrow$ the two lines are parallel.

25. Prove that $\begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix} = 0$

Ans.

$$\text{LHS} = \begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ x & y & z \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$= 0 (\because R_1 = R_2) + 2 \begin{vmatrix} a & b & c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$= 0 + 2(0) (\therefore R_1 = R_3)$$

$$= 0 = \text{RHS}$$

26. Find the value of λ for which the vectors $\vec{a} = 2\vec{i} + \lambda\vec{j} + \vec{k}$ **and** $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ **are perpendicular.**

Ans. When $\vec{a}$ and $\vec{b}$ are $\perp^r$ then $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \perp^r \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$(2)(1) + (\lambda)(-2) + (1)(3) = 0 \Rightarrow \lambda = 5/2$$

27. If $y = \dfrac{\tan x}{x}$ **find** $\dfrac{dy}{dx}$

Ans.

$$\text{Now } y = \frac{u}{v} \Rightarrow y' = \frac{vu' - uv'}{v^2}$$

$$u = \tan x \Rightarrow u' = \sec^2 x$$

$$v = x \Rightarrow v' = 1$$

$$\text{Now } y = \frac{u}{v} \Rightarrow y' = \frac{vu' - uv'}{v^2} = \frac{x\sec^2 x - \tan x(1)}{x^2}$$

$$= \frac{x\sec^2 x - \tan x}{x^2}$$

28. Evaluate $\int \sqrt{25x^2 - 9}\, dx$

Ans.

$$\text{Let } I = \int \sqrt{25x^2 - 9}\, dx$$

$$= \int \sqrt{(5x)^2 - 3^2 dx}$$

$$= \frac{1}{5}\left[\frac{5x}{2}\sqrt{(5x)^2 - 3^2} - \frac{3^2}{2}\log\left|5x + \sqrt{(5x)^2 - 3^2}\right|\right] + c$$

Therefore,

$$I = \frac{1}{5}\left[\frac{5x}{2}\sqrt{25x^2 - 9} - \frac{9}{2}\log\left|5x + \sqrt{25x^2 - 9}\right|\right] + c$$

29. If A and B are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$.

Ans.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A).P(B) \text{(since A and B are independent)}$$

$$\text{That is, } 0.9 = 0.4 + P(B) - (0.4)\,P(B)$$

$$0.9 - 0.4 = (1 - 0.4)\,P(B)$$

Therefore, $P(B) = \frac{5}{6}$

30. A rope of length 12 m is given. Find the largest area of the triangle formed by this rope and find the dimensions of the triangle so formed.

Ans. The largest triangle will be an equilateral triangle

$$\therefore \quad \text{side of the triangle} = \frac{12}{3} = 4\text{ m} = a$$

$$\text{Area of the triangle} = \frac{a^2\sqrt{3}}{4} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3} \text{ sq.m}$$

PART-III

III. Answer any seven questions. Question No. 40 is compulsory. **[7 × 3 = 21]**

31. Let $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it

(*i*) reflexive (*ii*) symmetric (*iii*) transitive (*iv*) equivalence

Ans. (*i*) (c, c) (*ii*) (c, a)

(*iii*) nothing (*iv*) (c, c) and (c, a)

32. Solve $2|x + 1| - 6 \leq 7$ and graph the solution set in a number line.

Ans. $2|x + 1| - 6 \leq 7$

$\Rightarrow \quad 2|x + 1| \leq 7 + 6 \; (= 13)$

$\Rightarrow \quad |x + 1| \leq \frac{13}{2}$

$\Rightarrow \quad x + 1 > \frac{-13}{2} \quad \text{(or)} \quad x + 1 < \frac{13}{2}$

$x + 1 > \frac{-13}{2}$ | $x + 1 < \frac{13}{2}$

$\Rightarrow \quad x > \frac{-13}{2} - 1 \left(= \frac{-15}{2}\right) \ldots. (1)$ | $\Rightarrow x < \frac{13}{2} - 1 \left(= \frac{11}{2}\right) \quad \ldots. (2)$

From (1) and (2) $\frac{-15}{2} \leq x \leq \frac{11}{2}$

33. If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in this list before the first word starting with B?

Ans. The required number of strings is the total number of strings starting with A and using the letters A, A, B, H, K, R, S $= \frac{7!}{2!} = 2520$

34. Find the sum up to n terms of the series: $1+\frac{6}{7}+\frac{11}{49}+\frac{16}{343}+\ldots$

Ans. Here $a = 1, d = 5$ and $r=\frac{1}{7}$

$$S_n=\frac{a-(a+(n-1)d)r^n}{1-r}+dr\left(\frac{1-r^{n-1}}{(1-r)^2}\right)$$

$$=\frac{1-(1+5(n-1))\left(\frac{1}{7}\right)^n}{1-\frac{1}{7}}+5\times\frac{1}{7}\left(\frac{\left(1-\frac{1}{7}\right)^{n-1}}{\left(1-\frac{1}{7}\right)^2}\right)$$

$$=\frac{1-\frac{5n-4}{7^n}}{\frac{6}{7}}+\frac{\frac{5}{7}\left(7^{n-1}-1\right)}{7^{n-1}\left(\frac{6}{7}\right)^2}=\frac{7^n-5n+4}{7^{n-1}6}+\frac{5\left(7^{n-1}-1\right)}{7^{n-2}\,36}$$

35. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of 45° with positive the x-axis.

Ans. Let p be the length of the perpendicular drawn from the origin to the required line.

The perpendicular makes 45° with the x-axis.

The equation of the required line is of the form,

$$x\cos\alpha+y\sin\alpha=p$$

$$\Rightarrow x\cos 45^\circ+y\sin 45^\circ=p$$

$$\Rightarrow x+y=\sqrt{2}\,p$$

This equation cuts the coordinate axes at A $\left(\sqrt{2}p,0\right)$ and B $\left(0,\sqrt{2}p\right)$.

Area of the ΔOAB is $\frac{1}{2}\times\sqrt{2}p\times\sqrt{2}p=36 \Rightarrow p=6$ ($\because$ p is positive)

Therefore the equation of the required line is $x+y=6\sqrt{2}$

36. If $A^T=\begin{pmatrix}4&5\\-1&0\\2&3\end{pmatrix}$ and $B=\begin{pmatrix}2&-1&1\\7&5&-2\end{pmatrix}$ verify that $(A-B)^T=A^T-B^T$

Ans. To verify $(A-B)^T = A^T-B^T$

$$A-B=\begin{pmatrix}4&-1&2\\5&0&3\end{pmatrix}-\begin{pmatrix}2&-1&1\\7&5&-2\end{pmatrix}$$

$$=\begin{pmatrix}4&-1&2\\5&0&3\end{pmatrix}+\begin{pmatrix}-2&1&-1\\-7&-5&2\end{pmatrix}=\begin{pmatrix}2&0&1\\-2&-5&5\end{pmatrix}$$

$$\therefore \quad (A-B)^T = \begin{pmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{pmatrix} \qquad (1)$$

$$\text{Also} \quad A^T = \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} \text{ and } B^T = \begin{pmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{pmatrix}$$

$$\therefore \quad A^T - B^T = \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} -2 & -7 \\ 1 & -5 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{pmatrix} \qquad (2)$$

Here $(1) = (2) \Rightarrow (A-B)^T = A^T - B^T$

37. For any vector $\vec{a}$ prove that $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2|\vec{a}|^2$.

Ans. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$|\vec{a}| = \sqrt{\vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2}$$

$$\vec{a}^{\,2} = \vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2$$

$$\text{Now} \quad \vec{a}\times\hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix} = [\vec{a}_3\hat{j} - \vec{a}_2\hat{k}]$$

$$|\vec{a}\times\hat{i}|^2 = (\vec{a}_3\hat{j} - \vec{a}_2\hat{k})\cdot(\vec{a}_3\hat{j} - \vec{a}_2\hat{k}) = \vec{a}_3^{\,2} + \vec{a}_2^{\,2}$$

$$\vec{a}\times\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 0 & 1 & 0 \end{vmatrix} = (-1)[\vec{a}_3\hat{i} - \vec{a}_1\hat{k}]$$

$$|\vec{a}\times\hat{j}|^2 = \vec{a}_3^{\,2} + \vec{a}_1^{\,2}$$

$$\text{and} \quad \vec{a}\times\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 0 & 0 & 1 \end{vmatrix} = \vec{a}_2\hat{i} - \vec{a}_1\hat{j}$$

$$|\vec{a}\times\hat{k}|^2 = a_2^{\,2} + a_1^{\,2}$$

$$\therefore \quad \text{LHS} = |\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$$

$$= \vec{a}_3^{\,2} + \vec{a}_2^{\,2} + \vec{a}_3^{\,2} + \vec{a}_1^{\,2} + \vec{a}_2^{\,2} + \vec{a}_1^{\,2}$$

$$= 2(\vec{a}_1^{\,2} + \vec{a}_2^{\,2} + \vec{a}_3^{\,2}) = 2\vec{a}^{\,2} = \text{RHS}$$

38. Given $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ find $\frac{dy}{dx}$

Ans. put $x = \tan\theta$

so $\frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

$y = \cos^{-1}(\cos 2\theta) = 2\theta$

$\Rightarrow \frac{dy}{d\theta} = 2$

Now $x = \tan\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec^2\theta$

$= 1 + \tan^2\theta$

$= 1 + x^2$

so $\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{2}{1+x^2}$.

39. A wound is healing in such a way that t days since Sunday the area of the wound has been decreasing at a rate of $-\frac{3}{(t+2)^2}$ cm^2 per day. If on Monday the area of the wound was 2 cm^2

(*i*) What was the area of the wound on Sunday ?

(*ii*) What is the anticipated area of the wound on Thursday if it continues to heal at the same rate ?

Ans. Let A be the area of wound at time 't'.

Hint:
Take $t = 0$ on Sunday
$t = 1$ on Monday
$t = 2$ on Tuesday
and so on ....

Given $\frac{dA}{dt} = \frac{-3}{(t+2)^2}$

$\int dA = -3\int \frac{1}{(t+2)^2}\,dt$

$A = -3\left[\frac{-1}{t+2}\right] + c$

$A = \frac{3}{t+2} + c$ ... (1)

By the given, condition area of the wound on monday is 2 cm^2

$\Rightarrow$ $A = 2;\quad t = 1$

$\Rightarrow$ $2 = \frac{3}{t+2} + c$

$c = 1$

$\therefore$ Area of wound at any day.

$\Rightarrow$ $1 \Rightarrow \quad A = \frac{3}{t+2} + 1$

(*i*) The area of the wound on Sunday

$t = 0 \Rightarrow A = \frac{3}{2} + 1 = \frac{5}{2} = 2.5 \text{ cm}^2$

(*ii*) The area of the wound on Thursday

$t = 4 \Rightarrow A = \frac{3}{6} + 1 = \frac{1}{2} + 1 = 1.5 \text{ cm}^2$

40. An integer is chosen at random from the first fifty positive integers. What is probability that the integer chosen is a prime or multiple of 4?

Ans. $S = \{1, 2, 3, ..., 50\}$ $\therefore n(S) = 50$

Let A be the event of getting prime number.

$\therefore \quad A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$

$n(A) = 15$, so $P(A) = 15/50$

Let B be the event of getting number multiple of 4

$\therefore B = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$

$n(B) = 12$, so $P(B) = 12/50$

Here A and B are mutually exclusive. (*i.e.,*) $A \cap B = \phi$

$\therefore P(A \cup B) = P(A) + P(B) = 15/50 + 12/50 = 27/50$

PART-IV

IV. Answer all the questions. **[7 × 5 = 35]**

41. (*a*) The formula for converting from Fahrenheit to Celsius temperatures is $y = \frac{5x}{9} - \frac{160}{9}$ **Find the inverse of this function and determine whether the inverse is also a function.**

Ans. $y = \frac{5x}{9} - \frac{160}{9}$

$y = \frac{5x - 160}{9}$

$9y = 5x - 160$

$9y + 160 = 5x$

$x = \frac{9y + 160}{5}$

$\therefore y = \frac{9x + 160}{5}$ (or) $f^{-1}(x) = \frac{9x}{5} + 32$

Yes it is also a function.

[OR]

(*b*) If $f : R \to R$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

Ans. Method 1:

One-to-one: Let $f(x) = f(y)$. Then $2x - 3 = 2y - 3$; this implies that $x = y$. That is, $f(x) = f(y)$ implies that $x = y$. Thus f is one-to-one.

Onto : Let $y \in R$. Let $x = \frac{y+3}{2}$. Then $f(x) = 2\left(\frac{y+3}{2}\right) - 3 = y$. Thus f is onto. This also can be proved by saying the following statement. The range of f is R (how?) which is equal to the co-domain and hence f is onto.

Inverse: Let $y = 2x - 3$. Then $y + 3 = 2x$ and hence $x = \frac{y+3}{2}$. Thus $f^{-1}(y) = \frac{y+3}{2}$. By replacing y as x, we get $f^{-1}(x) = \frac{x+3}{2}$

Method 2:

Let $y = 2x - 3$. Then $x = \frac{y+3}{2}$. Let $g(y) = \frac{y+3}{2}$.

Now $(g \circ f)(x) = g(f(x)) = g(2x-3) = \frac{(2x-3)+3}{2} = x$

$$(f \circ g)(y) = f(g(y)) = f\left(\frac{y+3}{2}\right) = 2\left(\frac{y+3}{2}\right) - 3 = y$$

Thus, $g \circ f = I_x$ and $f \circ g = I_y$

This implies that f and g are bijections and inverses to each other. Hence f is a bijection and

$f^{-1}(y) = \frac{y+3}{2}$. Replacing y by x we get, $f^{-1}(x) = \frac{x+3}{2}$

42. (*a*) If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots then prove that $ae = 2(b + f)$.

Ans. Let α be the common root

then $\alpha^2 - a\alpha + b = 0$ ....(1)

we are given that

$x^2 - ex + f = 0$ has equal roots.

So the roots will be α, β

Now sum of roots $= 2\alpha$

$= -(-e) \Rightarrow \alpha = e/2$

product of the roots

$\alpha \times \alpha = \alpha^2 f$

substituting α and α^2, values in (1) we get

$f - a\left(\frac{e}{2}\right) + b = 0$

$f - \frac{ae}{2} + b = 0$

$\frac{ae}{2} = b + f \Rightarrow ae = 2(b + f)$

[OR]

(*b*) Prove that $\cos\theta + \cos\left(\frac{2\pi}{3} - \theta\right) + \cos\left(\frac{2\pi}{3} + \theta\right) = 0$

Ans. We have

$$\text{LHS} = \cos\theta + \left[\cos\left(\frac{2\pi}{3} - \theta\right) + \cos\left(\frac{2\pi}{3} + \theta\right)\right]$$

$$= \cos\theta + 2\cos\frac{\frac{2\pi}{3} - \theta + \frac{2\pi}{3} + \theta}{2}\cos\frac{\frac{2\pi}{3} - \theta + \frac{2\pi}{3} + \theta}{2}$$

$$= \cos\theta + 2\cos\frac{2\pi}{3}\cos(-\theta) = \cos\theta + 2\left(-\frac{1}{2}\right)\cos\theta$$

$$\left[\because \cos\frac{2\pi}{3} = \cos 120° = -\frac{1}{2} \text{ and } \cos(-\theta) = \cos\theta\right]$$

$$= \cos\theta - \cos\theta = 0 = \text{RHS}$$

43. (*a*) Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85th string?

Ans. (*i*) Number of words formed = 5! = 120

(*ii*) The given word is THING

Taking the letters in alphabetical order G H I N T

To find the 85th word

The No. of words starting with G = 4! = 24

The No. of words starting with H = 4! = 24

The No. of words starting with	I = 4!	=	24
The No. of words starting with	NG = 3!	=	6
The No. of words starting with	NH = 3!	=	6
The No. of words starting with	NIGH = 1!	=	1
			85

So the 85th word is NIGHT **[OR]**

(b) A straight line passes through a fixed point (6, 8). Find the locus of the foot of the perpendicular drawn to it from the origin *O*.

Ans. Let the point (x_1, y_1) be (6, 8). and P (h, k) be a point on the required locus.

Family of equations of the straight lines passing through the fixed point (x_1, y_1) is

$y - y_1 = m(x - x_1) \Rightarrow y - 8 = m(x - 6)$

Since OP is perpendicular to the line (6.25)

$$m \times \left(\frac{k-0}{h-0}\right) = -1 \Rightarrow m = -\frac{h}{k}$$

Also P(h, k) lies on (6.25)

$$\Rightarrow k - 8 = -\frac{h}{k}(h-6) \Rightarrow k(k-8) = -h(h-6) \Rightarrow h^2 + k^2 - 6h - 8k = 0$$

Locus of P (h, k) is $x^2 + y^2 - 6x - 8y = 0$

44. (a) If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

Ans. In the equation of $\left(\frac{p}{2}+2\right)^8$, Number of terms = 8 + 1 = 9 (odd)

$\therefore$ There is only one middle term i.e. $\left(\frac{9+1}{2}\right)^{th}$ or 5th term

$$T_{r+1} = {}^8C_r \left(\frac{p}{2}\right)^r (2)^{8-r}$$

$$\therefore\ T_5 = T_{4+1} = {}^8C_4 \left(\frac{p}{2}\right)^4 (2)^{8-4} = 1120 \text{ (Given)}$$

$$\Rightarrow \frac{8!}{4!}\left(\frac{p}{2}\right)^4 (2)^4 = 1120$$

$$\Rightarrow \frac{8\times7\times6\times5}{4\times3\times2\times1} p^4 = 1120 \Rightarrow 70p^4 = 1120$$

$$\Rightarrow p^4 = \frac{1120}{70} = 16 \Rightarrow p^2 = 4, \text{ so } p = \pm 2$$

[OR]

(*b*) **Express the matrix** $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ **as the sum of a symmetric and a skew-symmetric matrices.**

Ans.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$$

Let $P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$

Now $P^T = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = P$

Thus, $P = \frac{1}{2}(A + A^T)$ is a symmetric matrix.

Let $Q = \frac{1}{2}(A - A^T)$

$$= \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

Then $Q^T = \frac{1}{2}\begin{bmatrix} 0 & -9 & -9 \\ 9 & 0 & 3 \\ 9 & -3 & 0 \end{bmatrix} = -Q$

Thus $Q = \frac{1}{2}(A - A^T)$ is a skew-symmetric matrix.

$$A = P + Q = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

Thus A is expressed as the sum of symmetric and skew-symmetric matrices.

45. (*a*) For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$

Ans. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$|\vec{a}| = \sqrt{\vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2}$$

and $$\vec{a}^2 = \sqrt{\vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2}$$

Now $$\vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix} = [\vec{a}_3\hat{j} - \vec{a}_2\hat{k}]$$

$$|\vec{a} \times \hat{i}|^2 = \left(\vec{a}_3\hat{j} - \vec{a}_2\hat{k}\right)\cdot\left(\vec{a}_3\hat{j} - \vec{a}_2\hat{k}\right) = \vec{a}_3^{\,2} + \vec{a}_2^{\,2}$$

$$\vec{a}\times\hat{j} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ 0 & 1 & 0\end{vmatrix} = (-1)[\vec{a}_3\hat{i}-\vec{a}_1\hat{k}]$$

$$|\vec{a}\times\hat{j}|^2 = \vec{a}_3^{\,2}+\vec{a}_1^{\,2}$$

$$\text{and } a\times\hat{k} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ 0 & 0 & 1\end{vmatrix} = \vec{a}_2\hat{i}-\vec{a}_1\hat{j}$$

$$|\vec{a}\times\hat{k}|^2 = a_2^{\,2}+a_1^{\,2}$$

$$\therefore \quad \text{LHS} = |\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$$

$$= \vec{a}_3^{\,2}+\vec{a}_2^{\,2}+\vec{a}_3^{\,2}+\vec{a}_1^{\,2}+\vec{a}_2^{\,2}+\vec{a}_1^{\,2}$$

$$= 2(\vec{a}_1^{\,2}+\vec{a}_2^{\,2}+\vec{a}_3^{\,2}) = 2\vec{a}^{\,2} = \text{RHS.}$$

[OR]

(b) If $y=e^{\tan^{-1}x}$, show that $(1+x^2)\,y''+(2x-1)\,y'=0$

Ans.

$$y = e^{\tan^{-1}x}$$

$$y' = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \quad y' = \frac{y}{1+x^2} \Rightarrow y'(1+x^2) = y$$

differentiating w.r.to x

$$y'(2x)+(1+x^2)(y'') = y'$$

(*i.e.*) $(1+x^2)\,y''+y'(2x)-y' = 0$

(*i.e.*) $(1+x^2)\,y''+(2x-1)\,y' = 0$

46. (a) Evaluate $\lim\limits_{x\to\infty}\dfrac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$.

Ans.

$$\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$$

$$= \lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\ldots\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left(1+\frac{10^{10}}{x^{10}}\right)}$$

$$= \lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}\ldots+\left(1+\frac{100}{x}\right)^{10}}{\left(1+\left(\frac{10}{x}\right)^{10}\right)}$$

$$= \frac{1+1+\ldots+1(100\text{ times})}{1+0} = \frac{100}{1} = 100$$

[OR]

(b) Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

Ans. $\frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x} = \frac{2^{\frac{5}{2}} - \left[(\cos x + \sin x)^2\right]^{\frac{5}{2}}}{1 - \sin 2x} = \frac{2^{\frac{5}{2}} - (1 + \sin 2x)^{\frac{5}{2}}}{2 - (1 + \sin 2x)}$

Therefore, $\lim_{x \to \frac{\pi}{4}} \frac{2^{\frac{5}{2}} - \left[(\cos x + \sin x)^2\right]^{\frac{5}{2}}}{2 - [1 + \sin 2x]} = \lim_{x \to \frac{\pi}{4}} \frac{2^{\frac{5}{2}} - [(1 + \sin 2x)]^{\frac{5}{2}}}{2 - (1 + \sin 2x)}$

Take $y = 1 + \sin 2x$. As $x \to \frac{\pi}{4}, y \to 2$

$$= \lim_{y \to 2} \frac{2^{\frac{5}{2}} - y^{\frac{5}{2}}}{2 - y}$$

$$= \frac{5}{2} \cdot 2^{\frac{5}{2} - 1} = \frac{5}{2} \times 2^{\frac{3}{2}} = 5\sqrt{2}$$

47. (a) Evaluate $\int \sin^{-1} x \, dx$

Ans.

$$\text{Let } I = \sin^{-1} x \, dx$$

$$u = \sin^{-1} x, \; dv = dx$$

$$\text{Then } du = \frac{1}{\sqrt{1 - x^2}}, \; v = x$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1 - x^2}} dx$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x + \frac{1}{2} \int \frac{dt}{\sqrt{t}}, \text{ where } t = 1 - x^2$$

$$= x \sin^{-1} x + \sqrt{t} + c$$

$$= x \sin^{-1} x + \sqrt{1 - x^2} + c$$

[OR]

(b) A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

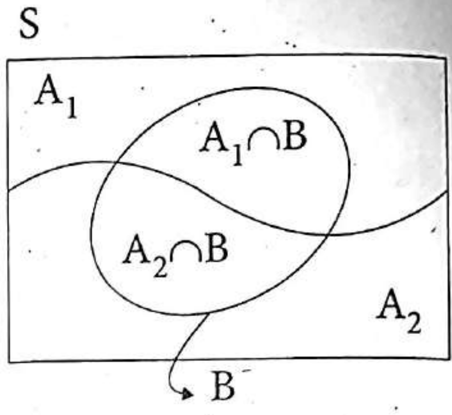
Ans. Let A_1 be the event that the items are produced by Machine-I, A_2 be the event that items are produced by Machine-II, Let B be the event of drawing a defective item. Now we are asked to find the conditional probability $P(A_2/B)$. Since A_1, A_2 are mutually exclusive and exhaustive events, by Bayes theorem,

$$P(A_2/B) = \frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2)}$$

We have,

$$P(A_1) = 0.40, \quad P(B/A_1) = 0.04$$

$$P(A_2) = 0.60, \quad P(B/A_2) = 0.05$$



$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$P(A_2/B) = \frac{(0.60)(0.05)}{(0.40)(0.04) + (0.60)(0.05)} = \frac{15}{23}$$

SAMPLE PAPER – 3

(SOLVED)

MATHEMATICS

Time. 2.30 Hours **Maximum Marks: 90**

PART-I

I. Choose the correct answer. Answer all the questions. [*Answers are in bold*] [20 × 1 = 20]

1. Let A and B be subsets of the universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is ....

   (*a*) A (*b*) A' (*c*) B **(*d*) N**

2. For any two sets A and B if $(A - B) \cup (B - A) =$ ....

   (*a*) $(A - B) \cup A$ (*b*) $(B - A) \cup B$

   (c) $(A \cup B) - (A \cap B)$ (*d*) $(A \cup B) \cap (A \cap B)$

3. The equations whose roots are numerically equal but opposite in sign to the roots of $3x^2 - 5x - 7 = 0$ is ....

   (*a*) $3x^2 - 5x - 7 = 0$ **(*b*) $3x^2 + 5x - 7 = 0$**

   (*c*) $3x^2 - 5x + 7 = 0$ (*d*) $3x^2 + x - 7 = 0$

4. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is ....

   (*a*) $2 \cos\theta$ (*b*) 1 **(*c*) 0** (*d*) $2 \sin\theta$

5. If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 + ax + b = 0$ then $\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$ is equal to ....

   (*a*) $\dfrac{b}{a}$ (*b*) $\dfrac{a}{b}$ **(*c*) $-\dfrac{a}{b}$** (*d*) $-\dfrac{b}{a}$

6. If $a^2 - aC_2 = a^2 - aC_4$ then the value of a is ....

   (*a*) 2 **(*b*) 3** (*c*) 4 (*d*) 5

7. If ${}^nP_r = 840$, ${}^nC_r = 35$ then $n =$ ....

   (*a*) 7 (*b*) 6 (*c*) 5 (*d*) 4

8. If $2x^2 + 3xy - cy^2 = 0$ represents a pair of perpendicular lines then $c =$ ....

   (*a*) -2 (*b*) $\dfrac{1}{2}$ (*c*) $-\dfrac{1}{2}$ **(*d*) 2**

9. If the n^{th} term of an A.P is $2n - 1$ then sum to n terms of that A.P. is ....

   (*a*) n^2 (*b*) $n^2 + 1$ (*c*) $2n - 1$ (*d*) $n^2 - 1$

10. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and $(A + B)^2 = A^2 + B^2$ then the values of a and b are ....

    (*a*) $a = 4, b = 1$ **(*b*) $a = 1, b = 4$** (*c*) $a = 0, b = 4$ (*d*) $a = 2, b = 4$

11. If the points $(x-2)$, $(5, 2)$, $(8, 8)$ are collinear then x is equal to ....

(*a*) -3 (*b*) $\frac{1}{3}$ (*c*) 1 **(*d*) 3**

12. In a regular hexagon ABCDEF if $\overline{AB}$ and $\overline{BC}$ are represented by $\vec{a}$ and $\vec{b}$ respectively then $\overline{EF}$ = ...

(*a*) $\vec{a}-\vec{b}$ (*b*) $\vec{a}$ **(*c*) $-\vec{b}$** (*d*) $\vec{a}+\vec{b}$

13. If $|\vec{a}+\vec{b}|=60, |\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then $|\vec{a}|$ is ....

(*a*) 42 (*b*) 12 **(*c*) 22** (*d*) 32

14. For $\vec{a}=\vec{i}+\vec{j}-2\vec{k}$, $\vec{b}=-\vec{i}+2\vec{j}+\vec{k}$ and $\vec{c}=\vec{i}-2\vec{j}+2\vec{k}$, the unit vector parallal to $\vec{a}+\vec{b}+\vec{c}$ is ....

(*a*) $\frac{\vec{i}+\vec{j}-\vec{k}}{\sqrt{3}}$ **(*b*) $\frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{3}}$** (*c*) $\frac{\vec{i}+\vec{j}+\vec{k}}{3}$ (*d*) $\frac{\vec{i}-\vec{j}+\vec{k}}{\sqrt{6}}$

15. The differential co-efficient of $\log_{10}x$ with respect to $\log_x 10$ is ....

(*a*) 1 **(*b*) $-(\log_{10}x)^2$** (*c*) $(\log_x 10)^2$ (*d*) $\frac{x^2}{100}$

16. $\frac{d}{dx}\left(e^{x+5\log x}\right)$ is ....

(*a*) $e^x x^4(x+5)$ (*b*) $e^x x(x+5)$ (*c*) $e^x+\frac{5}{x}$ (*d*) $e^x-\frac{5}{x}$

17. If $f(x)=x\tan^{-1}x$ then $f'(1)$ = ...

(*a*) $1+\frac{\pi}{4}$ **(*b*) $\frac{1}{2}+\frac{\pi}{4}$** (*c*) $\frac{1}{2}-\frac{\pi}{4}$ (*d*) 2

18. $\int \operatorname{cosec} x\,dx$ = ...

(*a*) $\log\tan\frac{x}{2}+c$ (*b*) $-\log(\operatorname{cosec}x+\cot x)+c$

(*c*) $\log(\operatorname{cosec}x-\cot x)+c$ **(*d*) all of them**

19. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(*a*) $P(A/B)=\frac{P(A)}{P(B)}$ (*b*) $P(A/B) < P(A)$ **(*c*) $P(A/B \geq P(A))$** (*d*) $P(A/B) > P(B)$

20. A number x is chosen at random from the first 100 natural numbers. Let A be the event of numbers which satisfies $\frac{(x-10)(x-50)}{x-30} \geq 0$, then P(A) is ....

(*a*) 0.20 (*b*) 0.51 **(*c*) 0.71** (*d*) 0.70

PART-II

II. Answer any seven questions. Question No. 30 is compulsory. [7 × 2 = 14]

21. Write the values of f at – 4, 1, –2, 7, 0 if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

Ans.

$$f(-4) = 4+4 = 8$$
$$f(1) = 1 - 1^2 = 0$$
$$f(-2) = 4+2 = 6$$
$$f(7) = 0$$
$$f(0) = 0$$

22. Solve $23x < 100$ when (*i*) x is a natural number (*ii*) x is an integer

Ans. $23x < 100$

$$\Rightarrow \quad \frac{23x}{23} < \frac{100}{23}$$

(*i.e.,*) $x > 4.3$

(*i*) $x = 1, 2, 3, 4$ ($x \in$ N)

(*ii*) $x = \ldots.$ –3, –2, –1, 0, 1, 2, 3, 4 ($x \in$ Z)

23. Expand $\frac{1}{5+x}$ in ascending powers of x.

Ans.

$$\frac{1}{5+x} = \frac{1}{5\left(1+\frac{x}{5}\right)} = \frac{1}{5}\left(1+\frac{x}{5}\right)^{-1}$$

$$= \frac{1}{5}\left\{1 - \frac{x}{5} + \left(\frac{x}{5}\right)^2 - \left(\frac{x}{5}\right)^3 \cdots\right\}$$

$$= \frac{1}{5} - \frac{x}{5^2} + \frac{x^2}{5^3} - \frac{x^3}{5^4} \ldots.$$

24. Find the nearest point on the line $2x + y = 5$ from the origin.

Ans. The required point is the foot of the perpendicular from the origin on the line $2x + y = 5$.

The line perpendicular to the given line, through the origin is $x - 2y = 0$.

Solving the equations $2x + y = 5$ and $x - 2y = 0$, we get $x = 2, y = 1$.

Hence the nearest point on the line from the origin is (2, 1).

Alternate method: Using the formula

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\frac{x-0}{2} = \frac{y-0}{1} = -\frac{(2(0)+1(0)-5)}{2^2+1^2} \Rightarrow \frac{x}{2} = \frac{y}{1} = 1 \Rightarrow (2,1)$$

PART-II

II. Answer any seven questions. Question No. 30 is compulsory. [7 × 2 = 14]

21. Write the values of f at $-4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2-x & \text{if } -2 \le x < 1 \\ x-x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

Ans.

$$\begin{aligned} f(-4) &= 4+4=8 \\ f(1) &= 1-1^2=0 \\ f(-2) &= 4+2=6 \\ f(7) &= 0 \\ f(0) &= 0 \end{aligned}$$

22. Solve $23x < 100$ when (*i*) x is a natural number (*ii*) x is an integer

Ans. $23x < 100$

$$\Rightarrow \quad \frac{23x}{23} < \frac{100}{23}$$

(*i.e.,*) $x > 4.3$

(*i*) $x = 1, 2, 3, 4$ $(x \in N)$

(*ii*) $x = \ldots\ -3, -2, -1, 0, 1, 2, 3, 4$ $(x \in Z)$

23. Expand $\frac{1}{5+x}$ in ascending powers of x.

Ans. $$\frac{1}{5+x} = \frac{1}{5\left(1+\frac{x}{5}\right)} = \frac{1}{5}\left(1+\frac{x}{5}\right)^{-1}$$

$$= \frac{1}{5}\left\{1-\frac{x}{5}+\left(\frac{x}{5}\right)^2-\left(\frac{x}{5}\right)^3 \ldots\right\}$$

$$= \frac{1}{5}-\frac{x}{5^2}+\frac{x^2}{5^3}-\frac{x^3}{5^4}\ldots.$$

24. Find the nearest point on the line $2x + y = 5$ from the origin.

Ans. The required point is the foot of the perpendicular from the origin on the line $2x + y = 5$.

The line perpendicular to the given line, through the origin is $x - 2y = 0$.

Solving the equations $2x + y = 5$ and $x - 2y = 0$, we get $x = 2, y = 1$.

Hence the nearest point on the line from the origin is (2, 1).

Alternate method: Using the formula

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$$

$$\frac{x-0}{2} = \frac{y-0}{1} = -\frac{(2(0)+1(0)-5)}{2^2+1^2} \Rightarrow \frac{x}{2} = \frac{y}{1} = 1 \Rightarrow (2,1)$$

25. Determine 3B + 4C − D if B, C and D are given by

$$B = \begin{pmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & -2 & 3 \\ -1 & 0 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 4 & -1 \\ 5 & 6 & -5 \end{pmatrix}$$

Ans. $3B + 4C - D = \begin{bmatrix} 6 & 9 & 0 \\ 3 & -3 & 15 \end{bmatrix} + \begin{bmatrix} -4 & -8 & 12 \\ -4 & 0 & 8 \end{bmatrix} + \begin{bmatrix} 0 & -4 & 1 \\ -5 & -6 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 13 \\ -6 & -9 & 28 \end{bmatrix}$

26. Find the constant b that makes g continuous on $(-\infty, \infty)$ $g(x) = \begin{cases} x^2 - b^2, \text{ if } x < 4 \\ bx + 20, \text{ if } x \geq 4 \end{cases}$

Ans. Since $g(x)$ is continuous,

$$\begin{aligned} \lim_{x\to 4^-} g(x) &= \lim_{x\to 4^+} g(x) \\ \lim_{x\to 4^-} (x^2 - b^2) &= \lim_{x\to 4^+} bx + 20 \\ 16 - b^2 &= 4b + 20 \\ b^2 + 4b + 4 &= 0 \\ (b+2)^2 &= 0 \\ b &= -2 \end{aligned}$$

27. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$

Ans. We differentiate both sides of the equation.

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2y\frac{dy}{dx} = 0$$

Solving for the derivative yields

$$\frac{dy}{dx} = -\frac{x}{y}$$

28. Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x} dx.$

Ans. $\int \frac{1}{\sin^2 x \cos^2 x} dx$

$$\begin{aligned} &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx \\ &= \int \sec^2 x dx + \int \text{cosec}^2 x dx \\ &= \tan x - \cot x + c \end{aligned}$$

29. If P(A) = 0.5, P(B) = 0.8 and P(B/A) = 0.8 find P(A/B) and P(A ∪ B)

Ans. Given P(A) = 0.5, P(B) = 0.8 and P(B/A) = 0.8

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = 0.8 \text{ (given)}$$

$$\Rightarrow \quad \frac{P(A \cap B)}{0.5} = 0.8$$

$$\Rightarrow \quad P(A \cap B) = 0.8 \times 0.5 = 0.4$$

(*i.e.,*) $P(A \cap B) = 0.4$

(*i*) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.8} = \frac{4}{8} = 0.5$$

(*ii*) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.5 + 0.8 - 0.4 = 0.9$$

So, $P(A/B) = 0.5$ and $P(A \cup B) = 0.9$.

30. Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

Ans. The angle between $\vec{a}$ and $\vec{b}$ using vector product is given by

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

Here $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1)$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{9+9+9} = \sqrt{9 \times 3} = 3\sqrt{3}$$

$$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\sin\theta = \frac{3\sqrt{3}}{\sqrt{6}\sqrt{6}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \pi/3$$

PART-III

III. Answer any seven questions. Question No. 40 is compulsory. **[7 × 3 = 21]**

31. If $(x^{1/2} + x^{-1/2})^2 = \frac{9}{2}$ find the value of $(x^{1/2} - x^{-1/2})$ for $x > 1$

Ans. Given $(x^{1/2} + x^{-1/2})^2 = \frac{9}{2} \Rightarrow \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = \frac{9}{2}$

(*i.e.,*) $x + \frac{1}{x} + 2\sqrt{x}\frac{1}{\sqrt{x}} = \frac{9}{2} \Rightarrow x + \frac{1}{x} + 2 = \frac{9}{2} \Rightarrow x + \frac{1}{x} = \frac{9}{2} - 2 = \frac{9-4}{2} = \frac{5}{2}$

Now $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2\sqrt{x}\frac{1}{\sqrt{x}} = x + \frac{1}{x} - 2 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

$\Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

32. If $\frac{n!}{3!(n-4)!}$ and $\frac{n!}{5!(n-5)!}$ are in the ratio 5 : 3 find the value of n.

Ans. We have $\frac{n!}{3!(n-4)!} : \frac{n!}{5!(n-5)!} = 5:3$

$$\Rightarrow \frac{n!}{3!(n-4)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3} \Rightarrow \frac{5\times4\times3!(n-5)!}{3!(n-4)(n-5)!} = \frac{5}{3} \Rightarrow \frac{20}{n-4} = \frac{5}{3}$$

$$\Rightarrow n-4 = 20\times\frac{3}{5} \Rightarrow n-4 = 12 \Rightarrow n = 16$$

33. Expand $(1+x)^{\frac{2}{3}}$ up to four terms for $|x|<1$.

Ans. Here $n = \frac{2}{3}$

$$\frac{n(n-1)}{2!} = \frac{\frac{2}{3}\left(\frac{2}{3}-1\right)}{2!} = \frac{\frac{2}{3}\left(\frac{-1}{3}\right)}{2} = \frac{-1}{9}$$

$$\frac{n(n-1)(n-2)}{2!} = \frac{\frac{2}{3}\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3!} = \frac{\frac{2}{3}\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)}{6} = \frac{4}{81}$$

Thus, $(1+x)^{\frac{2}{3}} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \ldots$

34. Find the equation of the line if the perpendicular drawn from the origin makes an angle 30° with x axis and its length is 12.

Ans. The equation of the line is $x\cos\alpha + y\sin\alpha = p$

here $\alpha = 30°$, $\cos\alpha = \cos 30° = \frac{\sqrt{3}}{2}$; $\sin\alpha = \sin 30° = 1/2$; $p = 12$.

So equation of the line is $x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 12$

(*i.e*) $\sqrt{3}x + y = 12\times2 = 24 \Rightarrow \sqrt{3}x + y - 24 = 0$

35. Prove that $\begin{vmatrix} \frac{1}{a^2} & bc & b+c \\ \frac{1}{b^2} & ca & c+a \\ \frac{1}{c^2} & ab & a+b \end{vmatrix} = 0$

Ans. $\begin{vmatrix} 1/a^2 & bc & b+c \\ 1/b^2 & ca & c+a \\ 1/c^2 & ab & a+b \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} 1/a & abc & a(b+c) \\ 1/b & abc & b(c+a) \\ 1/c & abc & c(a+b) \end{vmatrix}$ Multiply R_1, R_2, R_3 by a, b, c respectively.

$$= \frac{abc}{abc}\begin{vmatrix} 1/a & 1 & a(b+c) \\ 1/b & 1 & b(c+a) \\ 1/c & 1 & c(a+b) \end{vmatrix} \text{ Take } abc \text{ from } C_2$$

$$= \frac{1}{abc}\begin{vmatrix} bc & 1 & a(b+c) \\ ca & 1 & b(c+a) \\ ab & 1 & c(a+b) \end{vmatrix} \text{ Multiply } C_1 \text{ by } abc$$

$$= \frac{1}{abc}\begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix} \; C_3 \to C_3 + C_1$$

$$= \frac{(ab+bc+ca)}{abc}\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} \text{ Take } (ab+bc+ca) \text{ from } C_3$$

$$= \frac{(ab+bc+ca)}{abc}(0) \qquad [\because C_2 \text{ is identical to } C_3]$$

$$= 0$$

36. Find $\lim\limits_{t \to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$

Ans. We can't apply the quotient theorem immediately. Use the algebra technique of rationalising the numerator.

$$\frac{\sqrt{t^2+9}-3}{t^2} = \frac{\left(\sqrt{t^2+9}-3\right)\left(\sqrt{t^2+9}+3\right)}{t^2\left(\sqrt{t^2+9}+3\right)} = \frac{t^2+9-9}{t^2\left(\sqrt{t^2+9}+3\right)}$$

$$\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2} = \lim_{t\to 0}\frac{t^2}{t^2\sqrt{t^2+9}+3} = \lim_{t\to 0}\frac{1}{\sqrt{t^2+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}.$$

37. Find $\dfrac{dy}{dx}$ **where** $x = \dfrac{1-t^2}{1+t^2},\; y = \dfrac{2t}{1+t^2}$

Ans.

$$x = \frac{1-t^2}{1+t^2}; \frac{dx}{dt} = \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2}$$

$$= \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2}$$

$$y = \frac{2t}{1+t^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2)(2)-2t(2t)}{(1+t^2)^2} = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{2(1-t^2)}{(1+t^2)^2}\Big/\frac{-4t}{(1+t^2)^2} = \frac{t^2-1}{2t}$$

38. Evaluate $\int\left(5x^2-4+\frac{7}{x}+\frac{2}{\sqrt{x}}\right)dx$

Ans. $\int\left[5x^2-4+\frac{7}{x}+2(x)^{-1/2}\right]dx$

$$= \frac{5x^3}{3}-4x+7\log x+2\frac{(x)^{1/2}}{1/2}$$

$$= \frac{5}{3}x^3-4x+7\log x+4\sqrt{x}+c$$

39. What is the chance that leap year should have fifty three Sundays?

Leap Year:

Ans. In 52 weeks we have 52 Sundays. We have to find the probability of getting one sunday form the remaining 2 days the remaining 2 days can be a combination of the following S = {Saturday and Sunday, Sunday and Monday, Monday and Tuesday, Tuesday and wednesday, wednesday and Thursday, Thursday and Friday, Friday and Saturday}.

(*i.e*) $n(s) = 7$

In this n(A) = {Saturday and Sunday, Sunday and Monday}

(*i.e*) $n(A) = 2$

So, $P(A) = \frac{2}{7}$

40. Find x from the equation cosec (90° + A) + x cos A cot (90° + A) = sin (90° + A).

Ans. cosec (90° + A) = sec A, cot (90° + A) = – tan A

$$\text{LHS} = \sec A + x\cos A(-\tan A)$$

$$= \frac{1}{\cos A} - x\cos A\times\frac{\sin A}{\cos A} = \frac{1}{\cos A} - x\sin A$$

$$\text{RHS} = \sin(90° + A) = \cos A$$

$$\therefore \frac{1}{\cos A} - x\sin A = \cos A$$

$$\Rightarrow \frac{1}{\cos A} - \cos A = x\sin A \Rightarrow \frac{1-\cos^2 A}{\cos A} = x\sin A$$

$$\Rightarrow \frac{\sin^2 A}{\sin A\cos A} = x$$

$$x = \frac{\sin A}{\cos A} = \tan A$$

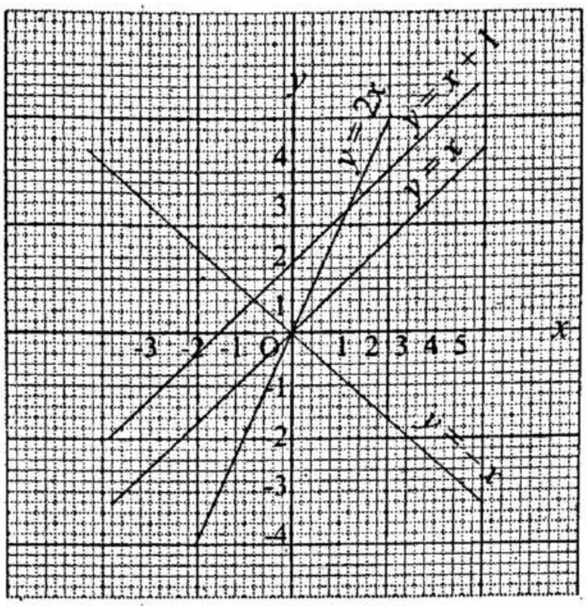
PART-IV

IV. Answer all the questions. **[7 × 5 = 35]**

41. (a) From the curve $y = x$, draw

(i) $y = -x$ (ii) $y = 2x$ (iii) $y = x + 1$ (iv) $y = \frac{1}{2}x + 1$ (v) $2x + y + 3 = 0$

Ans.



[OR]

(b) Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

Ans. $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

Here $a = -1$; $b = \sqrt{3}$; $c = \sqrt{2}$; $r = \sqrt{a^2 + b^2} = 2$

Thus, the given equation can be rewritten as

$$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6} = \sin\frac{\pi}{4}$$

$$\sin\left(\theta - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}$$

$$\theta - \frac{\pi}{6} = n\pi \pm (-1)^n\frac{\pi}{4}, n \in \mathbb{Z}$$

Thus, $\theta = n\pi + \frac{\pi}{6} \pm (-1)^n\frac{\pi}{4}, n \in \mathbb{Z}$

42. (a) Solve $\frac{x^2 - 4}{x^2 - 2x - 15} \le 0$

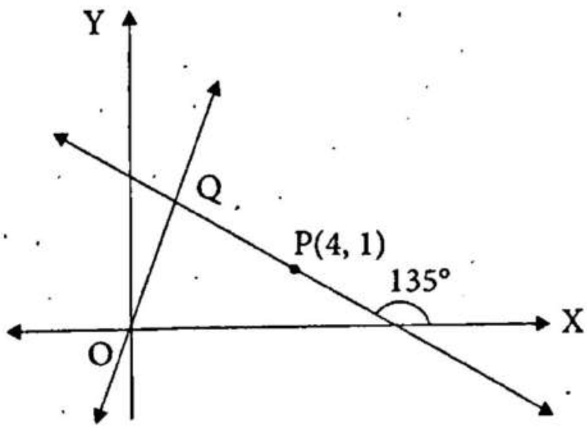
Ans. $\frac{x^2 - 4}{x^2 - 2x - 15} \le 0 \Rightarrow \frac{(x-2)(x+2)}{(x+3)(x-5)} \le 0$

$x - 2 \Rightarrow x = 2$; $x + 2 = 0 \Rightarrow x = -2$

$x + 3 = 0 \Rightarrow x = -3$; $x - 5 = 0 \Rightarrow x = 5$

plotting the points –3, –2, 2, 5 in the number line and taking the intervals

$$\frac{x-4}{\cos 135^\circ}=\frac{y-1}{\sin 135^\circ}$$



Suppose it cuts $4x-y=0$ at Q such that PQ = r Then, the coordinates of Q are given by

$$\frac{x-4}{\cos 135^\circ}=\frac{y-1}{\sin 135^\circ}=r$$

$$\Rightarrow \frac{x-4}{-1/\sqrt{2}}=\frac{y-1}{1/\sqrt{2}}=r$$

$$\Rightarrow x=4-\frac{r}{\sqrt{2}},\ y=1+\frac{r}{\sqrt{2}}$$

So, the coordinates of Q are $\left(4-\frac{r}{\sqrt{2}}, 1+\frac{r}{\sqrt{2}}\right)$

Clearly, Q lies on $4x-y=0$

$$\therefore\ 16-\frac{4r}{\sqrt{2}}-1-\frac{r}{\sqrt{2}}=0 \Rightarrow \frac{5r}{\sqrt{2}}=15 \Rightarrow r=3\sqrt{2}$$

Hence, required distance is $3\sqrt{2}$ units.

[OR]

(*b*) Evaluate $\lim_{x\to\infty} x\left[3^{\frac{1}{x}}+1-\cos\left(\frac{1}{x}\right)-e^{\frac{1}{x}}\right]$

Ans. Let $y=\frac{1}{x}$ as $x\to\infty, y\to 0$

$$\lim_{x\to\infty} x\left[3^{\frac{1}{x}}+1-\cos\left(\frac{1}{x}\right)-e^{\frac{1}{x}}\right]=\lim_{y\to 0}\frac{1}{y}(3^y+1-\cos y-e^y)$$

$$=\lim_{y\to 0}\frac{3^y-1}{y}+\frac{1-\cos y}{y}-\frac{e^y+1}{y}$$

$$=\lim_{y\to 0}\frac{3^y-1}{y}+\frac{2\sin^2 y/2}{y}-\frac{(e^y-1)}{y}$$

$$=\lim_{y\to 0}\left(\frac{3^y-1}{y}\right)+\frac{\sin^2\frac{y}{2}}{y/2}-\frac{(e^y-1)}{y}$$

$$=\lim_{y\to 0}\left(\frac{3^y-1}{y}\right)+\frac{\sin\frac{y}{2}}{y/2}\times\sin\left(y/2\right)-\left(\frac{e^y-1}{y}\right)$$

$$=\lim_{y\to 0}\left(\frac{3^y-1}{y}\right)+\lim_{y\to 0}\left(\frac{\sin y/2}{y/2}\right)\lim_{y\to 0}\left(\sin y/2\right)-\lim_{y\to 0}\frac{e^y-1}{y}$$

$$=\log 3+(1)(0)-(1)=(\log 3)-1=-1+\log 3=\log 3-1$$

45. (*a*) **Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.**

Ans. $\sqrt[3]{x^3+7}=\left(x^3+7\right)^{\frac{1}{3}}$

$$=\left[x^3\left(1+\frac{7}{x^3}\right)\right]^{\frac{1}{3}},\left(\left|\frac{7}{x^3}\right|<1 \text{ as } x \text{ is large}\right)=x\left(1+\frac{7}{x^3}\right)^{\frac{1}{3}}$$

$$=x\left(1+\frac{1}{3}\times\frac{7}{x^3}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{7}{x^3}\right)^2+...\right)=x\left(1+\frac{7}{3}\times\frac{1}{x^3}-\frac{49}{9}\times\frac{1}{x^6}+...\right)$$

$$=x+\frac{7}{3}\times\frac{1}{x^2}-\frac{49}{9}\times\frac{1}{x^6}+...$$

$$\sqrt[3]{x^3+4}=\left(x^3+4\right)^{\frac{1}{3}}$$

$$=\left[x^3\left(1+\frac{4}{x^3}\right)\right]^{\frac{1}{3}}=x\left(1+\frac{4}{x^3}\right)^{\frac{1}{3}},\ \left(\left|\frac{4}{x^3}\right|<1\right)$$

$$=x\left(1+\frac{1}{3}\times\frac{4}{x^3}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{4}{x^3}\right)^2+...\right)=x+\frac{4}{3}\times\frac{1}{x^2}-\frac{16}{9}\times\frac{1}{x^6}+...$$

Since x is large, $\frac{1}{x}$ is very small and hence higher powers of $\frac{1}{x}$ are negligible. Thus

$\sqrt[3]{x^3+7}=x+\frac{7}{3}\times\frac{1}{x^2}$ and $\sqrt[3]{x^3+4}=x+\frac{4}{3}\times\frac{1}{x^2}$. Therefore

$$\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}=\left(x+\frac{7}{3}\times\frac{1}{x^2}\right)-\left(x+\frac{4}{3}\times\frac{1}{x^2}\right)=\frac{1}{x^2}$$

[OR]

(*b*) **Evaluate $\sec^3 2x$**

Ans. $I = \int\sec^3 2x\,dx=\int\sec 2x\sec^2 2x\,dx$

Let $u = \sec 2x;\quad du=2\sec 2x\tan 2x\,dx$

$\sec^2 2x\,dx = dv$

$\therefore\quad v = \int\sec^2 2x\,dx=\left(\frac{\tan 2x}{2}\right)$

$I = \int\sec 2x\,d\left(\frac{\tan 2x}{2}\right)$

$$= (\sec 2x)\left(\frac{\tan 2x}{2}\right) - \int\left(\frac{\tan 2x}{2}\right)(2\sec 2x \tan 2x)\, dx$$

$$\therefore \quad I = \frac{1}{2}\sec 2x \tan 2x - \int \tan^2 2x \sec 2x\, dx$$

$$= \frac{1}{2}\sec 2x \tan 2x - \int (\sec^2 2x - 1)\sec 2x\, dx$$

$$I = \frac{1}{2}\sec 2x \tan 2x - \int \sec^3 2x\, dx + \int \sec 2x\, dx$$

$$I = \frac{1}{2}\sec 2x \tan 2x - I + \frac{1}{2}\log(\sec 2x + \tan 2x)$$

$$2I = \frac{1}{2}\{\sec 2x \tan 2x + \log(\sec 2x + \tan 2x)\}$$

$$\therefore \quad I = \frac{1}{4}[\sec 2x \tan 2x + \log(\sec 2x + \tan 2x)] + c$$

46. (*a*) Evaluate $y = \sin\left(\tan\left(\sqrt{\sin x}\right)\right)$

Ans. Put $u = \sqrt{\sin x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{\sin x}}(\cos x)$ ....(1)

Now $y = \sin(\tan u)$

Put $v = \tan u \Rightarrow \frac{dv}{du} = \sec^2 u$ ....(2)

Now $y = \sin v$

$$\frac{dy}{dv} = \cos v \qquad \text{....(3)}$$

$$\frac{dy}{dx} = \frac{dy}{dv}\times\frac{dv}{du}\times\frac{du}{dx} = \cos v[\sec^2 u]\frac{\cos x}{2\sqrt{\sin x}}$$

$$= \frac{\cos\left(\tan\sqrt{\sin x}\right)\sec^2\left(\sqrt{\sin x}\right)\cos x}{2\sqrt{\sin x}}$$

[OR]

(*b*) Evaluate $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

Ans. Let $u = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$$\Rightarrow \quad \frac{du}{dx} = 1 + \frac{1}{4\sqrt{x}}\left(\frac{2\sqrt{x}+1}{\sqrt{x+\sqrt{x}}}\right)$$

Now $y = \sqrt{u} \Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$

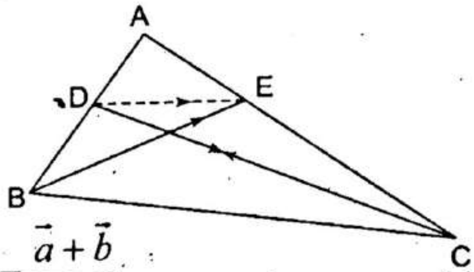
So $$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{1}{2\sqrt{u}}\left[1 + \frac{1}{4\sqrt{x}}\left(\frac{2\sqrt{x}+1}{\sqrt{x+\sqrt{x}}}\right)\right]$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[\frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$$

$$= \frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{x}\sqrt{x+\sqrt{x}}\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

47. (*a*) Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

Ans. In ΔABC,



$$\overrightarrow{OA} = \vec{a}$$

$$\overrightarrow{OB} = \vec{b} \text{ and}$$

$$\overrightarrow{OC} = \vec{c}$$

$$D = \text{mid point of AB} = \overrightarrow{OD} = \frac{\vec{a}+\vec{b}}{2}$$

$$E = \text{mid point of AC} = \overrightarrow{OE} = \frac{\vec{a}+\vec{c}}{2}$$

Now $$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \frac{\vec{a}+\vec{c}}{2} - \frac{\vec{a}+\vec{b}}{2}$$

$$= \frac{\vec{a}+\vec{c}-\vec{a}-\vec{b}}{2} = \frac{\vec{c}-\vec{b}}{2}$$

$$= \frac{\overrightarrow{BC}}{2}$$

$$\overrightarrow{DE} = \frac{\overrightarrow{BC}}{2} \Rightarrow \overrightarrow{DE} \parallel \text{ to } \overrightarrow{BC} \text{ and half of BC}$$ **[OR]**

(*b*) Given P(A) = 0.4 and P(A ∪ B) = 0.7. Find P(B) if (*i*) A and B are mutually exclusive (*ii*) A and B are independent events (*iii*) P(A / B) = 0.4 (*iv*) P(B / A) = 0.5

Ans. $P(A) = 0.4, P(A \cup B) = 0.7$

(*i*) When A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

(*i.e.,*) $$0.7 = 0.4 + P(B)$$

$$0.7 - 0.4 = P(B)$$

(*i.e.,*) $$P(B) = 0.3$$

(*ii*) Given A and B are independent

⇒ $$P(A \cap B) = P(A).\ P(B)$$

Now, $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(*i.e.,*) $$0.7 = 0.4 + P(B) - (0.4)\ (P(B)$$

(*i.e.,*) $$0.7 - 0.4 = P(B)\ (1 - 0.4)$$

$$0.3 = P(B)\ 0.6$$

$$\Rightarrow \quad P(B) = \frac{0\cdot 3}{0\cdot 6} = \frac{3}{6} = 0.5$$

(*iii*) $P(A/B) = 0.4$

(*i.e.,*) $\dfrac{P(A \cap B)}{P(B)} = 0.4$

$$\Rightarrow \quad P(A \cap B) = 0.4\,[P(B)] \qquad ...(i)$$

But We know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow \quad P(A \cap B) = 0.4 + P(B) - 0.7$$

$$= P(B) - 0\cdot 3 \qquad ...(ii)$$

from (*i*) and (*ii*) (equating R.H.S) We get

$$0.4\,[P(B)] = P(B) - 0.3$$

$$0.3 = P(B)\,(1 - 0.4)$$

$$0.6\,(P(B)) = 0.3 \Rightarrow P(B) = \frac{0\cdot 3}{0\cdot 6} = \frac{3}{6} = 0.5$$

(*iv*) $P(B/A) = 0.5$

(*i.e.,*) $\dfrac{P(A \cap B)}{P(A)} = 0.5$

(*i.e.,*) $P(A \cap B) = 0.5 \times P(A)$

$$= 0.5 \times 0.4 = 0.2$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad 0.7 = 0.4 + P(B) - 0.2$$

$$\Rightarrow \quad 0.7 = P(B) + 0.2$$

$$\Rightarrow \quad P(B) = 0.7 - 0.2 = 0.5$$

Sample Paper – 6

(UNSOLVED)

Time: 2.30 Hours **Maximum Marks: 90**

PART-I

I. Choose the correct answer. Answer all the questions. **[20 × 1 = 20]**

1. For non empty sets A and B if $A \subset B$ then $(A \times B) \cap (B \times A)$ is equal to ....
   (a) $A \cap B$ (b) $A \times A$ (c) $B \times B$ (d) none of these
2. The solution set of the inequality $|x-1| \geq |x-3|$ is ....
   (a) $[0, 2]$ (b) $[2, \infty)$ (c) $(0, 2)$ (d) $(-\infty, 2)$
3. The numer of solutions of $x^2 + |x-1| = 1$ is ....
   (a) 1 (b) 0 (c) 2 (d) 3
4. Which of the following is not true?
   (a) $\sin\theta = \frac{-3}{4}$ (b) $\cos\theta = -1$ (c) $\tan\theta = 25$ (d) $\sec\theta = \frac{1}{4}$
5. Let $f_k(x) = \frac{1}{k}[\sin^k x + \cos^k x]$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x) =$ ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
6. If A and B are co efficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively then $\frac{A}{B} =$ ....
   (a) $\frac{1}{2}$ b) $\frac{1}{n}$ (c) 1 (d) 2
7. The value of $15C_8 + 15C_9 - 15C_6 - 15C_7$ is ....
   (a) 0 (b) 1 (c) 2 (d) 3
8. The slope of the line which makes an angle 45° with the line $3x - y = -5$ are ....
   (a) $1, -1$ (b) $\frac{1}{2}, -2$ (c) $1, \frac{1}{2}$ (d) $2, \frac{-1}{2}$
9. The sum of the binomial co-efficients is ....
   (a) $2n$ (b) 2^n (c) n^2 (d) 1
10. If the square of the matrix $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ is the unit matrix of order 2 then α, β and γ should satisfy the relation ....
   (a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$
   (c) $1 - \alpha^2 + \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

13. One of the diagonals of parallelogram ABCD with $\vec{a}$ and $\vec{b}$ are adjacent sides is $\vec{a}+\vec{b}$. The other diagonal BD is ....

(a) $\vec{a}-\vec{b}$ (b) $\vec{b}-\vec{a}$ (c) $\vec{a}+\vec{b}$ (d) $\frac{\vec{a}+\vec{b}}{2}$

14. If (1, 2, 4) and (2, –3λ, –3) are the initial and terminal points of the vector $\vec{i}+5\vec{j}-7\vec{k}$ then the value of λ ....

(a) $\frac{7}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

15. If $y = mx + c$ and $f(0) = f'(0) = 1$ then $f(2) =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

16. The derivative of $\left(x+\frac{1}{x}\right)^2$ w.r.to. x is ....

(a) $2x-\frac{2}{x^3}$ (b) $2x+\frac{2}{x^3}$ (c) $2\left(x+\frac{1}{x}\right)$ (d) 0

17. If $f(x) = \begin{cases} ax^2 - b, & -1 < x < 1 \\ \frac{1}{|x|}, & \text{elsewhere} \end{cases}$ is differentiable at $x = 1$, then ....

(a) $a=\frac{1}{2}, b=\frac{-3}{2}$ (b) $a=\frac{-1}{2}, b=\frac{3}{2}$ (c) $a=-\frac{1}{2}, b=-\frac{3}{2}$ (d) $a=\frac{1}{2}$ $b=\frac{3}{2}$

18. $\int \sin 7x \cos 5x dx =$ ....

(a) $\frac{1}{2}\left[\frac{\cos 12x}{2}+\frac{\cos 2x}{2}\right]+c$

(b) $-\frac{1}{2}\left[\frac{\cos 12x}{2}+\frac{\cos 2x}{2}\right]+c$

(c) $-\frac{1}{2}\left[\frac{\cos 6x}{6}+\cos x\right]+c$

(d) $-\frac{1}{2}\left[\frac{\sin 12x}{2}+\frac{\sin 2x}{2}\right]+c$

19. $\int \frac{1}{e^x} dx =$ ....

(a) $\log e^x + c$ (b) $x + c$ (c) $\frac{1}{e^x}+c$ (d) $\frac{-1}{e^x}+c$

20. Two items are chosen from a lot containing twelve items of which four are defective. Then the probability that atleast one of the item is defective is ....

(a) $\frac{19}{33}$ (b) $\frac{17}{33}$ (c) $\frac{23}{33}$ (d) $\frac{13}{34}$

PART-II

II. Answer any seven questions. Question No. 30 is compulsory. **[7 × 2 = 14]**

21. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} \quad \frac{1+\sin\theta}{\cos\theta}$

Ans. $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$

$$= \frac{(\tan\theta + \sec\theta)[1-(\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta = \frac{1+\sin\theta}{\cos\theta}$$

22. Prove that the relation 'friendship' is not an equivalence relation on the set-of all people in chennai.

Ans. S = aRa (*i.e.*) a person can be a friend to himself or herself.

So it is reflextive.

$aRb \Rightarrow bRa$ so it is symmetric

aRb, bRc does not $\Rightarrow aRc$

so it is not transitive

$\Rightarrow$ it is not an equivalence relation

23. How many triangles can be formed by joining 15 points on the plane, in which no line joining any three points?

Ans. No. of non-collinear points = 15

To draw a Triangle we need 3 points

$\therefore$ Selecting 3 from 15 points can be done in $^{15}C_3$ ways.

$\therefore$ No. of Triangle formed = $^{15}C_3$

$$= \frac{15\times14\times13}{3\times2\times1} = 455$$

24. Expand $(2x+3)^5$

Ans. By taking $a = 2x$, $b = 3$ and $n = 5$ in the binomial expansion of $(a+b)^n$ we get

$$(2x+3)^5 = (2x)^5 + 5(2x)^4 3 + 10(2x)^3 3^2 + 10(2x)^2 3^3 + 5(2x)3^4 + 3^5$$

$$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243.$$

25. If $\lambda = -2$, determine the value of $\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2+1 \\ -1 & 6\lambda-1 & 0 \end{vmatrix}$

Ans. Given $\lambda = -2$

$\therefore \quad 2\lambda = -4;\ \lambda^2 = (-2)^2 = 4;\ 3\lambda^2 + 1 = 3(4) + 1 = 13$

$6\lambda - 1 = 6(-2) - 1 = -13$

So $$\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2+1 \\ -1 & 6\lambda-1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -4 & 1 \\ 4 & 0 & 13 \\ -1 & -13 & 0 \end{vmatrix}$$

expanding along R_1

$$0(0) + 4(0+13) + 1(-52+0) = 52 - 52 = 0$$

Aliter: The determinant value of a **skew symmetric matrix** is zero.

As repetition is not permitted, the 100^{th} place can be filled in 8 ways using remaining numbers and 10^{th} place can be filled in 7 ways. Hence, the required number of numbers is $1 \times 8 \times 7 \times 2 = 112$.

34. Find the $\sqrt[3]{126}$ approximately to two decimal places.

Ans. $\sqrt[3]{126} = (126)^{1/3} = (125+1)^{1/3} = \left\{125\left(1+\frac{1}{125}\right)\right\}^{1/3} = (125)^{1/3}\left[1+\frac{1}{125}\right]^{1/3}$

$$= 5\left[1+\frac{1}{3}\times\frac{1}{125}+\ldots\right]\left(\because \frac{1}{125}<1\right) = 5\left[1+\frac{1}{3}(0.008)\right] = 5(1+0.002666) = 5.01$$

35. Find the equation of the line through the intersection of the lines $3x + 2y + 5 = 0$ and $3x - 4y + 6 = 0$ and the point (1,1)

Ans. The family of equations of straight lines through the point of intersection of the lines is of the form $(a_1x + b_1y + c_1) + (a_2x + b_2y + c_2) = 0$

That is, $(3x + 2y + 5) + \lambda\,(3x - 4y + 6) = 0$

Since the required equation passes through the point (1,1), the point satisfies the above equation Therefore $\{3 + 2(1) + 5\} + \lambda\{3\,(1) - 4(1) + 6\} = 0 \Rightarrow \lambda = -2$

Substituting $\lambda = -2$ in the above equation we get the required equation as $3x - 10y + 7 = 0$

(verify the above problem by using two points form)

36. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$

Ans. $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$

Multiplying R_1 by a, R_2 by b, R_3 by c and dividing by abc we get

$$\frac{1}{abc}\begin{vmatrix} a(b+c) & abc & ab^2c^2 \\ b(c+a) & abc & a^2bc^2 \\ c(a+b) & abc & a^2b^2c \end{vmatrix}$$

$$= \frac{(abc)^2}{abc}\begin{vmatrix} ab+ac & 1 & bc \\ bc+ab & 1 & ca \\ ac+bc & 1 & ab \end{vmatrix} = (abc)\begin{vmatrix} ab+bc+ca & 1 & bc \\ ab+bc+ca & 1 & ca \\ ab+bc+ca & 1 & ab \end{vmatrix} (C_1 \to C_1 + C_3)$$

$$= (ab+bc+ca)(abc)\begin{vmatrix} 1 & 1 & bc \\ 1 & 1 & ca \\ 1 & 1 & ab \end{vmatrix} = 0 \;(\because C_1 = C_2)$$

37. Complete the following table using calculator and use the result to estimate $\lim_{x \to 2} \frac{x-2}{x^2-x-2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

Ans.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	0.345	0.334	0.3344	0.3332	0.3322	0.3225

$\therefore$ Limit is $0.333.... = 0.\overline{3}$

38. Differentiate $\frac{e^{3x}}{1+e^x}$ with respect to x

Ans.

$$y = \frac{e^{3x}}{1+e^x}$$

Let $y = \frac{u}{v}$; $y' = \frac{vu' - uv'}{v^2}$

Here $u = e^{3x} \Rightarrow u' = \frac{du}{dx} = e^{3x}(3) = 3e^{3x}$

$v = 1+e^x \Rightarrow v' = \frac{dv}{dx} = e^x$

Now $y' = \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$$= \frac{(1+e^x)(3e^{3x}) - e^{3x}(e^x)}{(1+e^x)^2}$$

$$= \frac{3e^{3x} + 3e^{4x} - e^{4x}}{(1+e^x)^2} = \frac{3e^{3x} + 2e^{4x}}{(1+e^x)^2}$$

39. Evaluate: e^x (tan x + log sec x)

Ans. Let $I = \int e^x (\tan x + \log \sec x)\, dx$

Take $f(x) = \log \sec x$

$$f'(x) = \frac{1}{\sec x} \times \sec x \tan x = \tan x$$

This is of the form $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + c$

$$\therefore \int e^x (\log \sec x + \tan x)\, dx = e^x \log|\sec x| + c$$

40. The position vectors of the vertices of a triangle are $\vec{i} + 2\vec{j} + 3\vec{k}$, $3\vec{i} - 4\vec{j} + 5\vec{k}$ and $-2\vec{i} + 3\vec{j} - 7\vec{k}$. Find the perimeter of a triangle.

Ans. Let A, B, C be the vertices of triangle ABC,

Then $\overrightarrow{OA} = \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{OB} = \vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{OC} = \vec{c} = -2\hat{i} + 3\hat{j} - 7\hat{k}$

Now, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (3\hat{i} - 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$$= (3\hat{i} - 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 2\hat{i} - 6\hat{j} + 2\hat{k}$$

$$|\overline{AB}| = \sqrt{4+36+4} = \sqrt{44} = AB$$

$$BC = \overline{OC} - \overline{OB} = (-2\hat{i} + 3\hat{j} - 7\hat{k}) - (3\hat{i} - 4\hat{j} + 5\hat{k})$$

$$= -2\hat{i} + 3\hat{j} - 7\hat{k} - 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$= -5\hat{i} + 7\hat{j} - 12\hat{k}$$

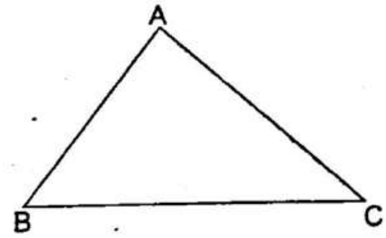
$$BC = \sqrt{25+49+144} = \sqrt{218} = BC$$

$$\overline{AC} = \overline{OC} - \overline{OA} = (-2\hat{i} + 3\hat{j} - 7\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -2\hat{i} + 3\hat{j} - 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$= -3\hat{i} + \hat{j} - 10\hat{k}$$

$$|\overline{AC}| = \sqrt{9+1+100} = \sqrt{110} = AC$$



$$\text{Perimeter of } \Delta ABC = AB + BC + AC$$

$$= \sqrt{44} + \sqrt{218} + \sqrt{110}$$

PART - IV

IV. Answer all the questions. **[7 × 5 = 35]**

41. (*a*) If $f : R \to R$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.

Ans.

$$P(x) = 3x - 5$$

$$\text{Let } y = 3x - 5 \Rightarrow 3x = y + 5$$

$$x = \frac{y+5}{3}$$

$$\text{Let } g(y) = \frac{y+5}{3}$$

$$\text{Now } g \circ f(x) = g[(f(x)] = g(3x-5)$$

$$= \frac{3x-5+5}{3} = x$$

$$\text{also } f \circ g(y) = f[g(y)] = f\left[\frac{y+5}{3}\right]$$

$$= 3\left[\frac{y+5}{3}\right] - 5 = y + 5 - 5 = y$$

Thus $g \circ f = I_x$ and $f \circ g = I_y$

f and g are bijections and inverse to each other. Hence f is a bijection and $f^{-1}(y) = \frac{y+5}{3}$

Replacing y by x we get $f^{-1}(x) = \frac{x+5}{3}$

[OR]

(*b*) **Prove that $\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{m-n}{m+n}\right)=\frac{\pi}{4}$**

Ans. $\text{LHS} = \tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{m-n}{m+n}\right)$

$$= \tan^{-1}\left(\frac{\frac{m}{n}-\frac{m-n}{m+n}}{1+\frac{m}{n}\times\frac{m-n}{m+n}}\right)=\tan^{-1}\left(\frac{m^2+mn-mn+n^2}{mn+n^2+m^2-mn}\right)$$

$$= \tan^{-1}\left(\frac{m^2+n^2}{m^2+n^2}\right)=\tan^{-1}(1)=\frac{\pi}{4}$$

42. (*a*) **Find the values of k so that the equation $x^2 = 2x(1+3k)+7(3+2k)=0$ has real and equal roots.**

Ans. The equation is $x^2 - x(2)(1+3k) - 7(3+2k) = 0$

The roots are real and equal

$\Rightarrow$ $\Delta = 0$ (*i.e.,*) $b^2 - 4ac = 0$

Here $a = 1$, $b = -2(1+3k)$, $c = 7(3+2k)$

So $b^2 - 4ac = 0 \Rightarrow [-2(1+3k)]^2 - 4(1)(7)(3+2k) = 0$

(*i.e.,*) $4(1+3k)^2 - 28(3+2k) = 0$

($\div$ by 4) $(1+3k)^2 - 7(3+2k) = 0$

$1 + 9k^2 + 6k - 21 - 14k = 0$

$9k^2 - 8k - 20 = 0$

$(k-2)(9k+10) = 0$

$\Rightarrow$ $k-2>0$ or $9k+10 = 0$

$\Rightarrow$ $k = 2$ or $k = \frac{-10}{9}$

To solve the quadratic inequalities $ax^2 + bx + c < 0$ (or) $ax^2 + bx + c > 0$ **[OR]**

(*b*) **If the roots of the equation $(q-r)x^2 + (r-p)x + (p-q) = 0$ are equal then show that p, q and r are in A.P.**

Ans. The roots are equal $\Rightarrow \Delta = 0$

(*i.e.*) $b^2 - 4ac = 0$

Hence, $a = q - r$; $b = r - p$; $c = p - q$

$b^2 - 4ac = 0$

$\Rightarrow$ $(r-p)^2 - 4(q-r)(p-q) = 0$

$r^2 + p^2 - 2pr - 4[qr - q^2 - pr + pq] = 0$

$r^2 + p^2 - 2pr - 4qr + 4q^2 + 4pr - 4pq = 0$

(*i.e.*) $p^2 + 4q^2 + r^2 - 4pq - 4qr + 2pr = 0$

(*i.e.*) $(p-2q+r)^2 = 0$

$$\Rightarrow \quad p-2q+r = 0$$

$$\Rightarrow \quad p+r = 2q$$

$\Rightarrow$ p, q, r are in A.P.

43. (*a*) Find the sum of all 4 digit-numbers that can be formed using the digits 1, 2, 3, 4 and 5 repetition not allowed?

Ans. The given digits are 1, 2, 3, 4, 5

The no. of 4 digit numbers

1000's	100's	10's	1's
5	4	3	2

$= 5 \times 4 \times 3 \times 2 = 120$

(*i.e*) ${}^5P_4 = 120$

Now we have 120 numbers

So each digit occurs $\frac{120}{5} = 24$ times

Sum of the digits $= 1+2+3+4+5 = 15$

Sum of number's in each place $= 24 \times 15 = 360$

Sum of numbers $=$

$$\begin{aligned} 360 \times 1 &= 360 \\ 360 \times 10 &= 3600 \\ 360 \times 100 &= 36000 \\ 360 \times 1000 &= 360000 \\ & 399960 \end{aligned}$$

[OR]

(*b*) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$ and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. Find $4\vec{a}\cdot\vec{b}+3\vec{b}\cdot\vec{c}+3\vec{c}\cdot\vec{a}$.

Ans. Given $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow$ $\vec{a}+\vec{b} = -\vec{c}$

So $(\vec{a}+\vec{b})^2 = \vec{c}^{\,2}$

(*i.e.,*) $a^2+b^2+2\vec{a}\cdot\vec{b} = \vec{c}^{\,2}$

$\Rightarrow$ $4+9+2\vec{a}\cdot\vec{b} = 16$

$\Rightarrow$ $2\vec{a}\cdot\vec{b} = 16-4-9 = 3$

$\vec{a}\cdot\vec{b} = 3/2$

Again $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow$ $\vec{a}+\vec{c} = -\vec{b}$

$(\vec{a}+\vec{c})^2 = \vec{b}^{\,2}$

$\vec{a}^{\,2}+\vec{c}^{\,2}+2\vec{a}\cdot\vec{c} = \vec{b}^{\,2}$

$4+16+2\vec{a}\cdot\vec{c} = 9$

$2\vec{a}\cdot\vec{c} = 9-4-16 = -11$

$\vec{a}\cdot\vec{c} = \frac{-11}{2}$ (*i.e.,*) $\vec{c}\cdot\vec{a} = \frac{-11}{2}$ $\qquad (\because \vec{a}\cdot\vec{c} = \vec{c}\cdot\vec{a})$

Also $\vec{a}+\vec{b}+\vec{c}=0$

$$\Rightarrow \quad \vec{b}+\vec{c} = -\vec{a}$$

$$\left(\vec{b}+\vec{c}\right)^2 = \vec{a}^2$$

$$\therefore \quad 9+16+2\vec{b}\cdot\vec{c} = 4$$

$$2\vec{b}\cdot\vec{c} = 4-9-16=-21$$

$$\vec{b}\cdot\vec{c}=\frac{-21}{2}$$

Here, $\vec{a}\cdot\vec{b} = 3/2;\ \vec{b}\cdot\vec{c} = -\frac{21}{2}$ and $\vec{c}\cdot\vec{a} = \frac{-11}{2}$

So, $4(\vec{a}\cdot\vec{b})+3(\vec{b}\cdot\vec{c})+3(\vec{c}\cdot\vec{a}) = 4\left(\frac{3}{2}\right)+3\left(\frac{-21}{2}\right)+3\left(\frac{-11}{2}\right)$

$$= 6-\frac{63}{2}-\frac{33}{2}=6-\frac{96}{2}=6-48=-42.$$

44. (*a*) If *a*, *b*, *c* are respectively the p^{th}, q^{th} and r^{th} terms of a G.P. show that $(q-r)\log a+(r-p)\log b+(p-q)\log c=0$.

Ans. Let the G.P. be $l, lk, lk^2, \ldots$

We are given $t_p=a, t_q=b, t_r=c$

$\Rightarrow a=lk^{p-1}\ ;\ b=lk^{q-1}\ ;\ c=lk^{r-1}$

$$a=lk^{p-1} \quad\Rightarrow\quad \log a=\log l+\log k^{p-1}=\log l+(p-1)\log k$$

$$b=lk^{q-1} \quad\Rightarrow\quad \log b=\log l+\log k^{q-1}=\log l+(q-1)\log k$$

$$c=lk^{r-1} \quad\Rightarrow\quad \log c=\log l+\log k^{r-1}=\log l+(r-1)\log k$$

LHS $= (q-r)\log a+(r-p)\log b+(p-q)\log c$

$= (q-r)[\log l+(p-1)\log k]+(r-p)[\log l+(q-1)\log k]+(p-q)[\log l+(r-1)\log k]$

$= \log l\,[p-q+q-r+r-p]+\log k\,[(q-r)(p-1)+(r-p)(q-1)+(p-q)(r-1)]$

$= \log l\,(0)+\log k\,[p(q-r)+q(r-p)+r(p-q)-(q-r+r-p+p-q)]$

$= 0 = $ RHS.

[OR]

(*b*) If $A=\begin{bmatrix}\frac{1}{2} & \alpha\\ 0 & \frac{1}{2}\end{bmatrix}$, prove that $\sum_{k=1}^{n}\det(A^k)=\frac{1}{3}\left(1-\frac{1}{4^n}\right)$

Ans. $$A = \begin{bmatrix}\frac{1}{2} & \alpha\\ 0 & \frac{1}{2}\end{bmatrix}$$

$$|A| = \begin{vmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{vmatrix} = \frac{1}{4} - 0 = \frac{1}{4}$$

$$A^2 = A \times A = \begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \alpha \\ 0 & \frac{1}{4} \end{pmatrix}$$

$$|A^2| = \begin{vmatrix} \frac{1}{4} & \alpha \\ 0 & \frac{1}{4} \end{vmatrix} = \frac{1}{4} \times \frac{1}{4} - 0 = \left(\frac{1}{4}\right)^2 = \frac{1}{4^2}$$

$$|A^k| = \frac{1}{4^k}$$

So, $\sum_{k=1}^{n} \det(A^k) = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots\ldots + \frac{1}{4^n}$

Which is a G.P with $a = \frac{1}{4}$ and $r = \frac{1}{4}$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{1}{4}\left[1-\left(\frac{1}{4}\right)^n\right]}{1-\frac{1}{4}}$$

$$= \frac{\frac{1}{4}\left[1-\frac{1}{4^n}\right]}{\frac{3}{4}} = \frac{1}{4} \times \frac{4}{3}\left[1-\frac{1}{4^n}\right]$$

$$= \frac{1}{3}\left[1-\frac{1}{4^n}\right]$$

45. (*a*) Find the equation of the straight line passing through intersection of the straight lines $5x - 6y = 1$ and $3x + 2y + 5 = 0$ and perpendicular to the straight line $3x - 5y + 11 = 0$.

Ans. Equation of line through the intersection of straight lines $5x - 6y = 1$ and $3x + 2y + 5 = 0$ is

$$5x - 6y - 1 + k(3x + 2y + 5) = 0$$

$$x(5 + 3k) + y(-6 + 2k) + (-1 + 5k) = 0$$

This is perpendicular to $3x - 5y + 11 = 0$

That is, the product of their slopes is -1

$$-\left(\frac{5+3k}{-6+2k}\right)\left(-\frac{3}{-5}\right) = -1$$

$$\Rightarrow \frac{15+9k}{-30+10k} = 1$$

$$\Rightarrow 15 + 9k = -30 + 10k$$

$$45 = k$$

Required equation is $5x - 6y - 1 + 45(3x + 2y + 5) = 0$

$$140x + 84y + 224 = 0$$

$$20x + 12y + 32 = 0$$

$$5x + 3y + 8 = 0$$

[OR]

(*b*) Integrate the following $\frac{\sqrt{x}}{1+\sqrt{x}}\,dx$

Ans. $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$ ...(1)

$$t = 1 + \sqrt{x} \quad ...(2)$$

$$\frac{dt}{dx} = 0 + \frac{1}{2\sqrt{x}}$$

$$2\sqrt{x}\,dt = dx \quad ...(3)$$

sub (2) and (3) in (1)

$$\Rightarrow \quad \int \frac{\sqrt{x}}{t} 2\sqrt{x}\,dt = 2\int \frac{x}{t}\,dt$$

$$= 2\int \frac{(t-1)^2}{t}\,dt$$

$$= 2\int \frac{(t^2 - 2t + 1)}{t}\,dt$$

$$= 2\int \left[t - 2 + \frac{1}{t}\right]dt = 2\left[\frac{t^2}{2} - 2t + \log t\right] + c$$

$$= 2\left[\frac{(1+\sqrt{x})^2}{2} - 2(1+\sqrt{x}) + \log\left|1+\sqrt{x}\right|\right] + c$$

$$= (1+\sqrt{x})^2 - 4(1+\sqrt{x}) + 2\log\left|1+\sqrt{x}\right| + c$$

46. (*a*) If $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **and** $v = \tan^{-1} x$, **find** $\frac{du}{dv}$

Ans.

$$u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) \text{ and } v = \tan^{-1} x$$

$$\frac{du}{dv} = \frac{du}{d\theta}\Big/\frac{dv}{d\theta}$$

To find $\frac{dv}{dx}$:

$$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta = 1 + \tan\theta$$

$$\frac{\sqrt{1+x^2}-1}{x} = \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \frac{\sec\theta - 1}{\tan\theta} = \frac{1}{\cos\theta} - 1\Big/\frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\therefore \quad \text{So } u = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\Rightarrow \quad \frac{du}{d\theta} = \frac{1}{2}$$

$$v = \tan^{-1} x \Rightarrow v = \tan^{-1}(\tan\theta) = 1$$

$$\frac{dv}{d\theta} = 1.$$

$$\frac{du}{dv} = \frac{du/d\theta}{dv/d\theta} = \frac{1/2}{1} = \frac{1}{2}$$

[OR]

(*b*) If $y = Ae^{6x} + Be^{-x}$ prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$

Ans. $y = Ae^{6x} + Be^{-x}$ .... (1)

$$y_1 = \frac{dy}{dx} = Ae^{6x}(6) + Be^{-x}(-1)$$

$$= 6Ae^{6x} - Be^{-x} \text{ .... (2)}$$

$$y_2 = \frac{d\ y}{dx} = 6Ae^{6x}(6) - Be^{-x}(-1)$$

$$= 36A\,e^{6x} + Be^{-x} \text{ .... (3)}$$

eliminating A and B from (1), (2) and (3) we get

$$\begin{vmatrix} y & A & B \\ y_1 & 6A & -B \\ y_2 & 36A & B \end{vmatrix} = 0$$

$$(i.e.,) \begin{vmatrix} y & 1 & 1 \\ y_1 & 6 & -1 \\ y_2 & 36 & 1 \end{vmatrix} = 0$$

$$y(6+36) - y_1(1-36) + y_2(-1-6) = 0$$

$$42y + 35y_1 - 7y_2 = 0$$

$$(\div \text{ by } -7)\ y_2 - 5y_1 - 6y = 0$$

$$(i.e.,)\ \frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$$

47. (*a*) Evaluate: $\lim\limits_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

Ans. $\lim\limits_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4} \times \frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1} \times \frac{\sqrt{x^2+16}+14}{\sqrt{x^2+16}+4}$

$$= \lim_{x\to 0} \frac{\left[(x^2+1)-1\right]\left[\sqrt{x^2+16}+4\right]}{(x^2+16-16)\left[\sqrt{x^2+1}+1\right]}$$

$$= \lim_{x\to 0} \frac{x^2\left[\sqrt{x^2+16}+4\right]}{x^2\left[\sqrt{x^2+1}+1\right]}$$

$$= \frac{4+4}{1+1} = \frac{8}{2} = 4$$

[OR]

(b) Urn-I contains 8 red and 4 blue balls and urn-II contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red.

Ans. Let A_1 be the event of selecting urn-I and A_2 be the event of selecting urn-II.

Let B be the event of selecting 2 red balls.

	Red balls	Blue balls	Total
Urn-I	8	4	12
Urn-II	5	10	15
Total	13	14	27

We have to find the total probability of event B. That is, P(B).

Clearly A_1 and A_2A_1 are mutually exclusive and exhaustive events.

We have

$$P(A_1) = \frac{1}{2}, P(B/A_1) = \frac{8c_2}{12c_2} = \frac{14}{33}$$

$$P(A_2) = \frac{1}{2}, P(B/A_2) = \frac{5c_2}{15c_2} = \frac{2}{21}$$

We know $P(B) = P(A_1).P(B/A_1) + P(A_2).P(B/A_2)$

$$P(B) = \frac{1}{2}\cdot\frac{14}{33} + \frac{1}{2}\cdot\frac{2}{21} = \frac{20}{77}$$