CUET (UG)

Mathematics Sample Paper - 10

Solved

Time Allowed: 50 minutes **Maximum Marks: 200**

General Instructions:

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

Section A

1. If $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the least positive integral value of k is **[5]**

a) 6 b) 4

c) 3 d) 7

2. If A = [a b], B = [-b -a] and C = $\begin{vmatrix} a \\ -a \end{vmatrix}$, then which is the correct statement. **[5]**

a) A = -B b) AC = BC

c) A + B = B + A d) CA = CB

3. The order of the single matrix obtained from **[5]**

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times2} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}_{2\times3} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}_{2\times3} \right\}$$ is

a) 2×3 b) 3×3

c) 3×2 d) 2×2

4. Find the slope of the normal to the curve $x = acos^3\theta, y = asin^3\theta$ at $\theta = \frac{\pi}{4}$ **[5]**

a) none of these b) -1

c) 1 d) 0

5. Minimum value of the function f(x) = $x^2 + x + 1$ is **[5]**

a) none of these
b) 3
c) $\frac{3}{4}$
d) 1

6. The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point **[5]**

a) $\left(a, \frac{a}{b}\right)$
b) (– a ,ba)
c) (a,-a)
d) (0,b)

7. $\int \frac{\log x}{x} dx = ?$ **[5]**

a) $-\frac{1}{2}(\log x)^2 + C$
b) $\frac{1}{2}(\log x)^2 + C$
c) $\frac{-2}{x^2} + C$
d) $\frac{2}{x^2} + C$

8. $\int \frac{e^x}{\sqrt{1+e^x}} dx = ?$ **[5]**

a) $\frac{1}{2}\sqrt{1+e^x} + C$
b) $2\sqrt{1+e^x} + C$
c) None of these
d) $\frac{1}{\sqrt{1+e^x}} + C$

9. $\int \tan^{-1}(\ cosec\, x - \cot x) dx = ?$ **[5]**

a) $\frac{-x^2}{2} + C$
b) $\frac{-x^2}{4} + C$
c) $\frac{x^2}{4} + C$
d) $\frac{x^2}{2} + C$

10. The area bounded by the curve y = $x^4 - 2x^3 + x^2 + 3$ with x-axis and ordinates corresponding to the minutes of y is **[5]**

a) 1
b) 4
c) $\frac{30}{9}$
d) $\frac{91}{30}$

11. General solution of $\left(1 + x^2\right) dy + 2xy\, dx = \cot x\, dx\ (x \neq 0)$ is **[5]**

a) $y(1 + x^2) = log|sinx| + c$
b) $y = (1 + x)^{-1} \log|\sin x| - C(1 + x^2)^{-1}$
c) $y = (1 + x)^{-1} \log|\sin x| + C(1 - x^2)^{-1}$
d) $y = (1 + x)^{-1} \log|\sin x| - C(1 - x^2)^{-1}$

12. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is **[5]**

a) $e^x + e^{-y} = C$

b) $e^{-x} + e^{-y} = C$

c) $e^{-x} + e^y = C$

d) $e^x + e^y = C$

13. The region represented by the inequation system x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3 is **[5]**

a) unbounded in first and second quadrants

b) bounded in first quadrant

c) None of these

d) unbounded in first quadrant

14. Let X be a discrete random variable. The probability distribution of X is given below: **[5]**

X	30	10	-10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E (X) is equal to

a) 6

b) 4

c) 3

d) –5

15. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. The probability that B and C hit and A does not hit is **[5]**

a) None of these

b) $\frac{7}{12}$

c) $\frac{1}{10}$

d) $\frac{2}{5}$

Section B

Attempt any 25 questions

16. Equivalence classes A_i satisfy **[5]**

A. No element of A_i is related to any element of A_j, $i \neq j$

B. No element of A_i is related to any element of A_i

C. Some elements of A_i are related to any element of A_j, , $i \neq j$

D. All elements of A_i are related to any element of A_j, $i \neq j_i$ are related to any element of A_j, $i \neq j$

a) B

b) C

c) A

d) D

17. $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=$ **[5]**

a) 0
b) None of these
c) -1
d) 1

18. If A is a square matrix, then A – A’ is a **[5]**

a) symmetric matrix
b) none of these
c) skew-symmetric matrix
d) diagonal matrix

19. The value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = 3A - B then the values of A and B are: **[5]**

a) A = 3abc, B = a + b + c
b) A = 2abc, B = a + b + c
c) A = abc, B = $a^3 + b^3 + c^3$
d) A = 0, B = $a^2 + b^2 + c^2$

20. The matrix $A=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is **[5]**

a) Orthogonal
b) Nonsingular
c) Idempotent
d) Nilpotent

21. $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$ = ? **[5]**

a) 2(a + b + c)
b) 0
c) $a^2b^2c^2$
d) a + b+ c

22. If x = f (t) cos t - f '(t) sint and y = f (t) sin t + f '(t) cost, then $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=$ **[5]**

a) none of these
b) f (t) - f " (t)
c) $\{f(t) + f''(t)\}^2$
d) $\{f(t) - f''(t))\}^2$

23. If f(x) = $\begin{cases} kx+5, \text{ when } x \leq 2 \\ x+1, \text{ when } x > 2 \end{cases}$ is continuous at x = 2 then k = ? **[5]**

a) -2

b) -1

c) 2

d) -3

24. Find the value of f(0), so that the function $f(x) = \frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, given by **[5]**

a) $a^{1/2}$

b) $-a^{1/2}$

c) $a^{3/2}$

d) $-a^{3/2}$

25. If $y^2 = ax^2 + b$, then $\frac{d^2y}{dx^2}$ is equal to **[5]**

a) None of these

b) $\frac{ab}{y^2}$

c) $\frac{ab}{y^3}$

d) $\frac{ab}{x^3}$

26. Number of points at which f(x) = $\frac{1}{\log|x|}$ is discontinuous is **[5]**

a) 1

b) 3

c) 2

d) 4

27. f(x) = 2x - $\tan^{-1}$ x - log $\{x+\sqrt{x^2+1}\}$ is monotonically increasing when **[5]**

a) x ∈ R

b) x > 0

c) x ∈ R - (0)

d) x < 0

28. The slope of the tangent to the curve x = a (cos θ + θ sin θ),y = a (sin θ – θ cos θ) at any point 'θ' is **[5]**

a) tan θ

b) – cot θ

c) – tan θ

d) cot θ

29. Function f(x) = a^x is increasing on R, if **[5]**

a) a > 0

b) a < 0

c) a > 1

d) 0 < a < 1

30. The minimum value of $\frac{x}{\log x}, x > 1$, is **[5]**

a) none of these
b) e
c) – e
d) $\frac{1}{e}$

31. $\int$ x log x dx = ? **[5]**

a) $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$
b) $x \log x + \frac{1}{2}x^2 + C$
c) $\frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C$
d) None of these

32. If f(a + b - x) = f (x), then $\int_a^b xf(x)dx$ is equal to **[5]**

a) $\frac{a+b}{2}\int_a^b f(x)dx$
b) $\frac{b-a}{2}\int_a^b f(x)dx$
c) $\frac{a+b}{2}\int_a^b f(b-x)dx$
d) $\frac{a+b}{2}\int_a^b f(b+x)dx$

33. $\int (x2^x)\, dx = ?$ **[5]**

a) $\frac{2^x}{(\log 2)^2}(x \log 2 - 1) + C$
b) None of these
c) $\frac{x\cdot 2^x}{(\log 2)} + \frac{2^x}{(\log 2)^2} + C$
d) $\frac{2^x}{(\log 2)}(x + \log 2) + C$

34. $\int \frac{\sin 2x}{\sin x}dx = ?$ **[5]**

a) 2 cos x + C
b) 2sin x + C
c) $\frac{1}{2}\sin x + C$
d) $\frac{1}{2}\cos x + C$

35. The area bounded by y =| sinx |, the x – axis and the line $|x| = \pi$ is **[5]**

a) none of these
b) 4 sq. units
c) 6 sq. units
d) 2 sq. units

36. Find the particular solution for (x + y)dy + (x –y) dx =0; y=1 when x =1 **[5]**

a) $\log\left(x^2 - y^3\right) - 2\tan^{-1}\frac{y}{x} = \frac{\pi}{2} - \log 2$
b) $\log\left(x^2 + y^2\right) - 2\tan^{-1}\frac{y}{x} = \frac{\pi}{2} + \log 2$
c) $\log\left(x^2 - y^2\right) + 2\tan^{-1}\frac{y}{x} = \frac{\pi}{2} + \log 2$
d) $log|x^2 + y^2| + 2tan^{-1}\frac{y}{x} = log2 + \frac{\pi}{2}$

37. The differential equation representing the family of curves y = a sin (λx + α) is **[5]**

a) $\frac{d^2y}{dx^2} - \lambda y = 0$
b) $\frac{d^2y}{dx^2} + \lambda y = 0$

c) $\frac{d^2y}{dx^2} - \lambda^2 y = 0$

d) $\frac{d^2y}{dx^2} + \lambda^2 y = 0$

38. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is: **[5]**

a) e^x

b) $\frac{x}{e^x}$

c) $\frac{e^x}{x}$

d) xe^x

39. If $|\vec{a}| = 10$, $|\vec{b}| = 2$, and $\vec{a} \cdot \vec{b} = 12$, then value of $|\vec{a} \times \vec{b}|$ is **[5]**

a) 5

b) 16

c) 14

d) 10

40. If $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$ is **[5]**

a) 2

b) 3

c) 0

d) None of these

41. Consider the following inequalities in respect of vectors $\vec{a}$ and $\vec{b}$. **[5]**

I. $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

II. $|\vec{a} - \vec{b}| \geq |\vec{a}| - |\vec{b}|$

Which of the above is/are correct?

a) Only II

b) Neither I nor II

c) Both I and II

d) Only I

42. If G is the intersection of diagonals of a parallelogram ABCD and O is any point, then $O\vec{A} + O\vec{B} + O\vec{C} + O\vec{D} =$ **[5]**

a) $3\overrightarrow{OC}$

b) $4\overrightarrow{OG}$

c) $2\overrightarrow{OG}$

d) $5\overrightarrow{OG}$

43. Let $\hat{a}$, $\hat{b}$ be two unit vectors and θ be the angle between them. What is $\cos\left(\frac{\theta}{2}\right)$ equal to? **[5]**

a) $\frac{|\hat{a}+\hat{b}|}{2}$

b) $\frac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{4}$

c) $\frac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{2}$

d) $\frac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{4}$

44. If a vector makes angles α, β and γ with the x axis, y axis and z axis respectviely then the value of $\left(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma\right)$ is **[5]**

a) 2
b) 0
c) 3
d) 1

45. The locus represented by xy + yz = 0 is **[5]**

a) A pair of perpendicular planes
b) A pair of perpendicular lines
c) A pair of parallel lines
d) A pair of parallel planes

46. Cartesian equation of a plane that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1$ and $A_2x + B_2y + C_2z + D_2 = 0$ is **[5]**

a) $(A_1x + B_1y - C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$
b) $(-A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$
c) $(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$
d) $(A_1x - B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$

47. You are given that A and B are two events such that P(B) = $\frac{3}{5}$, P(A | B) = $\frac{1}{2}$ and P(A ∪ B) = $\frac{4}{5}$, then P(A) equals **[5]**

a) $\frac{1}{2}$
b) $\frac{1}{5}$
c) $\frac{3}{5}$
d) $\frac{3}{10}$

48. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter driver, car driver, and truck driver are 0.01, 0.03, and 0.15, respectively. One of the insured persons meets with an accident. The probability that the person is a scooter driver, is **[5]**

a) $\frac{1}{52}$
b) $\frac{19}{52}$
c) $\frac{15}{52}$
d) $\frac{3}{52}$

49. 8 coins are tossed simultaneously. The probability of getting atleast 6 heads is **[5]**

a) $\frac{7}{64}$
b) $\frac{249}{256}$
c) $\frac{57}{64}$
d) $\frac{37}{256}$

50. Let X be a discrete random variable. Then the variance of X is **[5]**

a) $\sqrt{E(X^2) - (E(X))^2}$

b) $E(X^2) - (E(X))^2$

c) $E(X^2) + (E(X))^2$

d) $E(X^2)$

Solutions

Section A

1.
(d) 7

Explanation: $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = \cos^2\frac{2\pi}{7} - \sin\frac{2\pi}{7}\left(-\sin\frac{2\pi}{7}\right)$

$= \text{Cos}^2\ \frac{2\pi}{7} + \text{Sin}^2\ \frac{2\pi}{7}$

I = 1

I^k = I {K can be anything}

Let $\theta = \frac{2\pi}{7}$

$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & -\sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$

As $\{\cos^2\theta - \sin^2\theta = \cos^2\theta$ and $2\text{Sin}\ \theta\cos\theta = \sin 2\theta\}$

$= \begin{bmatrix} \cos 2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^4 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} \times \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta & -\sin 4\theta \\ \sin 4\theta & \cos 4\theta \end{bmatrix}$

Similarly, $A^7 = \begin{bmatrix} \cos 7\theta & -\sin 7\theta \\ \sin 7\theta & \cos 7\theta \end{bmatrix}$

Hence, $\theta = \frac{2\pi}{7}$

$7\theta = 2\pi$

Multiplying Cos & Sin, to LHS & RHS,

$\text{Cos}7\theta = \text{Cos}2\pi = 1$

$\text{Sin}7\theta = \sin 2\theta = 0$

$$\begin{bmatrix} \cos 7\theta & -\sin 7\theta \\ \sin 7\theta & \cos 7\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, k = 7

$A^7 = I$

Hence, k = 7.

2.

(c) A + B = B + A

Explanation: $\because$ A + B = B + A [by properties matrix addition, is communtative]

3.

(b) 3 × 3

Explanation: $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times 2} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}_{2\times 3} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}_{2\times 3} \right\}$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3\times 2} \begin{bmatrix} -1 & -1 & -21 \\ 1 & 0 & -20 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} -2 & -1 & -1 \\ 2 & 0 & -40 \\ 1 & -2 & -102 \end{bmatrix}_{3\times 3}$$

4.

(c) 1

Explanation: $x = a\cos^3\theta$, $y = a\sin^3\theta$

$$= \frac{dx}{d\theta} = -3acos^2\theta sin\theta, \frac{dy}{d\theta} = 3asin^2\theta cos\theta$$

Slope of tangent = $= \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3acos^2\theta sin\theta}{3asin^2\theta cos\theta} = -tan\theta$

Slope of normal is $\frac{-1}{\frac{dy}{dx}} = \frac{-1}{-tan\theta} = cot\theta$

When , Slope of normal $= cot\frac{\pi}{4} = 1$

5.

(c) $\frac{3}{4}$

Explanation: Given, f(x) = x^2 + x +1

⇒ f'(x) = 2x + 1

For minimum value of f(x) we have f'(x) = 0

⇒ 2x + 1 = 0 ⇒ x = $\frac{-1}{2}$

Now, f''(x) = 2 >0, hence the minimum of f(x) exist at x = $\frac{-1}{2}$ and minimum value = f($\frac{-1}{2}$) = $\frac{3}{4}$

6.

(d) (0,b)

Explanation: Given $y = be^{-x/a}....(i)$

Then, Slope of the tangent at any point (x,y) for the curve (i) is $\frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$

Slope of the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{dy}{dx} = \frac{-b}{a}$

If the line touches the curve then

$\frac{-b}{a}e^{-x/a} = \frac{-b}{a}$

⇒ $e^{-x/a} = 1$

⇒ $\frac{x}{a} = 0$

⇒ $x = 0$

Now, put x= 0 in (i), we get y = b

Hence, we have the line touches the curve at (0,b) which is a point on the Y-axis.

7.

(b) $\frac{1}{2}(\log x)^2 + C$

Explanation: Given Integral be: I =$\int \frac{\log x}{x}$

Let logx = u

$\Rightarrow \frac{1}{x}dx = du$

$\therefore \int \frac{\log x}{x}dx$

$= \int udu$

$= \frac{u^2}{2} + c$

$= \frac{(\log x)^2}{2} + c$

where c is the integrating constant.

8.

(b) $2\sqrt{1 + e^x} + C$

Explanation: Given: I = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, $1 + e^x = z^2$

$\Rightarrow e^x dx = 2zdz$

So,

$\int \frac{e^x}{\sqrt{1+e^x}}dx$

$= \int \frac{2zdz}{z}$

=2 ∫ dz

=2z+c

$= 2\sqrt{1 + e^x} + c$

where c is the integrating constant

9.

(c) $\frac{x^2}{4} + C$

Explanation: Given : $\int \tan^{-1}(\ cosec\ x - \cot x)dx = \int \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)dx$

$$= \int \tan^{-1}\left(\frac{2\sin\frac{x}{2}\sin\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)dx$$

$$= \int \tan^{-1}\left(\tan\frac{x}{2}\right)dx$$

$$= \int \frac{x}{2}dx$$

$$\frac{x^2}{4} + C$$

10.

(d) $\frac{91}{30}$

Explanation: The area bounded by the curve y = x^4 - $2x^3$ + x^2 + 3 with x-axis and ordinates

Minimum value of y when x = 0 is y = 3

Minimum value of y when x = 1 is y = 3

$$\Rightarrow \int_0^1 \left(x^4 - 2x^3 + x^2 + 3\right)dx$$

$$\Rightarrow \left[\frac{x^5}{5} - 2\frac{x^4}{4} + \frac{x^3}{3} + 3x\right]_0^1$$

$$\Rightarrow \frac{1}{5} - \frac{2}{4} + \frac{1}{3} + 3$$

$$\Rightarrow \frac{91}{30}$$

11. **(a)** $y(1 + x^2) = log|sinx| + c$

Explanation: $(1 + x^2)dy = (cotx - 2xy)dx$

$$\frac{dy}{dx} = \frac{cotx - 2xy}{1 + x^2}$$

$$\frac{dy}{dx} + \frac{2x}{1 + x^2}y = \frac{cotx}{1 + x^2}$$

It is a linear differential equation in y.

Therefore, Solution is

$$ye^{\int \frac{2xdx}{1+x^2}} = \int \frac{cotx}{1+x^2} e^{\int \frac{2xdx}{1+x^2}} dx + c$$

$$y(1 + x^2) = \int \frac{cotx}{1+x^2}(1 + x^2)dx + c$$

$$y(1 + x^2) = \int cotxdx + c$$

$$y(1 + x^2) = log|sinx| + c$$

12. **(a)** $e^x + e^{-y} = C$

Explanation: We have, $\frac{dy}{dx} = e^{x+y}$

$\Rightarrow \frac{dy}{dx} = e^x \times e^y$

separating variables

$\Rightarrow e^{-y}dy = e^x dx$

Integrating both sides

$\Rightarrow \int e^{-y}dy = \int e^x dx$

$\Rightarrow -e^{-y} = e^x + c$

$\Rightarrow e^x + e^{-y} = -c$

Or,

$e^x + e^{-y} = c$ (c is a constant)

13.

(b) bounded in first quadrant

Explanation: Converting the given inequations into equations, we obtain

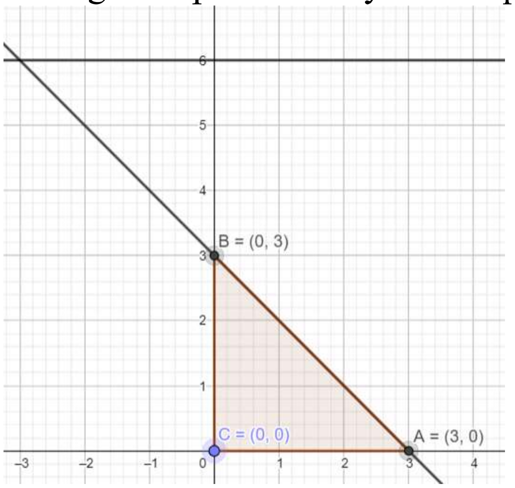
y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The

region below the line y = 6 will satisfy the given inequation.

The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation $x + y \leq 3$. So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by $x \geq 0$ and $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



14.

(b) 4

Explanation: E(X) = $30 \times \frac{1}{5} + 10 \times \frac{3}{10} - 10 \times \frac{1}{2} = 4$

15.

(c) $\frac{1}{10}$

Explanation: $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$

$P\left(B \cap C \cap A'\right) = P(B \cap C) - P(B \cap C \cap A)$

As the events are independent,

So, $P(B \cap C) = P(B) \cdot P(C) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

And $P(B \cap C \cap A) = P(B). P(C). P(A) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$

$P\left(B \cap C \cap A'\right) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Section B

16.

(c) A

Explanation: $A_1, A_2, A_3, \ldots \ldots \ldots \ldots A_k$ be subsets of a set A such that $\bigcup_{i=1}^{n} A_i = A$

and $A_i \cap A_j = \phi$ for $i \neq j$

17. **(a)** 0

Explanation: we know that $\cot^{-1}\left[\frac{xy+1}{x-y}\right] = \cot^{-1}x - \cot^{-1}y$

$\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$

$= \cot^{-1}a - \cot^{-1}b + \cot^{-1}b - \cot^{-1}c + \cot^{-1}c - \cot^{-1}a = 0$

18.

(c) skew-symmetric matrix

Explanation: The difference of a matrix A and its transpose is always skew – symmetric.

19.

(c) $A = abc, B = a^3 + b^3 + c^3$

Explanation: $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= abc - a^3 - b^3 + abc + abc - c^3$

$= 3abc - (a^3 + b^3 + c^3)$

$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - (a^3 + b^3 + c^3)$

$\therefore$ $A = abc, B = a^3 + b^3 + c^3$

20.

(c) Idempotent

Explanation: clearly for given matrix $A^2 = A$

Therefore idempotent

21.

(b) 0

Explanation: 0

22.

(c) $\{f(t) + f''(t)\}^2$

Explanation: x = f(t) cos t - f(t) sin t, and y = f(t) sint + f(t) cos t

Now,

$x \cos t + y \sin t = f(t)(\cos^2 t + \sin^2 t) = f(t)$ ..(i)

$x \sin t - y \cos t = -f'(t)(\cos^2 t + \sin^2 t) = -f'(t)$...(ii)

Differentiating (i) and (ii) w.r.t. to t we get

$\frac{dx}{dt}\cos t - x\sin t + \frac{dy}{dt}\sin t + y\cos t = f'(t)$

$\Rightarrow \frac{dx}{dt}\cos t + \frac{dy}{dt}\sin t = 0$ ....(iii) and $\frac{dx}{dt}\sin t + x\cos t - \frac{dy}{dt}\cos t + y\sin t = -f''(t)$

$\Rightarrow \frac{dx}{dt}\sin t - \frac{dy}{dt}\cos t = -f''(t) - f(t)$ ...(iv)

Solving (iii) and (iv) we get

$\frac{dx}{dt}$ = -sin t (f "(t) + f(t))

$\frac{dx}{dt}$ = cos t (f "(t) + f(t))

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(f''(t) + f(t)\right)^2$$

23.

(b) -1

Explanation: For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x = 2..

L.H.L = $\lim_{x \to 2^-}(kx + 5)$

$\Rightarrow \lim_{h \to 0}(k(2 - h) + 5)$

$\Rightarrow$ k(2 - 0) + 5 = 2k + 5

R.H.L = $\lim_{x \to 2^+}(x + 1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

$\Rightarrow$ 2 + 0 + 1

= 3

As f(x) is continuous,we get

$\because$ 2k + 5 = 3

k = -1.

24.

(b) $-a^{1/2}$

Explanation: $\lim\limits_{x \to 0} \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$

$= \lim\limits_{x \to 0} \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$

$= \lim\limits_{x \to 0} \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{a + x - (a - x)}$

$$x\frac{(\sqrt{a+x}+\sqrt{a-x})\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}$$

$$= \lim_{x\to 0} \frac{a^2-ax+x^2-\left(a^2+ax+x^2\right)}{2x} \times \frac{\sqrt{a+x}+\sqrt{a}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2}}$$

$$= \lim_{x\to 0} \frac{-2ax}{2x} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}$$

$$= -\frac{2a\sqrt{a}}{a+a}$$

$$= -\frac{2a\sqrt{a}}{2a}$$

$$= -\sqrt{a}$$

25.

(c) $\frac{ab}{y^3}$

Explanation: $y^2 = ax^2 + b \Rightarrow 2y\frac{dy}{dx} = 2ax \Rightarrow y\frac{dy}{dx} = ax$

$$\Rightarrow \frac{dy}{dx} = \frac{ax}{y} \Rightarrow \frac{d^2y}{dx^2} = \frac{ya-ax\frac{dy}{dx}}{y^2}$$

$$= \frac{ya-ax\frac{ax}{y}}{y^2} = \frac{a(y^2-ax^2)}{y^3} = \frac{ab}{y^3}$$

26.

(b) 3

Explanation: f(x) = $\frac{1}{\log|x|}$

f(x) is not defined for x = 0, -1 ,1

∴ f(x) is not continuous at x = 0, -1, 1

27. **(a)** x ∈ R

Explanation: x ∈ R

28. **(a)** tan θ

Explanation: Given $x = a(cos\theta + \theta sin\theta)$, $y = a(sin\theta - \theta cos\theta)$

$\frac{dx}{d\theta} = a[-sin\theta + \theta.\ cos\theta + sin\theta] = a\theta\cos\theta,\ \frac{dy}{d\theta} = a[cos\theta - (\theta.\ -\sin\theta + cos\theta)] = a\theta\sin\theta$

Slope of the tangent $= \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta sin\theta}{a\theta cos\theta} = tan\theta$

29. **(a)** a > 0
   Explanation: f(x) = ax
   f'(x) = a
   f(x) is increasing on $\frac{1}{2}$ if a > 0
30.
   (b) e
   Explanation: $f(x) = \frac{x}{\log x}$

   $\Rightarrow f'(x) = \frac{logx.1 - x.\frac{1}{x}}{(\log x)^2}$

   For maximum or minimum values of x we have f'(x) = 0

   $f'(x) = 0 \Rightarrow \frac{\log x - 1}{(\log x)^2} = 0 \Rightarrow (\log x - 1) = 0$

   $\Rightarrow \log x = 1 \Rightarrow x = e$

   Now, $f''(x) = (\log x - 1)\frac{-2}{(\log x)^3} + (\log x)^{-2}.\frac{1}{x}$

   $f''(e) = \frac{1}{e} > 0$

   Hence, f(x) has a minimum value f(e) = e.
31. **(a)** $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$
   Explanation: We have,
   $\int x.\ logxdx = logx.\int xdx - \int((\frac{d}{dx}logx)\int xdx) = \frac{1}{2}x^2logx - \frac{1}{4}x^2 + c$
32. **(a)** $\frac{a+b}{2}\int_a^b f(x)dx$
   Explanation: Given Integral is: $\int_a^b xf(x)dx$
   Let I = $\int_a^b xf(x)dx$
   as, f(x) = f(a + b - x)

$\Rightarrow I = \int_a^b (a + b - x)f(a + b - x)dx$

$\Rightarrow I = \int_a^b (a + b - x)f(x)dx$

$\Rightarrow I = \int_a^b (a + b)f(x)dx - \int_a^b (x)f(x)dx$

$\Rightarrow I = \int_2^b (a + b)f(x)dx - I$

$\Rightarrow 2I = \int_a^b (a + b)f(x)dx$

$\Rightarrow I = \frac{(a+b)}{2}\int_a^b f(x)dx$

33. **(a)** $\frac{2^x}{(\log 2)^2}$ (x log 2 - 1) + C

Explanation: $I = \int xI \cdot 2II^x dx = x \cdot \frac{2^x}{(\log 2)} - \int \frac{2^x}{(\log 2)} dx$

$= \frac{x \cdot 2^x}{(\log 2)} - \frac{2^x}{(\log 2)^2} + C$

34.

(b) 2sin x + C

Explanation: Given :

$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2\sin x\cos x}{\sin x} dx$

$= 2\int \cos x\, dx$

= 2 sin x + c

35.

(b) 4 sq. units

Explanation: Required area :

$= 2\int_0^{\pi} \sin x dx = 2[-\cos x]_0^{\pi} = 2[1 + 1] = 4 sq.\ units$

36.

(d) $log|x^2 + y^2| + 2tan^{-1}\frac{y}{x} = log2 + \frac{\pi}{2}$

Explanation: $\frac{dy}{dx} = \frac{y-x}{x+y}$ it is a homogenous equation Hence we put y=vx

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} let\ y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{vx - x}{vx + x}$$

$$x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$x\frac{dv}{dx} = \frac{v-1-v^2-v}{v+1}$$

$$\int\frac{(v+1)dv}{1+v^2} = -\int\frac{dx}{x}$$

$$\int\frac{vdv}{1+v^2} + \int\frac{dv}{1+v^2} = -\int\frac{dx}{x}$$

$$\frac{1}{2}log|1 + v^2| + tan^{-1}v = -logx + c$$

$$log|1 + v^2| + 2tan^{-1}v + 2logx = c$$

Resubstituting v=y/x we get

$$log|\frac{x^2+y^2}{x^2}| + 2tan^{-1}\frac{y}{x} + 2logx = c$$

When x=y=1 we get,

$$log|\frac{1^2+1^2}{1^2}| + 2tan^{-1}\frac{1}{1} + log1 = c$$

$$c = log2 + \frac{\pi}{2}$$

$$log|\frac{x^2+y^2}{x^2}| + 2tan^{-1}\frac{y}{x} + 2logx = log2 + \frac{\pi}{2}$$

$$log|x^2 + y^2| + 2tan^{-1}\frac{y}{x} = log2 + \frac{\pi}{2}$$

37.

(d) $\frac{d^2y}{dx^2} + \lambda^2 y = 0$

Explanation: Given, y = asin($\lambda x + \alpha$) ...(i)

On differentiating it w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\text{ a }\sin(\lambda\text{x} + \alpha)$$

= a cos($\lambda x + \alpha$)λ

$$\therefore \frac{dy}{dx} = a\lambda\cos(\lambda\text{x} + \alpha)$$

Again, differentiating d it w.r.t. x, we get

$\frac{d^2y}{dx^2} = a\lambda \frac{d}{dx} \cos(\lambda x + \alpha)$

$= a\lambda[\sin(\lambda x + \alpha)] \times \lambda$

$= -a\lambda^2 \sin(\lambda x + \alpha)$

$\Rightarrow \frac{d^2y}{dx^2} = \lambda^2 y$ [from Eq. (i)]

$\therefore \frac{d^2y}{dx^2} + \lambda^2 y = 0$

38.

(c) $\frac{e^x}{x}$

Explanation: We have, $\frac{dy}{dx} + y = \frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} + \frac{y(1-x)}{x}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = \frac{1}{x}$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$P = \frac{-(1-x)}{x}, Q = \frac{1}{x}$

I.F. $= e^{\int Pdx} = e^{-\int \frac{1-x}{x}dx}$

$= e^{x - \log x}$ or $\frac{e^x}{x}$

39.

(b) 16

Explanation: Given that, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$

Let θ be the angle between vector a and b.

Then, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 12 = 10 \times 2\cos\theta$

$\Rightarrow \cos\theta = \frac{12}{20} = \frac{3}{5}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}}$

$\Rightarrow \sin\theta = \pm\frac{4}{5}$

Now, $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow |\vec{a}\times\vec{b}| = 10\times 2\times\frac{4}{5} = 16.$

40.

(c) 0

Explanation: $\vec{r}\cdot\vec{a}= 0$

$\Rightarrow$ either $\vec{a} = 0$ or both are perpendicular to each other.

if $\vec{a}, \vec{b}, \vec{c}$are zero

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

and

if three vectors are non-zero

$\Rightarrow$ they are coplanar and perpendicular to $\vec{r}$.

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

41.

(c) Both I and II

Explanation:

I. Correct, because in a triangle, a sum of two sides is greater than the third side.

II. Correct, because in a triangle, a difference of the two sides always smaller than the third side. These two inequality is known as triangle inequality.

42.

(b) $4\overrightarrow{OG}$

Explanation: Let O be the origin and ABCD is a parallelogram with two diagonals AC and BD.

G is the midpoint of AC

$\Rightarrow \overrightarrow{OG} = \frac{\overrightarrow{OA}+\overrightarrow{OC}}{2}$

$2\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OC}$

G is also the midpoint of BD

$\Rightarrow \overrightarrow{OG} = \frac{\overrightarrow{OE}+\overrightarrow{OD}}{2}$

Adding both equations,

$4\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OB} + \overrightarrow{OD}$

43. **(a)** $\frac{|\hat{a}+\hat{b}|}{2}$

Explanation: Given $\hat{a}$ and $\hat{b}$ are unit vectors.

Now,

$|\hat{a} + \hat{b}|^2 = (\hat{a} + \hat{b}) \cdot (\hat{a} + \hat{b})$

$= |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$

$\Rightarrow |\hat{a} + \hat{b}|^2 = 2 + 2\cos\theta$

$= 2(1 + \cos\theta)$...(i)

Similarly,

$|\hat{a} + \hat{b}|^2 = 2(1 - \cos\theta)$....(ii)

From Eq. (i)

$$|\hat{a} + \hat{b}|^2 = 2 \times 2\cos^2\left(\frac{\theta}{2}\right)$$

$$|\hat{a} + \hat{b}|^2 = 2\cos\left(\frac{\theta}{2}\right)$$

$$\therefore \cos\left(\frac{\theta}{2}\right) = \frac{|\hat{a}+\hat{b}|}{2}$$

44. **(a)** 2

Explanation: Use this

$Cos^2\alpha + Cos^2\beta + Cos^2\gamma = 1$

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$= 2$

45. **(a)** A pair of perpendicular planes

Explanation: We have, xy + yz = 0

$\Rightarrow$ y(x + z) = 0

$\Rightarrow$ y = 0 and x + z = 0

Above are equations of planes

Normal to the plane y = 0 is $\hat{j}$

And normal to the plane x + z = 0 is $\hat{i} + \hat{k}$

Now $\hat{j} \cdot (\hat{i} + \hat{k}) = 0$

So, planes are perpendicular.

46.

(c) $(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$

Explanation: In the Cartesian coordinate system :

Cartesian equation of a plane that passes through the intersection of two given plane is given by :

$(A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0$

$(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2) = 0$.Which is the required solution.

47. **(a)** $\frac{1}{2}$

Explanation: We have,

$P(B) = \frac{3}{5}$, P(A | B) and $P(A \cup B) = \frac{4}{5}$

Now, We know that

$P(A|B) \times P(B) = P(A \cap B)$

[Property of conditional Probability]

$\Rightarrow \frac{1}{2} \times \frac{3}{5} = P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{3}{10}$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[Additive Law of Probability]

$\therefore \frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$

$\Rightarrow P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10}$

$\Rightarrow P(A) = \frac{1}{5} + \frac{3}{10}$

$\Rightarrow P(A) = \frac{2+3}{10}$

$\Rightarrow P(A) = \frac{5}{10} = \frac{1}{2}$

48. **(a)** $\frac{1}{52}$

Explanation: Let P(A) = P(scooter) = $\frac{2000}{12000} = \frac{1}{6}$

P(B) = P(car) = $\frac{4000}{12000} = \frac{1}{3}$

and P(C) = P (truck) = $\frac{6000}{12000} = \frac{1}{2}$

Let E = Event that person meets with accident.

Then, $P\left(\frac{E}{A}\right) = \frac{1}{100}, P\left(\frac{E}{B}\right) = \frac{3}{100}, P\left(\frac{E}{C}\right) = \frac{15}{100}$

$\therefore$ Required probability

$$= \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}} = \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}}$$

$$\frac{\frac{1}{6}}{\frac{1+6+45}{6}} = \frac{1}{52}$$

49.

(d) $\frac{37}{256}$

Explanation: . In a single throw, we have $P(H) = \frac{1}{2}$ and $P(\text{ not } H) = \frac{1}{2}$.

$\therefore \quad p = \frac{1}{2}, q = \frac{1}{2}$ and n = 8

Required probability = P(6 heads or 7 heads or 8 heads)
= P(6 heads) + P(7 heads) + P(8 heads)

$$= {}^8C_6 \cdot \left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{2}\right)^2 + {}^8C_7 \cdot \left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right)^1 + {}^8C_8 \cdot \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{28}{256} + \frac{8}{256} + \frac{1}{256}\right)$$

$$= \frac{37}{256}.$$

50.

(b) $E(X^2) - (E(X))^2$

Explanation: Since, the variance of a discrete random variable X is given by:

$Var(X) = E(X^2) - (E(X))^2$